EVOLUTION AND MASS DISTRIBUTION OF CENTRAL STARS OF PLANETARY NEBULAE

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1. INTRODUCTION

Considerable progress has been made in our understanding of the evolution of the central stars of planetary nebulae (NPN) compared to the situation five years ago at the Ithaca Symposium where Shaviv (1978) and Paczynski (1978) reviewed the subject. Shaviv stressed the necessity to start theoretical calculations with realistic initial models but doubted - in view of the loops in the HR diagram made by flashing stars - if the Harman-Seaton sequence could be taken as a single evolutionary sequence. Paczynski pointed out how strongly the theoretical rate of evolution depends on the stellar mass - a result which had appeared in his earlier calculations (1971) - and expected the existence of more flashing NPN's of the FG Sagittae type among the luminous ($L > 10^4 L_\odot$) central stars, for which the core mass luminosity relation ($M_c > 0.7 M_\odot$) combined with the core mass interpulse time relation predicts fairly short (2.10 yrs) intervals between flashing events. Weidemann, however, at the Symposium and shortly thereafter (1977a) concluded in view of the lower effective temperature derived by Pottasch et al. (1978) and the observed narrow mass distribution of white dwarfs around a 0.6 $M_\odot$ combined with the theoretical predicted horizontal tracks from the red giant branch towards the NPN region at a luminosity given by the core mass luminosity relation that the high luminosity part (and also the "upturn") of the Harman-Seaton sequence does not exist. He also proposed an increase in the distances by an average factor of 1.3 compared to the Seaton/Webster (Seaton, 1968) or Cahn/Kaler (1971) scale in order to bring the observed NPN on the 0.6 $M_\odot$ track in the HR diagram and to lower the NPN birth rates to a value compatible with white dwarf birth rates. Renzini (1979) took up Paczynski's remarks and results and studied the relation between

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evolutionary times of NPN and PN expansion carefully thereby predicting visibility ranges for PN as a function of the NPN mass. Since it appeared that - within a typical nebular life-time - only massive NPN could illuminate their nebulae at lower observed NPN luminosities ($M_{NPN} > 0.6\ M_\odot$ for $\log L/L_\odot < 2.6$) he concluded that the NPN do not form a single evolutionary sequence but devised a new scheme according to which the high luminosity NPN have small masses and belong to low mass progenitors whereas the low luminosity NPN have larger masses and belong to higher masses progenitors. He thus predicted differences in population characteristics for both groups of NPN, higher nebular masses and different chemical compositions for the fainter NPN, and the existence of an upturn caused essentially by NPN with masses of $0.7\ M_\odot$ whose nebulae are visible at the comparably highest luminosities of the NPN. A similar result emerged from Haerm and Schwarzschild's (1975) approach. They took off the envelope from AGB-models and got remnants of about $0.65\ M_\odot$ which evolved rapidly into the NPN region. Their lifetimes as luminous stars ($\sim 10^3\ L_\odot$), however, exceeded considerably that of any associated nebula, and again only more massive remnants would be able to explain low luminous NPN.

In the meantime Schoenberner (1979) had calculated evolutionary tracks for low mass stars all the way from central helium burning, through the asymptotic giant branch, without flash suppression, and steady mass loss included, towards the white dwarf region, thereby fulfilling Shaviv's call for more realistic initial models for NPN evolution. The existence of these calculations together with a general correlation between NPN location in the HR diagram and nebular radii - which had been established in the past - prompted Schoenberner and Weidemann (1981) to study the empirical material in order to check Renzini's predictions. They confirmed the general correlation between luminosity and nebular radii, but found by a comparison of nebula expansion time scales with theoretical time scales of NPN evolution the surprising result that the mass distribution of NPN appears to be even narrower than that of white dwarfs, essentially confined to $0.55 \leq M/M_\odot \leq 0.64$, and that within the observed sample there were only very few NPN with masses above $0.64\ M_\odot$. This implies that the HR positions of the NPN present essentially an evolutionary sequence and thus explains the general correlation between luminosities and nebular radii. However, even within the narrow mass range in which most NPN occur, differential effects of the Paczynski-Renzini scheme are considerable, in the sense that a $0.64\ M_\odot$ NPN evolves much faster than a $0.55\ M_\odot$ NPN. The upturn is absent since stars in that mass range reach already the observed low luminosities. For these low mass NPN thermal pulses are expected to be a rare event during a PN lifetime (interpulse time $\sim 10^5$ yrs) which makes Shaviv's (1978) and Paczynski's (1978) predictions irrelevant. In a more fully elaborated version of his investigation Schoenberner (1981) reached further conclusions, concerning the role of thermal pulses in the ejection of PN, the remnant hydrogen envelopes on NPN, the modus of burning and the time scales involved. Of highest importance with respect to the evolution on the asymptotic giant branch and the general picture of late stages of stellar evolution is the fact that enrichment of PN - as observed and studied in numerous papers in the past - with helium and other burning products occurs already at the low masses derived, in contrast to the canonical theory of stellar evolution, which predicted enrichment only for higher core masses above 0.8 $M_\odot$. This fact is in line with similar conclusions about lower luminosities of Mira's (considered to be progenitors of PN) and carbon stars. The questions and problems in this context are numerous: they are dealt with extensively in a forthcoming review by Iben and Renzini (1983), to which we refer.

In the following we shall present the essential steps and results of our investigation and the implications. Then we will discuss objections and outline future research.

2. EVOLUTION AND MASSES OF THE CENTRAL STARS

We first consider the empirical relation between absolute optical luminosities, M_V, and nebular radii, R_N, based on Cahn and Kaler (1971) distances, photometry by several sources (for details and quotations: see Schoenberner, 1981), and corrected for interstellar absorption. Figure 1 shows that relation for a local ensemble (Cahn and Wyatt, 1976) in order to avoid selection effects in favor of NPN with high luminosities. A gap appears at $M_V \sim 5$, which is present also in the total ensemble, and in older material (O'Dell, 1974) and more objects are below that gap. Uncertainties in the distances can not change the essential structure of the diagram. A radius histogram shows the

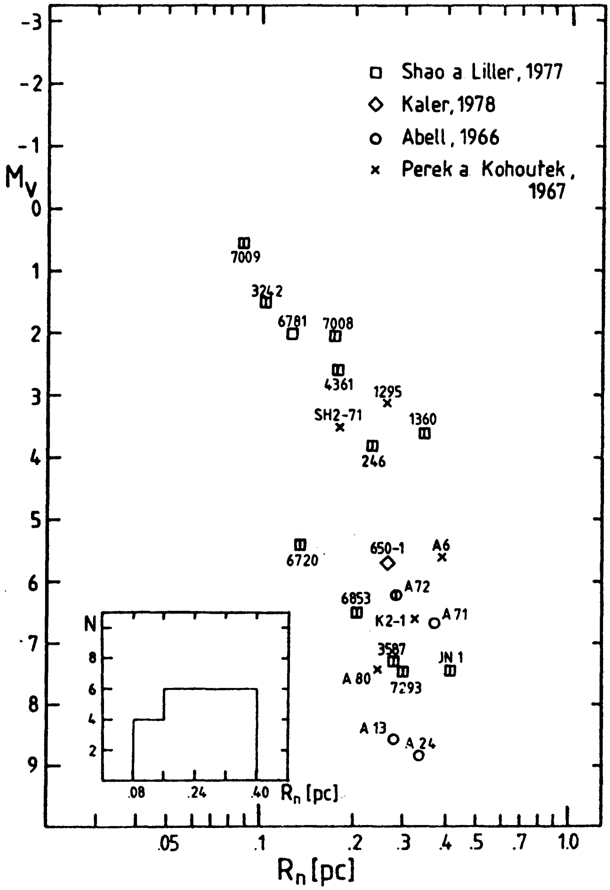

Figure 1. Absolute visual magnitudes for 22 central stars with optically thin nebulae from a local ensemble vs. nebular radii, together with a radii histogram.

number of NPN per radius interval to be nearly constant and thus suggests constant expansion velocity.

In Fig. 2 a larger ensemble is superposed with theoretical evolutionary tracks computed by Schoenberner (1979). In doing this, we converted the bolometric luminosities of the models into absolute magnitudes M_v and plotted these vs. ages. It is assumed that the nebular shell is formed in a time very short compared to the nebular lifetime, and that the remnant is able to maintain its thermal equilibrium. The NPN ages are counted from $T_{eff} = 10^{3.7}$. The evolution of these post-AGB models is accelerated by mass loss (stellar wind) only for $T_{eff} < 10^4$ K (for more details, see Schoenberner, 1981,1982). We want to emphasize the great sensitive of the fading time on the NPN mass (= core mass of the AGB-progenitor), as displayed in the Figure. The reason is the large increase of luminosity and the large decrease of available fuel as one goes from low to higher NPN masses (Schoenberner, 1982). The fading of these models is much faster than in Paczynskis calculation, and they reach lower luminosities during the PN lifetime. A 0.6 $M_\odot$ NPN may thus be found (within a PN event) at log $L/L_\odot$ = 2.0. The reason for that difference has been explained in detail by Schoenberner (1981). The tie-in with the observed NPN is made by the assumption of constant nebular expansion, with v = 20 km/s, changes for different expansion velocities are minor, as indicated by the arrows in the lower left corner. The thick broken line excludes NPN for which the nebulae are probably optically thick, so that the Shklovsky distances (Cahn and Kaler, 1971) do not apply. The distances are here increased by a factor of 1.2 compared to the CK distances. The thin broken lines give effective temperatures taken from the model calculations.

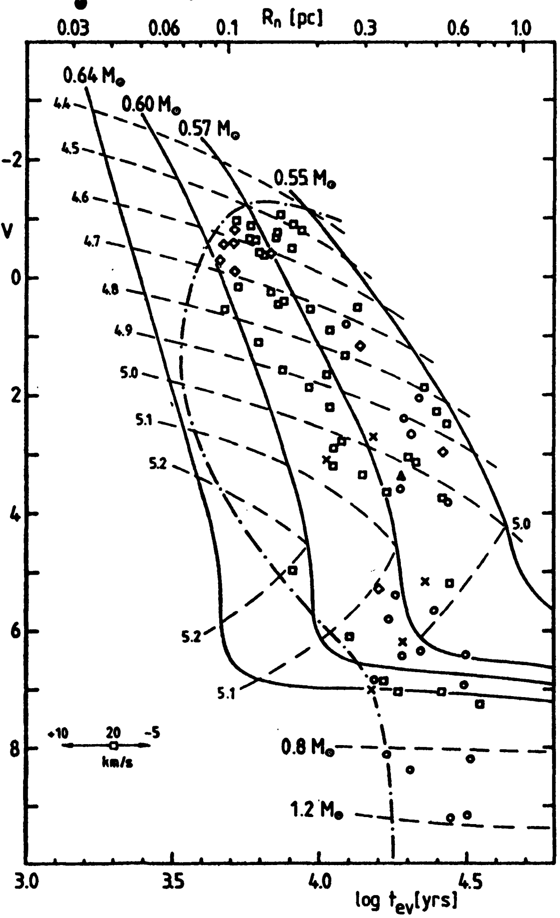

Figure 2. Absolute visual magnitudes vs. ages of observed central stars and post-AGB models. For explanations, see text.

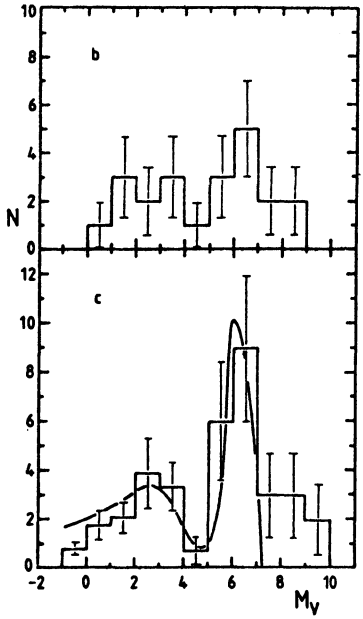

Figure 3. Above: Local ensemble of Fig. 1; Bottom: Ensemble of Fig. 2, corrected empirically for different sample volumina. The curved line gives the theoretical prediction of a 0.57 $M_\odot$ model.

The Fig. 2 shows that all displayed NPN are covered by the evolutionary tracks from 0.55 $M_\odot$ to 1.2 $M_\odot$. The gap is now explained by accelerated fainting during the extinction of the hydrogen shell source after a phase of quiet hydrogen burning. The important conclusion is that the PN-ejection and the subsequent central star evolution take place between two successive thermal pulses, with the outcome that no further pulse will (in most cases) occur. The observed positions indicate evolution of the NPN with a luminosity independent narrow mass distribution. There are only very few NPN with probable masses above 0.64 $M_\odot$, concluded from their (uncertain) positions at the lowest luminosities. Evolutionary tracks with thermal pulses, at the moment of PN ejection or during PN lifetime, show a completely different behaviour (Schoenberner, 1981). Not only would the empirical material not display a luminosity independent mass distribution, but also the gap could not be explained. A comparison of theoretical and empirical luminosity functions (Fig. 3) which is independent of the nebular expansion velocity, and from the choice of the zero point of NPN evolution, also agreees only for NPN evolution with quiet hydrogen burning. The luminosity function (local ensemble) shows more objects at faint luminosities, contrary to what would be expected if these NPN were the products of high mass (and therefore rare) progenitors.

The mass distribution, as evident from Fig. 2, corrected for selection effects, is extremely narrow (Fig. 4). We can identify only about 25% of the faint NPN ($M_V > 5$) with a possible mass in excess of 0.64 $M_\odot$ (this fraction varies sensitively with the assumed distance scale!). The maximum is at exactly the same value (0.58 $M_\odot$) which was found for the DA white dwarfs (Koester, et al., 1979), however, the white dwarf distribution appears to be broader (broken lines). Although the true WD mass distribution may be even narrower (due to observational uncertainties) there are definitely white dwarfs with $M < 0.55$ $M_\odot$. We thus presume that the progenitor of white dwarfs with $M < 0.55$ $M_\odot$ do not go through the PN stage, but evolve directly from the horizontal branch to the WD region, thereby crossing the sdO region (Hunger et al., 1981).

The steep decline towards higher NPN masses should be real even if there may be more higher mass NPN which were not in our ensemble, hidden

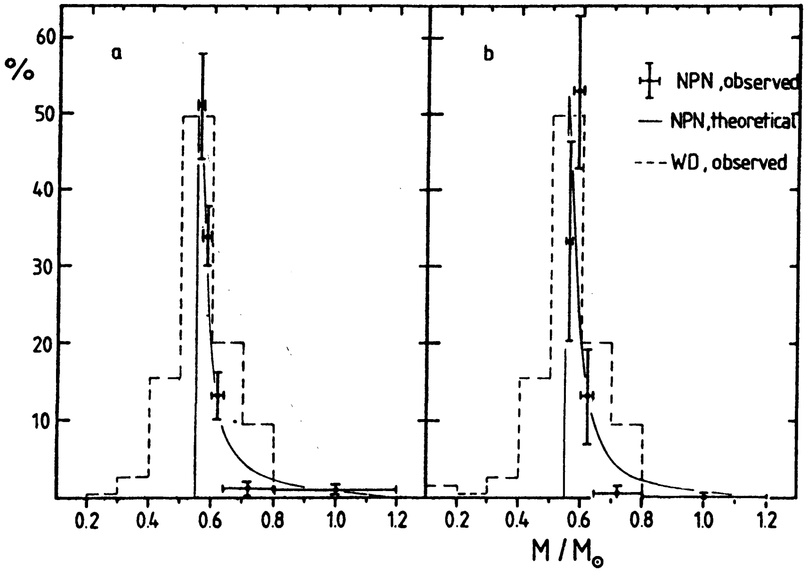

Figure 4: Left: Mass distribution, for the (corrected) ensemble of Fig. 2; Right: The same for the local ensemble. For comparisons, the observed (DA) white dwarf distribution (Koester et al., 1979) as well as the predicted according to the Barkat and Tuchman (1980) ejection scheme.

in massive PN's or undetected or with unmeasured central stars. The reason is: already the 0.60 $M_\odot$ model predicts 3.5 times more NPN with $M_V > 5$, than with $M_V < 5$. More massive NPN stay for almost the total PN-lifetime at luminosities $M_V > 7$. They must be really rare in order to account for the paucity of observed objects in that luminosity range, even if their number has been considerably underestimated. Note that this follows only from the appearance of the NPN luminosity function (Fig. 3) and the differentially evolutionary behaviour of post-AGB models with different masses. Theoretically, a very narrow mass distribution can be predicted, if PN's are ejected according to the shock mechanism proposed by Barkat and Tuchman (1980) combined with steady mass loss on the AGB with a Reimers scale factor $\eta \geq 1$.

Consequences for the initial/final mass relation in stellar evolution have been discussed by Weidemann (1981). That this function runs very flat for progenitor masses from the galactic turnoff up to at least 2.5 $M_\odot$ has been concluded already from white dwarfs which occur in clusters with known turn-off masses (see Weidemann, 1977b, 1979). Recent investigations by Koester and Reimers (1981), Reimers and Koester (1982) have demonstrated that white dwarfs do occur up to progenitor masses of 7 $M_\odot$ with WD masses up to 1.2 $M_\odot$. Thus there remains little doubt that the majority of all stars evolve through the PN stage and leave remnants with small masses around 0.6 $M_\odot$. More massive NPN's or white dwarfs are as rare as progenitor stars with, say $M > 4\ M_\odot$, i.e., about 20% or less, depending on the initial mass function and the past rate of star formation (see Koester and Weidemann, 1980). In the white dwarf case efforts to confirm higher-than-average masses (Weidemann and Koester, 1980, Schulz and Wegner, 1981, Weidemann, 1982) brought as yet meager results, but further studies can be made.

In the NPN case one has to search for and investigate especially low luminosity objects - as predicted by the visibility criteria of Renzini. Spectroscopic analysis has been possible up to now only for a few fairly bright central stars. The result by Mendez et al. (1981) confirm the surface gravities and temperatures expected by our method, which is independent of NPN temperature.

In the cases in which reliable temperature determinations are available a consistency test can be made. If the theoretically derived temperature differs, a change in the distance can always enforce agreement. Schoenberner (1981) showed that the smaller distances proposed by Acker (1978) are inconsistent in that the ages of the central stars and the ages derived from the nebular diameter are more discrepant.

Enrichment - and therefore dredge-up - evidently occurs at the small core masses corresponding to the range of our observed ensemble. And the low excitation Pop II objects - in Kaler's notation - do not form a separate group by mass. Schoenberner (1981) also finds no correlation between excitation parameter and temperature of the exciting star (see his Fig. 15) and concludes that mechanical heating of the surface layers must be responsible for the high He ionization.

3. DISCUSSIONS

Before we start with a discussion of several objections which have been raised or could be made against the presented evolutionary scheme, let us make a few remarks. The precise knowledge of effective temperatures of NPN - besides the distances - is essential for the determinations of meaningful bolometric luminosities. Up to now, these temperatures are known - if at all - only for a very limited number of objects. Our scheme is independent of temperature and uses only visual luminosities in order to study NPN evolution. We have to adopt, however, a mean value for PN expansion velocities. But this assumption is not as bad as it appears at the first glance: Only very few of measured PN expansion velocities exceed the mean of 20 km/s by more than a factor of two on either side (Robinson et al., 1982), whereas the fading time of NPN varies already by a factor of 10 between $0.55\ M_\odot$ and $0.64\ M_\odot$. Moreover, when using the luminosity function of NPN, we are even free from expansion velocities and absolute timescales. We thus believe that our approach is at present more reasonable than any representation which uses temperatures and luminosities.

Let us now first discuss the reliability of the evolutionary tracks. Additional computations have been made for low mass stars in the meantime by Kovetz and Harpaz (1981) and Iben (1982) which reproduce the Schoenberner (1979) results, especially the fact that a 0.6 $M_\odot$ NPN reaches low luminosities during a nebular lifetime. The reasons for the different results of Paczynski (1971) have been explained by Schoenberner (1981), see also Iben and Renzini (1982) . The luminosity at the horizontal part of the tracks correspond to that given by the core mass luminosity relation for quiet hydrogen burning at the asymptotic giant branch. Since in a post flash phase - before the hydrogen shell has been fully reactivated - the luminosity is lower for a considerable fraction of the interpulse time, one could object to the use of these tracks if PN ejection would occur with equal probability sometimes during the interpulse interval. But, as shown by Schoenberner (1981), only post-AGB models with a fully activated H-shell and a degraded He-shell are able to account for the observed central star luminosity function. Additionally, it is reasonable to assume that the star has to exceed a threshold luminosity in order to enter the region of enhanced mass loss rates, i.e., the PN-ejection region. Then it has to gain at least the preflash luminosity when recovering from that flash. This occurs when about half of the interpulse time has been elapsed. But then the hydrogen burning shell is already fully activated, and the helium already degraded. The remaining luminosity increase is then $\log L/L_\odot \sim 0.1$ at $\log L/L_\odot \sim 3.3$ and $\log L/L_\odot \sim 0.04$ at $\log L/L_\odot \sim 3.7$, respectively. The use of our tracks may then lead to an underestimate of NPN masses (on the average) by only 0.001 $M_\odot$ and 0.003 $M_\odot$, respectively. The core-mass luminosity relation itself is well established and agrees numerically between different authors as shown by the following values for $M_c = 0.6\ M_\odot$, $\log L/L_o = 3.71, 3.78, 3.79, 3.80, 3.72$ for Gingold (1974), Schoenberner (1979), Kovetz and Harpaz (1981), Wood and Zarro (1981), and Iben (1982), or 3.68 for the more crude models of Paczynski (1971).

Important for the appearance of the tracks in Fig. 2 is the fading time, i.e, the time a post-AGB model needs to reach a limiting luminosity during a PN lifetime. This time depends on the available fuel, which, in turn, depends on the envelope composition, for the metal poor post-AGB models of Gingold (1974) and Iben (1982) have much higher envelope masses at appropriate positions in the HR-diagram. For instances, the 0.6 $M_\odot$ model of Iben ($Z = 0.001$) is burning twice the amount of hydrogen (with the same luminosity) than the corresponding Schoenberner (1979) Pop I model ($Z = 0.021$), thus needing also twice the time to evolve through corresponding positions in the HR-diagram. Fortunately, the limiting luminosity does not depend on the envelope mass. Thus the 0.6 $M_\odot$ track of Iben (1982) has to be placed in Fig. 2 as follows: displaced by 0.3 dex to the right from the original 0.6 $M_\odot$ track. This then would shift the NPN mass distribution peak from 0.58 $M_\odot$ to 0.61 - 0.62 M_o. We think, however, that a Pop I composition is appropriate for most PN and that the application and labelling of the tracks in Fig. 2 is therefore justified. As long as any composition differences are not as drastic as in the example above, we can neglect them.

It is exactly the high sensitivity of the fading time on M_c which secures the narrow mass distribution derived, even if individual distances, magnitudes or nebular expansion velocities are uncertain. This leads us to the second objection: the distances used are based on the Shklovsky method and therefore uncertain. True, but as long as these uncertainties are within a factor of, say 2, the pattern of Figs. 1-3 will not be changed. Vice versa, the narrow confinement of the NPN with optically thin nebulae points to the applicability of the Shklovsky method (see Schoenberner, 1981). This is supported by a recent study of Daub (1982). The consistence test has shown that the smaller distances and PN radii, derived by Acker (1978), give incompatible expansion and NPN evolution ages. Expansion velocities would have to be lowered by factors between 5 and 10, or the NPN evolution accelerated by the same factors in order to reach agreement. Both possibilities must be ruled out: expansion velocities, if not 20 km/s, do not differ very much (see Robinson et al., 1982, for a recent study), again the corresponding changes in the derived mass distribution of Fig. 2 are minor. Continued mass loss of NPN, partly derived from IUE observations, is not sufficiently strong as to accelerate the evolution, however may be important for the thickness of remnant hydrogen layers on the final white dwarfs. If it were strong enough to accelerate the NPN evolution significantly, this would result in even lower NPN masses (the tracks in Fig. 2 would be shifted to the left).

The final objection concerns selection effects. Our ensemble has been selected by existing photometry of the central stars. Indeed, there may be many cases in which NPN are not visible, or too faint to be measured. Especially Pottasch (1981) has selected 12 objects with very faint NPN, $m_v \sim 18$-19, and has presented an HR diagram which shows several hot NPN at effective temperatures far beyond the upper limit of our ensemble, 150 000 K. Aside from the fact that his temperature determinations are very uncertain, the existence of hot NPN with high luminosities is in contradiction to evolutionary tracks which predict such high temperatures only for massive NPN with short evolutionary time scales which pass this region before they are able to ionize the nebulae. Furthermore, the visual magnitudes are extremely faint and uncertain in these cases. In 5 of his 17 objects photometry by Kaler (1978) and Kohoutek and Martin (1981) yields luminosities which are several magnitudes higher. However, it is almost certain that there are more NPN at faint luminosities which have not yet been detected. A striking example is 158 + 17°1 (Purgathofer and Weinberger, 1980). Observational efforts should be made to increase the local ensemble and to measure the luminosities and other parameters of these faint NPN. Similarly, efforts to determine nebular abundances should be concentrated on a more local ensemble, in order to confirm or refute our result that enrichment occurs independent of the position in the HR diagram.

Finally, we want to discuss the role of thermal pulses during the PN lifetime in more detail. When an AGB-star ejects its envelope between successive thermal pulses, then the remnant may experience a thermal pulse as long as its hydrogen shell remains active. Now the ratio between the lifetime of the H-shell during the NPN-evolution and the thermal pulse period is about 0.1 for masses above 0.6 $M_\odot$, independent of mass. Stars with cores below 0.58 $M_\odot$ do not experience full amplitude flashes, and the flash period increases with core mass (Schoenberner, 1979), contrary to the interpulse period-core mass relation (valid only for well developed thermal pulses). Thus, for NPN near the lower mass limit of 0.55 $M_\odot$ we expect a 30-40% chance for them to experience a final thermal pulse (Schoenberner, 1982). Altogether, we estimate that 15% of all NPN will undergo a (final) thermal pulse, and that pulse - not necessarily with full amplitude - will result in an expansion of the PN-nucleus to giant dimensions for some 10^3 years (see for details Schoenberner, 1979). To see such an event during a PN lifetime (3.10^4 yr) is rather unlikely: about only 1 out of 100 NPN is to be expected to display a rapid evolution through the HR-diagram. In fact, up to now only one object is known which can be identified to be in this evolutionary phase: the late type central star FG Sge. Its distance is known (Herbig and Boyarchuk, 1968), and its luminosity of $10^{3.6}$ $L_\odot$ corresponds well to a post-AGB model with, say, 0.6 $M_\odot$, which displays rapid evoution to low effective temperatures which is driven by a final thermal pulse (Schoenberner, 1979). The expansion age of the PN is about 5000 yrs, and its ionization indicates that the central star was at least as hot as 55000 K some 10^2 years ago (Harrington and Marionni, 1976). We conclude that in this case the formation of the PN - and the creation of a contracting NPN - took place shortly before the onset of a thermal pulse (the interpulse period is $\sim 10^5$ yrs). Thus the very existence of the FG Sge appears to be an observational proof that i) thermal pulses really exist, ii) the PN formation occurs during the interpulse phase of quiet hydrogen burning, i.e., during the thermal equilibrium phase of an AGB-star.

During the post-flash recovering phase, a FG Sge-like nucleus will evolve through the same region of the HR-diagram and with a similar fading time than normal nuclei. Thus 15 or 20% NPN out of the ensembles in Figs. 1 and 2 might be in this stage, but we cannot detect them because that fraction is too small. If the final flash occurs at very high effective temperatures of the NPN, say at $T_{eff} \gtrsim 10^5$ K, the convective helium shell cuts into the hydrogen envelope, and a more or less complete mixing of the hydrogen occurs (Schoenberner, 1979, Iben et al., 1982). It might be possible to explain the existence of two peculiar PN, A 78 and A 30, which show helium enriched matter, by such an event. But their position in Fig. 2 can also be explained by assigning them a nucleus of ~ 0.55 $M_\odot$. Such a model evolves slowly enough to allow for high central star luminosity and large PN radius. The only method to distinguish between both modes of evolution is provided by a precise temperature determination of the nucleus because the post-flash cooling track of the Iben et al. (1982) 0.6 $M_\odot$ model displays - during the PN-lifetime - higher effective temperatures (~ 0.1 dex) than a 0.55 $M_\odot$ track with quiet hydrogen burning. In any way such a mixing flash should be rather rare: From evolutionary timescales one may estimate that only 30% of all final thermal pulses occur at $T_{eff} \gtrsim 10^5$ K, or with other words, only a few percent of all NPN are expected to experience a flash driven mixing.

4. FINAL REMARKS

Although we have shown that a general correlation between M_v and nebular radius does exist, and that the HR positions present in essence an evolutionary sequence we want to point out that even within the narrow NPN mass range derived there are important differential effects which explain, for example, Kaler and Hartkopf's (1981) finding of two contrasting Abell PN. Within our scheme, their results for Abell 43 and Abell 50 can be easily interpreted by minor mass differences. From the data given we obtain M (Abell 43) = 0.56 $M_\odot$ and M (Abell 50) = 0.60 $M_\odot$. Indeed the more massive NPN has reached lower luminosities within a shorter timescale than the less massive comparison objects. Our scheme predicts also effective temperatures as well, and we find $T_{eff} \sim 110000$ K for A 50, while Kaler and Hartkopf determined the Zanstra temperatures to be 104000 K and 125000 K for H and He II, respectively. We have a similar agreement for A 43. We conclude that our scheme is consistent - even without knowing individual expansion velocities - and we encounter "with a magnifying glass" within our ensemble exactly the differential effects which were fundamental for the set up of Renzini's evolutionary scheme. Furthermore, A 50 is considered to be a Pop II object according to Kaler and Hartkopf (1982) and consequently we could not accept any substantially higher mass for its nucleus than that derived above. The mere fact that a low mass Pop II NPN appears as faint as $M_v \sim 7$ supports our statement that most of the faint nuclei have indeed such low masses.

Clearly, it will be rewarding to continue investigations of this kind and to improve the observational material on which further tests of the present interpretation of NPN evolution can be made.

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SEATON: I do not think you should be so shy of temperatures! We can make good progress in determining temperatures of individual PN nuclei. Their distances represent a much harder problem.

RENZINI: I expect that the real mass distribution is somewhat wider towards higher masses for the simple reason that your sample is compiled according to the availability of M_V. Since there are many PN with visually undetected nuclei, and since the more massive a PN nucleus the fainter it will appear, your sample suffers from this obvious selection effect.

SCHÖNBERNER: The mass distribution which I presented is valid for observed nuclei of a local ensemble. Only further observations will show if this sample is representative.