ASTRONOMY FROM WIDE-FIELD IMAGING

Part Fourteen:

MAPPING THE LARGE-SCALE STRUCTURE

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MAPPING THE LARGE-SCALE STRUCTURE

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ABSTRACT. The nearby galaxy distribution suggests a remarkable structure in which large voids are delineated by dense walls of galaxies in a cell-like pattern. The nearby voids range in diameter from ~ 10 to ~ $50h^{-1}$ Mpc. Deeper surveys appear to be consistent with the nearby distribution and show no evidence of voids larger than ~ $100h^{-1}$ Mpc. We might thus have reached the scale where the universe becomes homogeneous. The size of the largest inhomogeneities in the galaxy distribution is an important issue because it can put tight constraints on the theoretical models when confronted by the high degree of isotropy of the microwave background radiation.

Comparison of the various existing redshift surveys emphasizes the need for systematic redshift surveys over significant areas of the sky out to intermediate and large distances. Although deep pencil-beam surveys are best suited for probing a large number of voids and walls, understanding the nature of the intercepted peaks and valleys in terms of large-scale structure requires that the angular coverage of the surveys be larger than the galaxy auto-correlation length. If this condition is not satisfied, the size of the voids and the density contrast of the walls can be overestimated.

1. Introduction

The first clues towards the understanding of the matter distribution in the Universe and the measurement of its density have been obtained by mapping the distribution of its major light emitting components, the galaxies. One of the fundamental questions of observational cosmology raised by the properties of the galaxy distribution is the problem of the missing mass detected in increasing amounts at larger and larger scales. The integrated counts of galaxies taking into account the mass-to-light ratio of individual galaxies yield low mass density estimates in agreement with the primordial nucleosynthesis predictions (Binney & Tremaine 1987). However, dynamical studies of systems of galaxies on scales from cluster cores to superclusters (Binney & Tremaine 1987) suggest the presence of a non-baryonic dark matter component more uniformly distributed than the galaxies, representing a dominating fraction of the matter in the Universe, and approaching the closure density.

Answering the question of whether galaxies trace the underlying mass requires a detailed knowledge of the galaxy distribution. One of the major properties of the distribution is the presence of structures at nearly the largest scales examined. Historically, the enhancements in particular regions of the galaxy distribution were first detected in angular catalogues — giving

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the positions on the sky — and led to the discovery of individual structures as for example the Local Supercluster (de Vaucouleurs 1958). General maps over significant fractions of the celestial sphere (Oort 1983) have clearly demonstrated the inhomogeneity of the galaxy distribution and pointed to the necessity of measuring the lacking third coordinate, the distance to the galaxies.

Velocity measurements of significant numbers of objects over the past 20 years have confirmed the existence of inhomogeneities on the so-called 'supercluster' scales, ranging from 10 to $100h^{-1}$ Mpc (with a Hubble constant of $H_0 = 100h$ km/s/Mpc [Oort 1983; Hubble 1934]). The distances to the objects are obtained by measuring their redshifts: the displacement of known absorption and/or emission lines of hydrogen and other heavier elements with respect to their rest-frame wavelengths. The redshift yields a recession velocity which is converted into a distance using the Hubble law (Hubble 1934). With the advent of digital detectors such as Charged Coupled Devices (CCD) and with the use of multi-fibre spectrographs allowing simultaneous measurement of tens — and more recently hundreds — of spectra, redshift surveys of rapidly increasing numbers of galaxies out to fainter and fainter limiting magnitudes are being obtained.

In the following, I describe the major properties of the nearby galaxy distribution and discuss the issue of fair sample. The theoretical context for understanding the formation of the observed structures is briefly reviewed, and the main statistical tools for characterizing the distribution are mentioned. I then attempt to provide an overview of the new and undergoing redshift surveys probing the distribution out to increasing distances. The question of the typical and largest scales for the structures in the observed galaxy distribution is addressed. Particular attention is brought to the importance of the survey definition in interpreting the limits derived from the data.

2. The Nearby Galaxy Distribution

The recent extension of the CfA redshift survey to $m_{B(0)} = 15.5$ is one of the largest existing galaxy catalogues (Huchra 1990). It suggests a new picture of the galaxy distribution, in which galaxies are distributed in thin sheet-like or shell-like structures surrounding vast voids (de Lapparent et al. 1986). The survey is based on a merge version of the Zwicky et al. catalogue (Zwicky et al. 1961-1968) and the Nilson catalogue (Nilson 1973). Figure 1 shows the Zwicky catalogue. The catalogue covers only the northern sky. The region with no galaxies having the shape of an arc is due to the disk of the Milky Way which obscures background galaxies. Notice the presence of apparently under-dense and over-dense regions. The Virgo cluster of galaxies is located at 12^h30^m and ~ 10°. The Coma cluster, more distant and therefore smaller in angular diameter, is near 13^h and ~ 30°. Some of the under-dense regions appear to extend linearly over several hours in right ascension.

The redshifts for the CfA survey are obtained with a dedicated telescope measuring several hundreds of redshifts every year. The redshifts are acquired for declination strips, thus allowing statistical analyses of partial data by restricting them to the complete slices. Figure 2 shows pie-diagrams of three adjacent 'slices' of the CfA survey in the northern galaxy cap (Huchra et al. 1990; de Lapparent et al. 1991). The first of these three slices (de Lapparent et al. 1986) (at the top) contains the Coma cluster in its centre. The elongation of the Coma cluster along the line-of-sight is caused by the velocity dispersion of galaxies in the gravitational potential of the cluster, which is included in the observed redshift as an added term to the recession velocity. A large fraction of the galaxies in the slices are also part of groups of galaxies (Ramella et al. 1989)

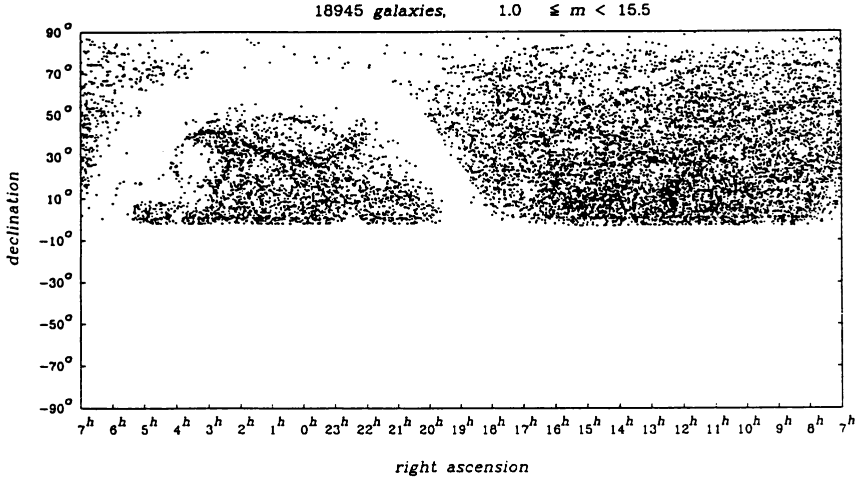

Figure 1. Map of the projected galaxy distribution from the Zwicky et al. catalogue for galaxies with $m_{B(0)} = 15.5$.

causing analogous 'fingers-of-god' with typical velocity dispersions of ~ 250 km/s^{-1}.

Several under-dense regions were suggested by earlier redshift surveys with more than ~ 100 objects, but yet were too restricted in angular coverage for probing the full structure (Oort 1983). A sparse but systematic survey of the Boötes region clearly shows the existence of a large void of $60h^{-1}$ Mpc in diameter (Kirshner et al. 1981; Kirshner et al. 1987), raising the question of whether such structures are frequent. The maps in Fig. 2 confirm that voids are indeed common features of the galaxy distribution, with diameters ranging from 10 to $50h^{-1}$ Mpc. The 2-dimensional or sheet-like nature of the high-density regions of the map can be demonstrated quantitatively by statistical tools for measuring geometrical properties (de Lapparent et al. 1991). The persistence of the structures from one slice to the other, as if one were performing cuts through a cell-like network, is clearly visible in Fig. 2.

Figure 3 shows a perpendicular cut through the three slices of Fig. 2 in the right-ascension range $15^h \leq \alpha \leq 16^h$. This other view of the data shows the inclination of the sheet located near $V \simeq 10{,}000$ km/s^{-1} and delineating the void centred at $\alpha \simeq 15^h$ and $V \simeq 7000$ km/s^{-1} in the slices of Fig. 2. This sheet is part of the 'Great Wall' structure (Geller & Huchra 1989) discussed below.

Figure 3 shows the remarkable sharpness of the detected sheets of galaxies in the CfA slices. This characteristic property of the data stimulated a renewed interest for models where hydrodynamical processes play an important role in the formation of structure (Ikeuchi 1981; Ostriker & Cowie 1981; Weinberg et al. 1989). The sharpness of the structures can be measured quantitatively. In the CfA slices, the FWHM of the sheets is ~ 500 km/s^{-1} (de Lapparent et al. 1991), comparable to the expected velocity thickening due to the groups of galaxies (Ramella et al. 1989) (for the sheets perpendicular to the line-of-sight). Large velocity streaming motions have been detected in the nearby universe (Dressler et al. 1987) and one may wonder whether they can distort or emphasize the structures seen in Fig. 2 on scales > $10h^{-1}$ Mpc. However, direct distance measurements for edge-on spirals on the near and far edges of the large void outlined in Fig. 3 show that its true spatial diameter is nearly identical to its diameter in redshift

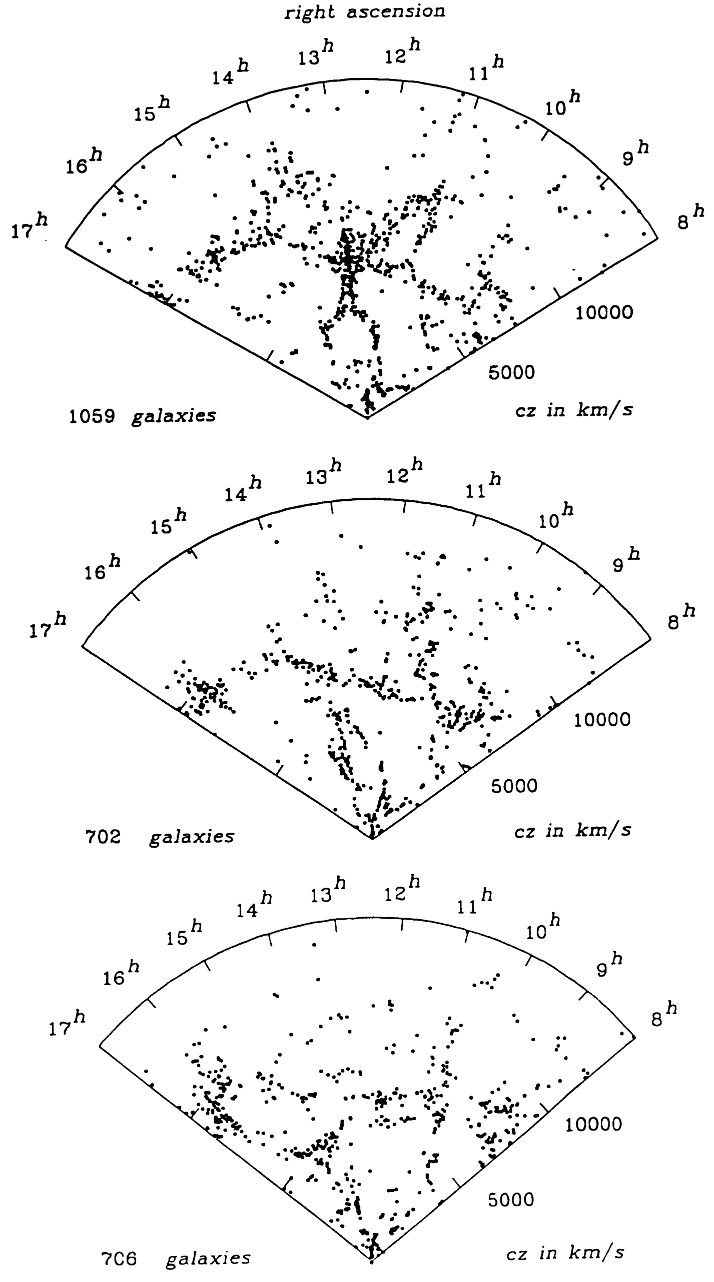

Figure 2. Maps of velocity versus right ascension for the galaxies with $cz \leq 15{,}000$ km/s^{-1} in three slices of the CfA redshift survey. In declination, the slices are bounded by $26.5° \leq \delta \leq 32.5°$, $32.5° \leq \delta \leq 38.5°$, and $38.5° \leq \delta \leq 44.5°$.

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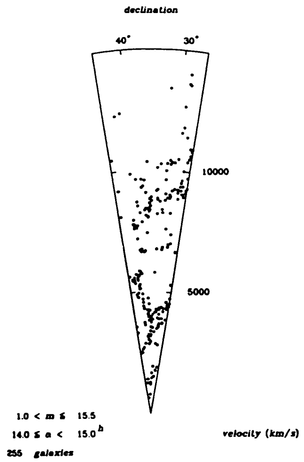

Figure 3. Perpendicular cut through the slices of Fig. 2 in the right ascension range $15^h \leq \alpha \leq 16^h$. Declination as a function of velocity in km/s^{-1} is plotted for the 255 galaxies in the region; the declination range is bounded by $26.5° \leq \delta \leq 44.5°$.

space (Bothun et al. 1992). This study demonstrates that this under-dense structure is not a dynamical artefact of the redshift map. Moreover, in N-body simulations designed to match the data (Melott 1987; White et al. 1987), the generated networks of sheets and voids have very similar appearances in velocity and real space; the general topology is preserved and the significantly distorted structures are the centres of rich clusters.

The CfA redshift survey extension is now complete over 12 slices of 6° in declination (6 slices in the northern galactic cap, and 6 in the southern galactic cap) amounting to more than 10,000 redshifts over the northern sky (Vogeley et al. 1994). Other extensive surveys have been completed, some with similar depth and angular coverage as the CfA redshift survey (Haynes & Giovanelli 1988; da Costa et al. 1988; Pellegrini et al. 1989), others with a depth twice larger and a smaller angular range (2 - 5 hours in right ascension) (Parker 1992; Broadbent et al. 1992). All these surveys show analogous structures as in the CfA redshift survey and with a similar range of sizes. Voids appear to be the physical units and the coherence on larger scales seems to be a geometric effect (although the existence of structure on larger scale than the voids remains to be tested from larger catalogues). One example is the 'Great Wall' (Geller & Huchra 1989), this remarkably coherent structure running from $\alpha \simeq 17^h$ and $V \simeq 10{,}000$ km/s^{-1} to $\alpha \simeq 8^h$ and $V \simeq 7{,}000$ km/s^{-1} and containing the Coma cluster. The 'Great Wall' is visible in all three slices of Fig. 2 as well as in other adjacent slices, spanning the full declination range $14.5° \leq \delta \leq 50.5°$. Its spatial dimensions are ~ 170 x 60 x $10h^{-3}$ Mpc^3 (in right ascension, declination and redshift respectively).

A full 360° panoramic map of the northern sky (Geller & Huchra 1989) shows that the Great Wall could extend across the plane of the Milky Way into the southern galactic cap and connect to the Pisces-Perseus chain (Haynes & Giovanelli 1986), located in the ranges $22^h \leq \alpha \leq 3^h$ and $25° \leq \delta \leq 45°$ at $V \sim 5{,}000$ km/s^{-1}. This chain is clearly visible in Fig. 1. Note that the largest over-dense structures are best detected in these surveys in the velocity range 5,000 – 10,000 km/s^{-1} (resulting from the limiting magnitude of ~ 15.5), which means that the significance of the Great Wall and the Pisces-Perseus chain is artificially enhanced relative to that for other structures in the survey (for example structures parallel to the line-of-sight are less striking due to their varying contrast with velocity).

The fact that the Great Wall and the Pisces-Perseus chain, despite the wide angle covered in right-ascension, dominate the redshift distribution averaged over all angles for $V \leq 15{,}000$ km/s^{-1} is nevertheless symptomatic of a more profound result: these surveys do *not* represent fair samples of the galaxy distribution (Peebles 1973). As can be seen for example in the top slice of Fig. 2, the largest under-dense region centred at $\alpha \simeq 15^h$ and $V \simeq 7{,}000$ km/s^{-1} has a diameter (~ 5,000 km/s^{-1}) comparable to the characteristic depth of the survey (~ 10,000 km/s^{-1}).

Calculation of the mean galaxy density in the CfA slices also illustrates quantitatively that they are not fair samples. Under some assumptions on the spectrum of void diameter, it can be shown that the uncertainty in the determination of the mean galaxy density ρ is directly related to the number N of large voids in the survey (de Lapparent et al. 1988). If F is the fraction of the survey volume occupied by the largest void, and under assumptions on the behaviour of the spectrum of void diameters at small radius,

$$\frac{\delta\rho}{\rho} \propto \frac{F}{\sqrt{N}} \tag{1}$$

In Fig. 2, we measure $F \simeq 0.25$, and the presence of a unique large void of diameter $50h^{-1}$ Mpc ($N = 1$) yields

$$\frac{\delta\rho}{\rho} \sim 0.25 \tag{2}$$

The mean density derived from these CfA slices is thus poorly determined. Note that using several adjacent slices as those shown in Fig. 2 does not improve the determination as compared to using only one slice because the slices thickness in declination is small compared to the diameter of the large voids. The adjacent slices cut through mostly the same structures and provide redundant information. This feature nevertheless demonstrates the efficiency of the slice-like surveys for probing the large-scale structure.

Examination of the CfA redshift survey slices and other comparable surveys emphasizes the need for larger samples of the galaxy distribution containing many of the typical structures. Such surveys are necessary for reliable measurement of the average statistical properties of the distribution, and for a better description of the clustering at the largest scales. In the following sections, I briefly recall the theoretical context for understanding the formation of the observed structures and emphasize the importance of the statistical characterization of the structures for confrontation with the models. I then discuss some of the more recent and/or undergoing surveys which bring new clues on the galaxy distribution.

3. Comparison of the Data with the Theoretical Models

3.1 THEORETICAL FRAMEWORK FOR THE FORMATION OF THE LARGE-SCALE STRUCTURE

A variety of theoretical models have been proposed to reproduce the typical features of the galaxy distribution. The most widely explored models by N-body simulations are the so-called pancake — or adiabatic — models (Zeldovich 1970). These have been successful in reproducing many properties of the observed distribution. The major assumption in these models is that structures form by gravitational collapse of gaussian primordial fluctuations. A spectrum of fluctuations with sufficient power at large scale for producing the observed structures is used (White et al. 1987). However, in order to comply with the extremely small anisotropies observed in the cosmic microwave background and imprinted by the matter fluctuations before decoupling, the pancake models require a flat universe ($\Omega_0 = 1$; this is independently required by inflation [Guth 1981]). Because the nucleosynthesis predictions limit the baryonic matter density well below the closure density (Smith et al. 1993), several kinds of non-baryonic particles have been suggested as dark matter candidates (Primack 1993).

For the models, one of the challenging properties of the galaxy distribution to be matched is the high-density contrast of the structures. The pancake models with hot dark matter (Melott 1987) produce many structures at the largest observed scales, but lack the small-scale clustering generated successfully by the cold dark models (White et al. 1987). High-contrast structures emerge naturally from models with non-gaussian initial fluctuations (for example textures [Cen et al. 1991]); the presence of such fluctuations can be tested in the data using statistical indicators such as the skewness (Juszkiewicz et al. 1993). In the pancake models, the high contrast is generated by biased galaxy formation in the sites where the density fluctuations reach above a given threshold (Kaiser 1984) invoking some poorly understood mechanism. Thus galaxies form only at the peaks of the matter distribution and leave large empty regions. In this picture, most of the matter is dark and nearly uniformly distributed, and the voids of the galaxy distribution are filled with dark matter at densities corresponding to $\Omega_0 \simeq 1$.

Another important issue is that of the typical and largest size of the inhomogeneities in the observed distribution, which put tight constraints on the pancake models. Recent statistical analyses (power-spectrum and genus [Vogeley et al. 1992; Park et al. 1992a]; counts-in-cells [Saunders et al. 1991]; see also the review [Juszkiewicz 1993]) indicate an excess of power at the largest scales in the observed galaxy clustering over the cold dark matter model, but the large error bars due to the limited volume probed by the existing data call for larger surveys (see also Bahcall et al. 1993). The size of the largest structures in the cold dark matter models is limited by the microwave background isotropy. Models of gravitational evolution of voids directly demonstrate the tight constraints which the anisotropies in the microwave background radiation and the characteristics of the void-dominated topology of the observed galaxy distribution put on the initial conditions and the formation mechanism of the structures.

For structures originating from Gaussian initial fluctuations and evolving under gravity in a flat universe ($\Omega_0 = 1$), the galaxy distribution is expected to become a network of close-pack voids (Dubinski et al. 1993). In fact, it can be shown that the level of isotropy of the microwave background radiation on angular scales larger than 1 degree (Meinhold & Lubin 1991; Meyer et al. 1991) puts direct constraints on the observed void sizes in the galaxy distribution: a detailed calculation (Blumenthal et al. 1992) shows that the largest voids should have a diameter $< 80h^{-1}$ Mpc, and there should be at most one void in the whole Hubble volume with diameter $> 130h^{-1}$

Mpc. Because the predicted void sizes are comparable to or larger than the largest void sizes which can be detected in the existing wide-angle surveys (see section 2), larger data samples are required for testing this calculation and its assumptions. Moreover, a Harrison-Zeldovich spectrum of primordial fluctuations normalized to produce the microwave background anisotropies cannot be reconciled with the typical void size of 5000 km/s^{-1} seen in the nearby redshift surveys, unless basic underlying assumptions of the model (i.e. the gravitational evolution of voids, the flatness of the universe etc.) are called into question (Piran et al. 1993).

Note that the presence of large amounts of dark matter inside the voids raises the question of whether they contain baryonic matter within the nucleosynthesis limits, and whether this matter formed light-emitting systems which may be observationally detectable. The cold dark matter model predicts that the voids in the galaxy distribution could be filled with dwarf and/or low-surface brightness galaxies (Frenk et al. 1988). Statistical analyses of the relative distribution of intrinsically bright and faint galaxies in the data suggest that the fainter galaxies are distributed more uniformly (Salzer et al. 1990; Santiago & da Costa 1990). These indications of a segregation in the distribution of galaxies in the high- versus low-density regions are however controversial (Eder et al. 1989; Bingelli et al. 1990). Moreover, there has been so far no evidence for existence of a significant population of galactic size objects in the voids of the galaxy distribution: in redshift maps of dwarf and low-surface brightness galaxies (Thuan et al. 1991; Schombert et al. 1992) and in direct searches for hydrogen-rich but optically faint galaxies in the voids (Weinberg et al. 1991), all detected objects — which were absent from the CfA and the other analogous redshift surveys — fall along the high-density structures delineating the voids. Therefore, if $\Omega_0 \simeq 1$ and unless the voids contain undetected baryonic matter in the form of 1) stellar objects (brown dwarves for instance [Aubourg et al 1993; Alcock et al. 1993]), or 2) extremely faint and/or low surface brightness objects which we would have completely missed from our optical and radio catalogues, all the dark matter in the voids could be non-baryonic.

3.2 STATISTICAL CHARACTERIZATION OF THE LARGE-SCALE STRUCTURE

In order to reliably compare the theoretical models with the data, quantitative measures of clustering are required. Because the structures in the CfA slice are highly asymmetric and contrasted, these measures must depend on the high-order moments of the distribution. Various such statistical measures have been applied to the CfA slices for objective detection and/or characterisation of the structures. Appropriate binning and smoothing of the distribution in the CfA slices make the galaxies percolate, thereby defining the voids and walls and allowing the estimation of the following fundamental parameters of the structure (de Lapparent et al. 1991): 1) at the percolation threshold, the galaxies occupy ~ 25% or less of the survey volume, thus providing a measure of the filling factor for these data; 2) the corresponding surface density of galaxies is ~ $0.3h^2$ gal/Mpc^2 with $M_{B(0)} < -18$; 3) variations of the counts-in-cell with cell-size indicate that the structures are consistent with a network of 2-dimensional structures, i.e. walls; 4) the average thickness of the walls is 500 km/s^{-1} FWHM. These various measures can allow direct comparison with other surveys and N-body simulations. For example, the filling factor for the galaxies puts limits on the relative distribution of galaxies and dark matter in the pancake models. These measures are also necessary for designing new surveys for efficient mapping of the large-scale structure.

Another interesting approach is the wavelet analysis which provides an objective detection of the structures in the data maps (Slezak et al. 1993). It locates the detected structures as a function of the scale at which the data is envisioned and measures their significance level. This statistic is useful for designing further observational programmes (for instance follow-up observation inside a given void in another wavelength-band), but most importantly it should allow to constrain the spectrum of void diameters, which is closely related to the spectrum of initial perturbations. Other sophisticated statistics which, in contrast, can be analytically derived from standard gaussian perturbation models, have also been considered for application onto the CfA or analogous data: topological measures (Gott et al. 1989; Ryden et al. 1989; Park et al. 1992a; Park et al. 1992b; Vogeley et al. 1994); fractal analyses (Martinez & Jones 1990; Colombi et al. 1992); void statistics and count probabilities (Vogeley et al. 1991; Maurogordato et al. 1992).

4. Constraints from New Surveys

4.1 WIDE-ANGLE DEEP SURVEYS

In the perspective of a better description of the galaxy clustering at the largest scales, both the existing wide-angle surveys and the theoretical models point to the necessity of obtaining larger catalogues. As mentioned above, although the CfA catalogue is one of the largest existing surveys, it is not a fair sample of the galaxy distribution. This urge for larger samples has led to the initiation of several new redshift programmes during the past decade.

The under-going survey at the Las Campanas Observatory probes structures at larger distance and suggests that we might have reached the maximum size for the inhomogeneities (Shectman et al. 1992). The goal of this survey is to sparsely map regions of the southern sky to a limiting magnitude of $r_{gunn} \leq 18$, corresponding to an effective depth of ~ $400h^{-1}$ Mpc. The survey is two-fold: CCD images are obtained in a chequer-board pattern with a periodicity of ~5° and an overall sampling rate of ~15% of the galaxies in the region; multi-fibre spectroscopic observations of the detected galaxies subsequently yield the redshift catalogue. Two declination strips of 1.5° in declination by 5^h in right ascension have however been completed to an average 60% sampling factor by filling the initially un-surveyed regions of the chequer-board. The diagram for one of the two slices is shown in Fig. 4. The distribution shows essentially similar structures as in the CfA slices of Fig. 2: voids are a common feature of the distribution and are delineated by sharp structures where the galaxies lie.

The remarkable feature of the map in Fig. 4 is that the largest voids seem to be comparable in size and at most a factor of 2 larger than the largest voids detected in the CfA slices, that is that they have a diameter in the range ~ 50 - $100h^{-1}$ Mpc. Other redshift slices obtained for this catalogue confirm the impression that the typical void sizes are not significantly larger than ~ $100h^{-1}$ Mpc. Statistical measures should be applied to the various samples to confirm quantitatively this visual impression. If when completed this survey does not uncover larger voids, then we might have reached the scale where the galaxy distribution becomes homogeneous.

The impression of weaker clustering and less contrast in the structures seen in the Las Campanas survey as compared to those in the CfA slices is caused by the partial sampling in the Las Campanas survey (~ 60%). The effect is strong because the walls of galaxies are so sharp, and it might affect some of the statistical characterizations of the distribution. A distribution with weaker density gradients would be less affected by partial sampling. The problem of partial sampling is further discussed in the next sub-section. An analogous survey to the Las Campanas programme is running at ESO and will be free from the difficulties caused by sparse sampling.

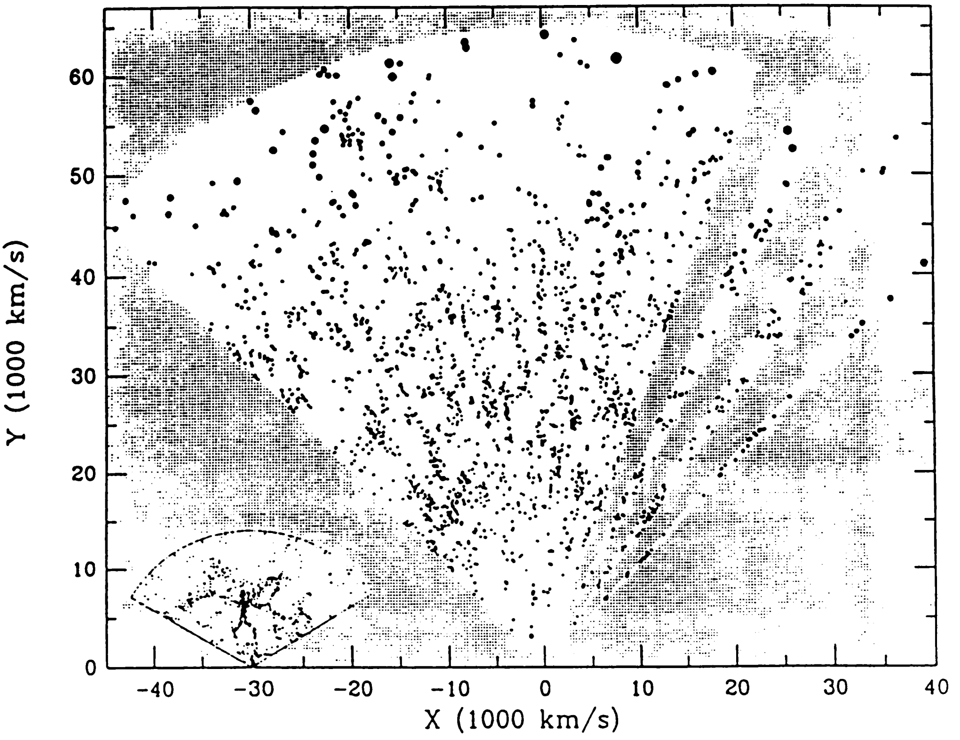

Figure 4. Maps of velocity versus right ascension for the galaxies in a declination slice bounded by $10^h15^m \leq \alpha \leq 15^h27^m$ in right ascension and by $-6.75° \leq \delta \leq -5.25°$ in declination. Angles and velocities are converted into cartesian coordinates in km/s^{-1}. An insert shows a slice of the CfA survey (top diagram in Fig. 2) for comparison of the scales. From Shectman et al. (1992).

The goal of the ESO survey is to obtain a complete magnitude-limited sample to $m_B = 19.4$, in a 32°-long by 1.5°-wide strip of the sky near in the southern galactic hemisphere (Vettolani et al. 1993). Preliminary maps with incomplete data display structures similar to those seen in the Las Campanas survey, both in shape and size.

The Muenster survey (Schuecker & Ott 1991), following a different philosophy supports the view provided by the Las Campanas and ESO surveys. While the latter surveys are aimed at measuring the detailed characteristics of the galaxy clustering, on both intermediate and large scales, the spirit of the Muenster survey is to measure a large number of redshifts in the most efficient manner: by using objective prism plates (photographic plates), where the image of each galaxy is dispersed by a low dispersion prism. Figure 5 shows the obtained redshift distributions in two regions near the southern galactic pole. The poor spectral resolution results in a large uncertainty in the measured redshifts, of order of 3000 - 5000 km/s^{-1}. These redshift uncertainties are comparable to the void sizes and therefore all structures on these or smaller scales are smoothed out. In the CfA slices, the uncertainty is of order of 30 km/s^{-1}, much smaller than the

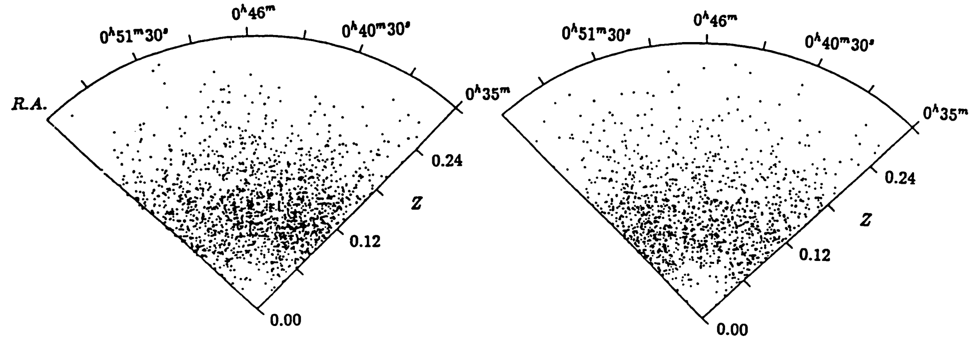

Figure 5. Maps of redshift versus right ascension for the galaxies with $15 \leq m_J \leq 20$ in two declination slices bounded by $-30° \leq \delta \leq -28°30'$ (diagram to the left) and $-31°30' \leq \delta \leq -30°$ (diagram to the right). The maps contain 1952 and 1228 galaxies respectively. From Schueker et al. (1991).

thickness of the sheets and hence than the diameter of the voids. However, the Muenster survey is adequate for detecting structures at larger scale. Visual inspection of the maps in Fig. 5 shows that at this resolution and within the depth of the survey (~ $500h^{-1}$ Mpc), the distribution deviates weakly from a uniform distribution. Therefore, no voids significantly larger than $100h^{-1}$ Mpc in diameter — but this also should be checked quantitatively — are detected by the Muenster survey.

4.2 CONFIGURATION OF REDSHIFT SURVEYS

The different redshift maps described above illustrate how the detected structures in a sample depend fundamentally on the configuration of the survey. We now emphasize how detection of voids and sheet-like structures in redshift surveys requires a careful choice of the combination of the following survey parameters: 1) limiting magnitude (defining the depth); 2) effective sampling; 3) precision of the redshifts; 4) solid angle on the sky.

The limiting magnitude is the most trivial parameter. The Las Campanas survey shows how by increasing the depth one can probe a larger number of structures if not larger ones. The effect is even more striking when comparing the data for the earlier CfA redshift survey (Huchra et al. 1983) first completed to $m_{B(0)} \leq 14.5$, yielding an effective depth of $60h^{-1}$ Mpc, with the extension to $m_{B(0)} \leq 15.5$. Figure 6 shows one of the slices in Fig. 2 restricted to the galaxies with $m_{B(0)} \leq 14.5$. Because the depth of the shallower survey is nearly equal to the size of the largest voids, the sheet-like structure is hardly visible. Comparison with Fig. 2 shows that the far edge of the void centred at $\alpha \simeq 15^h$ and $V \simeq 7{,}000$ km/s^{-1} and being part of the Great Wall is too distant to be included in the shallow survey of Fig. 6. The sample is in fact barely large enough to contain one such structure.

An efficient way of probing a large region of the sky and therefore a large number of structures is to perform a sparsely sampled survey. The existence of the Boötes void was

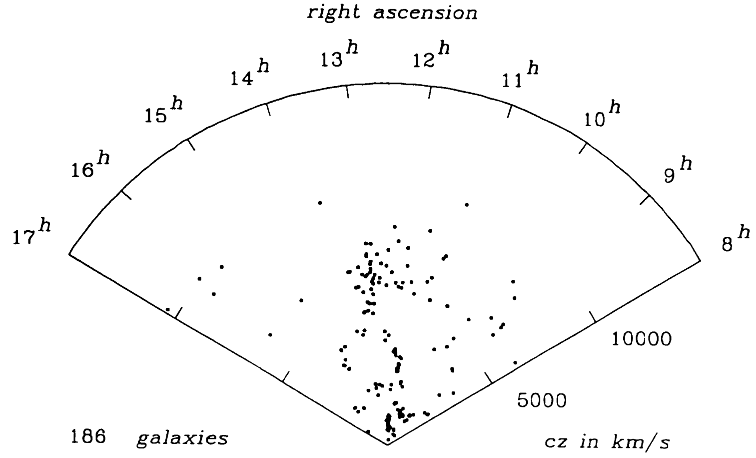

Figure 6. Map of velocity versus right ascension for the top slice of Fig. 2 restricted to the 183 galaxies with $m_{B(0)} \leq 14.5$ and $cz \leq 10{,}000$ km/s^{-1}. The declination range is $26.5 \leq \delta \leq 32.5°$.

confirmed by such a survey with a sampling rate of 2% over the area of the void (Kirshner et al. 1987). Although the resulting redshift maps show the under-dense region corresponding to the void in which an empty sphere of diameter $60h^{-1}$ Mpc can be enclosed, the sheets of galaxies defining the edges of the void are not visible (see the corresponding map [Kirshner et al. 1987]). Moreover, with this low sampling rate, galaxies located within the void might have been missed, thus biasing downward the density-contrast of the structure. For understanding the effect of sparse sampling, one can arbitrarily remove random galaxies from the CfA slices (Geller 1988). It appears that with a sampling rate of 10%, the voids are of course still present but the delineating surfaces are poorly defined. A quantitative manner to formulate this problem is the following: for reliable detection and definition of the voids, the separation between the galaxies must be small compared to the void diameter (Politzer & Preskill 1986).

The Muenster redshift survey demonstrates how the precision in the redshift measurement puts a lower limit on the detected size for the voids. The velocity errors also affect the properties of the high-density regions. Because the sheets of galaxies have a FWHM thickness of ~ 500 km/s^{-1} (or $5h^{-1}$ Mpc), velocity errors larger than ~ 100 km/s^{-1} will significantly widen the sheets perpendicular to the line-of-sight and erase small scale structures within the sheets. These errors are typical of those obtained in redshift surveys to faint limiting magnitude, and lead us to the discussion of the fourth parameter relevant to the survey configuration: the solid angle on the sky.

All surveys discussed above cover several tens of degrees on the sky at least in one dimension and are thus called wide-angle surveys. These surveys allow detailed mapping of the individual voids and sheets of galaxies, and are necessary for measuring the statistical properties of the structures related to their geometry (for example, the topology of the networks of sheets and voids). These measures are based on the clustering properties over the entire scale range from 10 to $100h^{-1}$ Mpc. However, typical measures at the largest scales, namely the scale of the voids can be obtained by mapping the redshift distribution over a small area on the sky (typically a fraction of a square degree) and to a large distance. Such surveys — described in the next sub-section — offer an efficient way of probing a large number of voids and walls.

4.3 ULTRA-DEEP PENCIL-BEAM PROBES

A deep pencil-beam survey recently completed (Broadhurst et al. 1990) confirms that at higher redshift the large-scale clustering keeps its major properties. The data were acquired in two diametrically-opposed probes pointing to the northern and southern galactic poles respectively. The redshift distributions are shown in Fig. 7. Because a deep probe is essentially a one dimensional survey, the voids and walls are detected in the redshift distribution by an alternation of peaks and valleys. In Fig. 7, the typical separation between the most marked peaks appears to be of order of $130h^{-1}$ Mpc. Moreover, Fourier transform and pair-separation analyses indicate a periodic signal at this scale in the data (Broadhurst et al. 1990). This result has been interpreted as indication of a periodicity in the galaxy distribution. Deep probes shot through periodic distributions as those generated by Voronoi foams (a method for constructing geometric distributions with a cell-like structure [van de Weygart 1991]) do yield a periodic alternation of peaks and valleys as seen in the data (Subbarao & Szalay 1992). However, the periodic nature of the observed distribution has been questioned (Kaiser & Peacock 1991), suggesting that the Fourier transform is not an adequate tool for reliably detecting periodicity in a given signal nor for measuring its significance level.

Establishing a quantitative correspondence between the peaks and valleys detected in a deep probe and the sheets and voids intercepted along the line-of-sight requires careful examination of the sample definition. The surface density of galaxies in the sheets as measured from the CfA slices is ~ 0.3 galaxy h^2 Mpc^{-2} at the median redshift of the sample (de Lapparent et al. 1991) (this value remains unchanged for the sheets at the median redshift of a sample with a different limiting magnitude). Therefore, the deep-probe survey of Fig. 7 with its effective beam width of ~ $5h^{-1}$ Mpc at $z \simeq 0.2$ can detect on the average only several galaxies per wall at that depth. It was demonstrated that galaxies cluster within the sheets and that about half of the galaxies in the slices belong to dynamical groups of galaxies with more than 3 members (Ramella et al. 1989; Ramella et al. 1990). The clustering scale-length corresponds to the correlation length measured from the correlation for the galaxies and is of order of $7h^{-1}$ Mpc (de Lapparent 1988). Therefore the number of galaxies detected by a beam width of $5h^{-1}$ Mpc is expected to fluctuate by significant amounts.

Simulations of many narrow probes with similar spatial extent through the full area of the Great Wall would in nearly half of the cases detect no galaxy at all, or in other words miss the Great Wall (Ramella et al. 1992). These simulations further show that a beam width of at least ~ $12h^{-1}$ Mpc is required for detecting any intervening sheet of galaxies with comparable surface density as the sheets in the CfA survey. Analogous simulations also show that the vast majority of the high-contrast peaks (with a density ratio ≥ 3 over the smooth curves in Fig. 7) correspond to previously identified groups of galaxies (de Lapparent 1991). The Coma cluster in turn yields a peak with a contrast larger than 10. Moreover, the average size for the empty intervals in the redshift distribution in these test probes is ~ $120h^{-1}$ Mpc, a large over-estimate of the true void size in the CfA slices. Therefore, simple tests using the CfA slices suggest that many peaks in Fig. 7 might correspond to groups of galaxies, and that their strong density contrast is not representative of the average underlying distribution within the intercepted walls (for the average value of the galaxy surface density in the walls, the contrast should be ~ 1.3). Moreover, the typical void size of ~ $130h^{-1}$ Mpc in Fig. 7 is probably an overestimate because the narrow beam-width misses some of the intervening sheets.

Note that for a given solid angle on the sky the beam-width varies with distance, and the

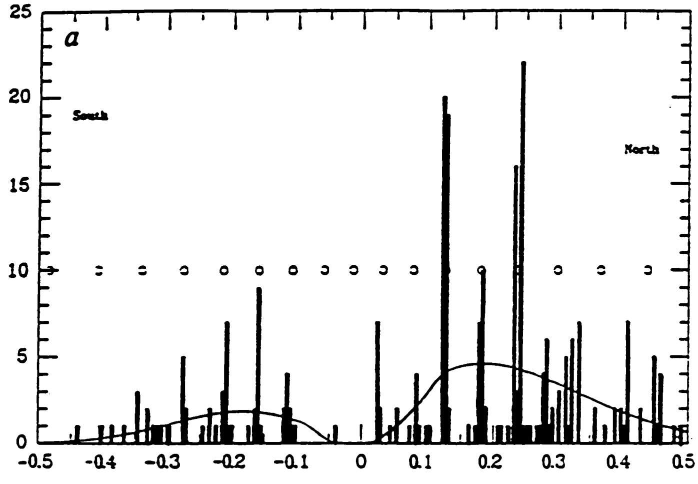

Figure 7. Redshift histogram for the galaxies with $B < 22.5$ in two deep pencil-beam probes. The negative values of the redshift were introduced to indicate the probe pointing to the southern galactic pole. The smooth curves represent the expected distributions if the galaxy were distributed uniformly. Circles are drawn every $128h^{-1}$ Mpc in co-moving coordinates to illustrate the suggested periodicity. From Broadhurst et al. 1990.

details of the expected number of objects per sheet as a function of redshift must be scaled accordingly (de Lapparent et al. 1991). Nevertheless, the fundamental result that narrow probes can miss a significant fraction of walls along the line-of-sight remains unchanged.

In order to obtain more reliable limits on the void sizes, another deep pencil-beam survey was started at the European Southern Observatory (ESO) facilities (de Lapparent et al. 1993). The survey has the configuration of a thin strip of ~ 0.2° x 2° reaching out to a redshift of ~ 0.6 (an order of magnitude deeper than the CfA slices), and will contain ~ 700 galaxies. The survey should cut through ~ 30 voids of average diameter 5,000 km/s^{-1}. We will obtain better constraints on the spectrum of void diameters than those derived from the survey in Fig. 7 because the ESO survey was designed to sample efficiently structures similar to those found in the CfA slice: at the median redshift of 0.3, the full survey will probe an area of 2.5 x $25h^{-2}$ Mpc^2, sufficient for detecting most sheets with a surface density comparable to that for the Great Wall (Ramella et al. 1992).

So far, the spectra of nearly half of the objects for the programme have been obtained. The redshift distribution shows the usual pattern of peaks alternating with valleys, but this pattern is nowhere as regular as in Fig. 7. However, a slice-diagram of the data shows a highly structured distribution along the line-of-sight which strikingly resembles that seen in the nearby surveys: an alternation of thin structures which are spatially extended across the sky, with voids with typical diameters of order of $50h^{-1}$ Mpc. Although the survey is still incomplete, the intercepted structures are remarkably sharp. Several statistical analyses of the data are in preparation and should yield preliminary measures of the size of the structures (Bellanger et al. 1993). The important new result of this ESO pencil-beam survey is that the galaxy distribution at $z < 0.6$

appears similar to the nearby distribution, and that the typical void size may be significantly smaller than the $128h^{-1}$ Mpc scale suggested by the survey in Fig. 7, in good agreement with the various wide-angle surveys.

5. Conclusions and Prospects

The CfA redshift survey slices show that the galaxy distribution is characterized by sharp sheet-like structures delineating voids of diameters ranging from ~ 10 to ~ $50h^{-1}$ Mpc. The physical units seem to be the voids and the coherence of sheets extending across the entire survey, like the 'Great Wall', appears to be mainly geometric. To reproduce such structures, the theoretical models for the formation of large-scale structure must assume large amounts of dark matter reaching closure density. This matter must be nearly uniformly distributed and if the limits of the nucleosynthesis predictions are to be met, must be for the most part non-baryonic. In this picture, the galaxy formation must be biased towards the densest peaks of the matter distribution. Therefore the voids of the galaxy distribution could be filled with dark — and partly non-baryonic — matter. So far, all observational searches for baryonic matter within these voids in the form of galactic-size systems have not led to any detection.

Both the observed galaxy distribution and the comparison of the N-body models with the data suggest that structures exist at the largest detectable scales in the observations which might be absent from the models (scales of order of 50 - $100h^{-1}$ Mpc). However reliable confrontation of the data with the models requires larger catalogues for better constraining of the *typical* behaviour of the galaxy distribution at these scales.

Deep pencil-beam surveys can efficiently probe through a large number of voids as well as uncover larger structures exceeding the extent of the nearby surveys. The typical void size of $130h^{-1}$ Mpc suggested by one of these deep surveys could however be an overestimate. Due to the intrinsic clustering of galaxies within the intervening walls and the narrow beam size of the probe (comparable to the galaxy correlation length), the survey might fail to detect some of the existing sheets and probably detects others as peaks with an enhanced density contrast relative to the values for structures like the Great Wall. Failure to detect all the intervening sheets of galaxies can lead to a systematic and artificial increase of the void sizes in the survey. Another deep pencil-beam survey designed to cover a sufficiently wide angle on the sky appears to be free of such biases. This new survey shows a similar large-scale structure as in the nearby surveys, with a typical void size of apparently ~ $50h^{-1}$ Mpc (to be measured quantitatively by statistical analyses).

Wide-angle surveys to intermediate distances show no evidence for structures on scales larger than ~ $100h^{-1}$ Mpc in diameter. It is worth emphasizing that so far, all existing surveys are consistent with a same description of the galaxy distribution, namely a cell-like galaxy distribution in which galaxies lie along walls separated by voids with diameters in the range of 10 - $100h^{-1}$ Mpc. Discovery of larger voids or structures on larger scales in under-going or future surveys would tighten the challenge already posed to the theoretical models.

Because the largest under-going surveys are probing regions an order of magnitude larger in extent than the largest detected voids, we are now entitled to ask whether we have reached the scale where the galaxy distribution becomes homogeneous. Before we can answer this question with reasonable confidence, larger and deeper redshift surveys are necessary. Medium size telescopes with wide-field CCD imaging capabilities will be important tools for providing the necessary galaxy catalogues, and the high multiplex gain of the multi-object spectrographs being built will allow us to obtain within a reasonable time span large redshift samples. The planned wide-angle catalogues such as the Sloan Digital Sky Survey (managed by the Astrophysical Research Consortium) will provide fair samples of the universe and appropriate data bases for studying with a high confidence level the detailed statistical properties of the galaxy distribution. Deep surveys which can sample more efficiently a large number of structures will yield the complementary constraints on the upper limits for the structures' sizes, and will allow us to check for possible evolutionary effects related to the large-scale structure.

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