7 - MECANIQUE CELESTE (CELESTIAL MECHANICS)

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A - INTRODUCTION

Celestial Mechanics has undergone, in the past three years, a considerable development that has been marked by a significant increase of published papers and by the recent organization of several specialized colloquia or symposia. The reasons of this increase of interest may be found among the following circumstances :

- The discovery of new dynamical situations in the Solar system: ellipticity of ringlets, shepherd satellites.
- The increase of interest in the problem of the dynamical evolution of the solar system or of its sub-systems.
- The present necessity to include relativistic effects in the theory of motion of celestial bodies.
- New promising results in the analysis of the properties of dynamical systems.

Several books or monographs in Celestial Mechanics were published during these three years:

- Akim, E.L., Bazhinov, I.K., Pavlov, V.P. and Pochukaev, V.N., (1984), "The gravitational field of the Moon and the motion of its artificials satellites", in russian, Mashinostroenie, Moscow.
- Bordovitzina, T.V. (1984), "Modern numerical methods in the problems of Celestial Mechanics", in russian, Nauka, Moscow.
- Dragomir, V.C., Ghitan, D.N., Mihailescu, M.S. and Rotoru, M.G. (1984), "The theory of Earth's shape", Elsevier, Amsterdam.
- Duboshin, G.N. (1983), "Celestial Mechanics: Methods of the theory of motion of artificial celestial bodies", in russian, Nauka, Moscow.
- Emelyanov, N.V. (1983), "Methods for the construction of algorithms and programs in problems of Celestial Mechanics", in russian, Nauka, Moscow.
- Ferraz-Mello, S. (1983), "Dynamics of the Galilean satellites", russian translation, Mir, Moscow.
- Kane, T.R., Likins, P.W. and Levinson, D.A. (1982), "Spacecraft dynamics", Mc. Graw Hill, New York.
- Kovalevsky, J. (1984), "Introduction to Celestial Mechanics", chinese translation.
- de Luca, N. (1982), "Mecânica Celeste", in portuguese, Curitiba.
- Nazarenko, A.I. and Skrebushevsky, B.S. (1981), "Evolution and stability of satellite systems", in russian, Mashinostroenie, Moscow.
- Roy, A.E. (1982), "Orbital motion", 2nd edition, Heyden and Sons, Philadelphia; russian translation of the first edition (1981), Mir, Moscow.
- Szebehely, V. (1982), "Theory of orbits: the restricted problem of three bodies", russian translation, Nauka, Moscow.
- Timoshkova, E.I. and Kholshevnikov, K.V. (1982), "Asymptotic methods in Celestial Mechanics", in russian, Itogui Nauki i techniki, ser.astron., vol.20, VINITI, Moscow.

The great diversity of research under way in the various fields of Celestial Mechanics does not permit to draw a complete picture of its development in the few pages that have been allocated to the commission report. So, following the example of the 1982 report by Y. Kozai, and in harmony with the wishes of the organizing

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committee, no attempt was made to give a quasi-exhaustive bibliography. We chose to critically describe in more details the achievements in some specific domains of Celestial Mechanics, leaving aside large fields of interest in which quite important contributions have been made. Hopefully, they will be included in the next report as a part of a six year development description. In the present report, we put the emphasis on two domains: the dynamics and the evolution of the solar system and the structure of the solutions of dynamical systems in Celestial Mechanics and particularly in the 3 or n-body problem. In doing so, no mention is made of the achievements in relativistic Celestial Mechanics (this will be done in the incomming IAU symposium N°114), resonance theory, systems involving finite bodies (including artificial satellite motion, structure or rotation of celestial bodies), methods for solving the equations (asymptotic methods, various developments, Hamiltonian mechanics, perturbations theory, etc...), variable masses, etc...

In particularly thank J. Chapront, J. Hadjidemetriou and C. Marchal who have accepted to review the achievements in their respective fields and have greatly contributed to this report, and also all the colleagues who sent their reports.

N.B. In the course of the report, A and A. Abstracts numbers are used except for the most usual titles for which an abreviated designation followed by the volume number (Colloquium or Symposium number for IAU Coll. or Symp.) and the page number is used. In addition, results presented in three recent non IAU colloquia are indicated as follows :

BER : "The Big Bang and Georges Lemaître"
A. Berger ed. (Louvain, Oct. 1983), Reidel Publ.Co.

SZB : "The stability of the Solar System"
V. Szebehely ed. (Cortina d'Ampezzo, Aug. 1984), in press.

FME : Resonances in the motion of planets, satellites and asteroids"
S. Ferrza-Mello and W. Sessin eds. (Sào Paulo, Nov. 1984), in press.

B - PLANETARY AND LUNAR THEORIES

(Main contributor : J. Chapront)

The last three years have been characterized, in these fields, by the introduction in the "Connaissance des Temps" of the planetary and lunar theories recently constructed in the Bureau des Longitudes and a series of improvements and accuracy analyses of these theories.

I. SECULAR VARIATION PLANETARY THEORIES

These theories are characterized by the presence of time in the coefficients of the periodic terms. They are valid for an interval of time of the order of one thousand years. The planetary theories constructed at Bureau des Longitudes have resulted in the solutions TOP82 for giant planets (Simon, AA 120.197) and VSOP82 for telluric and giant planets (Bretagnon, AA 114.278). Since the 1984 edition of the Connaissance des Temps, the solutions TOP82 and VSOP82 replace the theories by Le Verrier and Gaillot in the computation of the ephemerides of the Sun and major planets. The solution VSOP82 was compared to planetary observations (transits of Mercury and Venus, occultations of Venus and Mars) over the interval 1717-1973 by Krassinky et al (submitted to AA). The relativistic drifts that have been introduced for the perihelion of Mercury have been verified to an accuracy of a second of arc. The ephemerides of Mars constructed from an earlier solution (VSOP80) have been compared to 1622 observations performed between 1935 and 1976 by Niimi (33.042.090). A set of constants of integration was determined for the Earth and Mars. A particular attention was directed towards the modelling of the phase effect.

Presently, the improvement of these new theories is undertaken either in trying to improve the precision or in extending the interval of validity. The effects of lunar perturbations on the barycenter of the Earth-Moon system and on all major planets have been recomputed with a better precision. Perturbations due to Ceres, Pallas and Vesta on the major planets have been determined up to the second order of masses (Bretagnon, CM in press). On an interval of 1000 years around J2000, the accuracies range between 0.″001 and 0.″1 according to the planet. The interval of validity of the theories of Jupiter and Saturn has been extended to 6000 years around J2000 using methods that have been developed during the construction of TOP82 (Simon and Bretagnon, AA 138.169). The accuracy of the theory of Jupiter is better than 8" in the interval - 4000 to +8000. For Saturn, it is better than 23". The interval of validity of the theories of the Sun, Mercury, Venus and Mars has also been extended to 6000 years using, in particular, secular developments deduced from the general theory of Laskar (AA in press). For the Sun and these planets, the accuracy ranges between 1" and 5" over all the interval -4000 to +8000 years.

J. Chapront (CM in press) has proposed a new method of solving the problem of the motion of the Neptune-Pluto. He renounced the successive approximation method using Fourier series developments in mean longitudes which diverges. If one accepts to limit the validity of the solution to a thousand years - and this eliminates the 3.2 resonance problem -, the solution may be represented in terms of mixed series called Fourier-Chebyshev series (Chapront, CM 28.415).

II. GENERAL PLANETARY THEORIES

These theories provide long periodic variations of orbital elements of planets and their validity covers a time interval of the order of one million years. They have been developed in different directions. Bretagnon (CM in press) has refined his 1974 previous solution, introducing relativistic and lunar perturbations. Duriez (25.42.100) has constructed an autonomous system limited to the four giant planets. Laskar has continued this work (AA in press) and has included all the eight planets. He performed a numerical integration of the autonomous system over one million years instead of looking for a solution in quasi-periodic functions of time that have a poor convergence. The relativistic and lunar perturbations were included. The accuracy obtained allows a development of the solution in powers of time around the origin which has permitted to obtain the secular variations of the elements that were used to extend VSOP82 over 6000 years.

An important effort was initiated by Kamel in order to compute litteral expressions for a general theory of the motion of giant planets. It was prepared by the development of powers of the inverse distance between two planets (Kamel MP 26.339) and by the elimination of the critical terms in Jupiter-Saturn and Uranus-Neptune theories (Kamel MP 24.137, 27.407, 28.221 and Kamel and Bakry MP 24.261). A theory to the second order of masses of the Jupiter-Saturn system was computed using Poincaré variables and the Von-Zeipel method up to the fourth order in the eccentricities and inclinations (Kamel and Bakry MP 27.417 and 28.113).

III. NUMERICAL THEORIES AND METHODS

Most of the international planetary ephemerides are now based upon the numerical integration on the solar system performed in JPL (see Standish CM 26.181) and called DE200/LE200. In USSR, work is progressing in improving high accuracy numerical theories of the motion of inner planets and the Moon (Krassinsky and Sveshnikov CM 26.171; Krassinsky et al. AA in press; Sveshnikov 32.041.001, Trudy ITA 19). They are based upon soviet radar observations (1961-80), Washington meridian observations of the Sun (1961-70), passages of Mercury on the Sun (1717-1913), occultations of stars and inner planets by the Moon (1713-1980), laser observations of the Moon in McDonald (1970-1973). Among the results already obtained a number of parameters have been improved such as the motion of the equinox, the tidal acceleration of the mean motion of the Moon, Universal time scale during the XVII-XIXth centuries, the relativistic motion of the perihelion of Mercury, the secular variation of the so-

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lar radius, etc... During this work, rare but very valuable modern photoelectric observations of occultations of stars by planets were used. A simple such observation has an efficiency that is equivalent to a long series of classical astrometric observations. Planning of such observations in the future, might be very useful for improving the theories in Celestial Mechanics. In parallel, in the Institute for Theoretical Astronomy of Leningrad, a general information center on the motion of bodies of the solar system is being set up.

Kinoshita and Nakai (CM in press) have integrated the motion of the outer planets for five million years. The libration of Pluto's perihelion about 90° with a period of 4 million of years was assessed. It avoids close approaches to Neptune. This integration was used by Milani (CM in press) who traced the time variations of c^2h (angular momentum squared times energy) for various sub-systems and discovered a secular resonance locking between the perihelion of Uranus and the aphelion of Jupiter with a 1.100.000 years period. This locking is controlled by a complex 4 planets interaction.

Much work was done in the domaine of improving the method of numerical integration of the equations of Celestial Mechanics. They are developed by Bordovitsina in 30.021.22 and in a monograph (see Section A). Many experiments are described in this book. Most of the participants to these experiments showed their preference to the method of Everhart. The representation of the motion of planets or of the Moon by series of finite functions such as Chebyshev polynomials was studied in several places. The Bureau des Longitudes algorithm has been improved and a single formula may represent one year of the motion of the Earth with the desired accuracy. Batrakov and Fursenko (31.042.32) have developed a theory of Chebyshev slipnes to represent such motions; Schwartz and Walker have continued their studies of the applications of recurrence methods (CM 27.191) and Dvorak and Hanslmeier used Lie series method for numerical integration (AA 132.203 and 34.042.122), and found that this method is very advantageous.

IV. LUNAR THEORY

The semi-analytical theory of the motion of the Moon ELP2000-82 constructed by Chapront-Touzé and Chapront (CM 26.082 and AA 124.50) was adjusted to the JPL.LE 200 numerical integration. The angular and radius vector residuals do not exceed respectively 0".01 and 15 meters in a century. This solution was introduced in Connaissance des Temps starting 1984 in replacement of the Brown-Eckert theory. Recently, improvements were brought by the computation of perturbations due to the shape of the Moon and the effects induced by the libration (Chapront-Touzé AA 119.256). It is shown that, if third or higher order harmonics of the potential are taken into account, physical libration has to be introduced, otherwise secular drift appears. Kubo (Report of Hydrographic Res., Japan, 18) also computed perturbations due to the shape of the Moon and compared his results with ELP and Henrard's series. They agree better with EPL. Relativistic perturbations were evaluated by Lestrade and Chapront-Touzé (AA 116.75) and by Brumberg and Ivanova (Trudy ITA, 19). Both solutions agree. Comparisons of ELP with occultations of stars by the Moon have been made by Krassinsky et al (AA in press) and by Sôma (submitted to CM). The first authors discuss the value of the secular acceleration which is in good agreement with lunar laser data. The second author discusses the values of the lunar and solar parameters. The tidal term obtained is not inconsistent with the modern occultation observations and the motions of the perigee and the nodes coincide with the theoretical values.

Analytical theories of the libration of the Moon have been recently improved. Eckhard (29.094.023 and IAU 63.193) has developed a theory that includes planetary perturbations, Earth's flattening and the effect of the Earth on the fourth order harmonics of the lunar potential. Moons (CM 26.131 and 33.094.23) has developed a more general theory, retaining more physical parameters in a litteral form and her expressions are more precise. However, it is not yet completed and some perturbations are still missing. The difference between the common parts of both theories is of the order of 2 meters on the surface of the Moon and are mainly due to two resonant terms of the perturbation in parallax.

C - SMALL BODIES IN THE SOLAR SYSTEM

(main contributor : J. Kovalevsky)

The detection of many new satellites in the Jupiter and Saturn systems and the analysis of the detailed structure of the ring systems have given rise to a number of theories to explain the new dynamical features so discovered. In parallel many efforts have been devoted to the understanding of some features of the solar system from the dynamical evolution point of view.

I. MOTION OF NATURAL SATELLITES

The dynamical characteristics of natural satellites and their consequences on the theory of their motion have been described by Kozai (CM 23.265). Many new orbits of satellites have since been completed. However, the results will not be reviewed here and we shall consider here only general problems or methods.

Galilean satellites. The work on the theory of the motion continues in several directions. Arlot (AA 107.305) has obtained new integration constants for the Sampson-Lieske theory. The possibility to extend a general planetary theory to a resonant case and particularly to the Galilean satellites was shown by Duriez (CM 26.231) and the results agree in a first approximation with known results. After the elimination of all periodic terms to the third order, Brown (CM 23.203) obtained and solved the equations for the secular effects on the elements. New values of the semimajor axes were obtained by Vu (CM 26.265), while a complete second order theory, based on Sagnier approach has been constructed by Thuillot and Vu (34.099.112). In particular, the influence of dynamical parameters on the secular terms was computed by Thuillot (CM in press).

Saturn satellites. The dynamics of the newly discovered satellites have been subject to numerous studies in the frame of the discussion of horse-shoe librating orbits (Dermott and Murray, IC 48.12; Yoder et al. IC 53.431). Harrington and Seidelmann (IC 47.97) showed numerically that the system is stable and that the satellites never approach one to another by more than 6° during the 3000 days libration of S3 around S1. The dynamics of the co-orbiting satellites was studied by Spiris and Waldfogel (SZB in press) who showed that the problem reduces to the Hill's problem with appropriate boundary conditions at infinity. They predicted that S1 and D3 exchange orbits at close encounters, while S26 and S27 do not. I. Stellmacher (CM 28. 381) applied her algorithm for the construction of periodic orbits in the Mimas-Tethys case, the satellites being considered as two oscillators coupled by gravitational interaction. Bec-Borsenberger derived a litteral theory of the motion of Phoebe (CM 26.271).

II. RING DYNAMICS

The fine structure of Saturn rings found by Voyager spacrafts as well as the ellipticity of some Saturn and Uranus rings has brought to Celestial Mechanics a number of new problems which have now found satisfactory answers (Aknes 31.100.058 and SZB in press; Goldreich and Tremaine 32.091.025). The dynamics of elliptical rings for which a non zero inclination was found, have been discussed by Borderies, Goldreich and Tremaine (AJ 88.226). They show that by self-gravity, the ring may maintain by external forces (planetary oblatenes or satellite action). The remarkable satellite-ring interaction known as "shepherd satellites" by Goldreich and Tremaine in 1979 has been analysed and the role of dissipation was clarified by

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Greenberg (IC 53.207). Borderies et al. discussed globally the dynamics of elliptical rings taking into account external forces due to the oblateness of the planet and the sphepherd satellites, self-gravity and viscous forces due to interparticle collisions (AJ 88.1560). The picture is as follows: the mean eccentricity grows under the action of corotation resonances with the sphepherd satellites and is damped by viscosity so that an equilibrium is established. The uniform precession is induced by self-gravity.

The general ring picture has been also refined by Borderies et al. (IC 55.124) who computed the stress tensor due to particle collisions and explained sharp edges of rings and the decay of density waves. Other tentative explanations exist also for the radial diffusion or density and bending waves (Borderies, CM in press) and fine ring structure enhanced by viscosity (Michel 31.100.43). Salo (37.100.63) has applied the Hameen-Anttila's theory of bimodal gravitating systems to obtain steady state solutions for the optical thikness of Saturn's rings. The elliptic properties of particles determine the behaviour of gaps while the existence of ringlets would depend on the size and density of the particles.

The accretion of centimeter sized particles into large aggregates has been proved to be possible in Saturn's rings on a time scale of weeks by Davis, Weidenschilling et al. (37.100.44). These aggregates are later disrupted by tidal stresses, so that the mass of the ring system is processed through a population of large "dynamic ephemeral bodies" which are continously forming and desintegrating. Seidelmann, Harrington and Szebehely (IC 58.169) have studied analytically and numerically the dynamical behaviour of the E ring which extends from the orbit of Mimas to the orbit of Rhea and consequently interacts with Tethys, Dione and their Trojan companions. These detailed studies do not superseed the classical interpretation of ring density distribution by resonance with satellites. For instance, Wiesel (IC 51.149), including also Saturn oblateness, has explained in detail the structure of Cassini division. Other investigation in this direction have been made by Lissaper and Cuzzi (AJ 87.1051) who tabulated all the resonance locations and strengths from the most recent and reliable values of the masses and orbital parameters. These theories have also their equivalent in the studies of the structure of Uranus rings (Aksnes, IAU 75 in press).

III. TROJAN PLANETS AND ASSOCIATED PROBLEMS

Garfinkel (31.098.038; CM 30.373) is constructing a general analytical theory of the motion of Trojan planets. Formal long periodic orbits have been obtained and their periods expressed in terms of a mass parameter and of the normalised Jacobian constant. The motion of perihelion of Trojan orbits was investigated by Erdi (CM 24.377; IAU 47.165) and Erdi and Varadi (34.098.059). Several studies of horseshoe periodic orbits have been made in connection with this problem (Dermott and Murray, IC 48.1 and 12; Taylor, AA 103.288).

IV. EVOLUTIONARY PROBLEMS

Under the word evolution, are meant the dynamical processes that tend to modify the fundamental structure of planetary or satellite systems: secular drifts in semi-major axis, trapping into resonance, escapes from, or capture into a system, etc... Many investigations have been made in the recent years in an attempt to understand the present structure of the solar system (an introduction to the problem is found in Burns, 32.091.053).

Asteroidal belt. Several different models have been proposed to explain the Kirkwood gaps in the distribution of Minor Planets as a result of gravitational or cosmological processes. Hadjidemetriou (CM 27.305 and 31.042.051) and the same author with Ichtiaroglou (IAU 74.141 and AA 131.20) have based their study on the behaviour of resonant solutions of the non-averaged plane restricted problem. Unstable regions have been identified in the phase space. Wisdom (AJ 87.577) starts from the same model for the 3.1 resonance, then he models high frequency terms as Dirac impulsions and constructs a mapping of the phase space on itself. This permits him to study the behaviour of an asteroid over much longer times. He finds that the eccentricity oscillates and its maximum value increases so that the planet reaches the orbit of Mars and leaves the resonant region. In a second paper (IC 56.51) he traces the chaotic region in the elliptic restricted region: it outer boundary coincides with the boundary of the 3/1 Kirkwood gap in the actual distribution of asteroids. The extension to other gaps is not straight forward because of the specific role played by Mars. Other attempts have been to explain the gaps in introducing non conservative effects. Torbett and Smoluchowski (IC 44.722, AA 110.43 and 127.345) and Gonczi et al. (IC 51.639) have investigated the effect of resonance sweeping due to a proto-solar accretion disk or Pointing-Robertson effect, by numerical integration. Henrard and Lemaître (IC 55.482 and BER 217) and Lemaître (IAU 74.189 and CM in press) developed an analytical model that they applied to the gravitational effects of the proto-solar nebula. The adiabatic invariant theory and the introduction of an area index (the algebraic area enclosed by the trajectory) are used to monitor the evolution of the orbit in time. Comparing the statistical results of the evolution of a number of asteroids initially resonant with the actual distribution has shown a satisfactory agreement.

Satellites. Several papers were devoted to the time evolution of satellite systems. Sinclair (IAU 74.19) has reexamined the tidal hypothesis for Saturn resonant satellites. For Mimas-Tethys, this assumption is consistent with their anomalously high inclinations. It is not sufficient for the Enceladus-Dione system, where one should in addition consider tidal dissipation within Enceladus. Henrard has developed the theory of capture in a general one degree of freedom Hamiltonian system and connected it to the adiabatic invariant theory (CM 27.3 and 31.042.054). He applied it to the orbital evolution of the Galilean satellites (IC 53.55), while the same theory was applied by Borderies and Goldreich to capture probabilities for the j+1:j and j+2:j orbit-orbit resonance problem (CM 32.127).

For the non-resonant evolution of satellites, Mignard has completed his study of the past secular evolution of the Earth-Moon system (MP 24.189) and of the Martian satellites (MN 194.365). In both cases, the dissipation in the satellite is a critical parameter in distinguishing various evolutionaly paths. He shows that, in any case, Deimos has not undergone a significant tidal evolution, while Phobos may have evolved from a high eccentricity orbit. Kovalevsky (SZB and FME in press) has shown that planetary terms of the lunar theory may undergo a passage through resonance while the tidal friction makes the lunar semi-major axis to evolve. If its coefficient is sufficiently large such a term may become resonant and stop the evolution of the semi-major axis during several libration periods, then an escape from the resonant region occurs. For a given term the same situation may repeat a great number of time while the Earth's eccentricity undergoes long periodic variations. A model of capture into and escape from resonance is being studied by Sidlichovsky.

Comets. The evolution of cometary orbits at close approaches with planets has been studied by several authors. Alekseev and Osipov (Ergod.Th. and Dyn.Sys.2,263) have estimated rigorously the variation of parameters during a close approach to Jupiter. Nakamura (IC 45.529 and IAU 74.97) has estimated, by numerical methods, the final orbital distribution of extinct comets. The depletion of the Oort cloud by a dynamical process has been studied by several authors. Fernandez (IC 42.406 and 47.470) has studied numerically the perturbations of cometary orbits by stars and outer planets. Weissman (33.102.034 and IAU 61.637) developed a model of stellar perturbation in terms of a velocity impulse and obtained rates of depletion and of formation of comets entering the planetary system. Rémy (IAU 83 in press) has removed two assumptions from the Weissman model: the original eccentricities are assumed to hase a wide distribution between 0 and 1 and not only close approaches are considered. She estimated analytically the main depletion of long periodic comets.

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D - PERIODIC ORBITS AND STABILITY

(Main contributor : J.D. Hadjidemetriou)

Research in periodic orbits and stability has been influenced by recent developments in non-linear dynamics and this interaction has been proven very fruitful. Integrability, chaotic motion, infinite bifurcation, Lyapounov characteristic exponents have been successfully used in the study of old problems of Celestial Mechanics. Resonance phenomena and their relation to the stability of planetary systems were widely studied in connection with a variety of problems (see also the section on the small bodies in the solar system). The study of dynamical systems with three degrees of freedom attracted the attention of many investigators. Finally, the stability of planetary or stellar systems was also widely studied by a variety of methods and definitions of stability. The method of work varies from abstract mathematical proofs to numerical computations.

I. METHODS OF NON-LINEAR DYNAMICS

Bifurcation theory. Bifurcations of families of periodic orbits in a dynamical non integrable system are now studied in connection with the understanding of the onset of chaotic motion. This work started after a paper by Feigenbaum (J.Stat.Phys.19,25) who found that dissipative systems have infinite period doubling bifurcations and that this leads to chaotic motion. This result was extended to Hamiltonian systems, which mostly interest Celestial Mechanics, and also to area preserving mappings. It was found that, in this latter case, there exists a universal ratio of the successive intervals between the (infinite) bifurcations, equal to 8.72 (Benettin et al., lett.Nuovo Cim., 28,1 and Bountis, Physica 3D,577 for area preserving mappings). Such a sequence of infinite period doubloing patchwork bifurcations has been found by Contopoulos and Pinotsis (AA 133.49) for two simple families of periodic orbits of the restricted circular three body problem, and the same universal ratio was found. Successive bifurcations in Hamiltonian systems have been also studied by Contopoulos in letters to Nuovo Cimento (30.498; 37.149 and 38.257). Infinite period doubling bifurcations have also been studied by Heggie (CM 29.207) for a family of periodic orbits of a dynamical system with two degrees of freedom, representating two coupled oscillators. These bifurcations were of different nature than those mentionned above, and it was proved that no quantitative universality exists in this case. The bifurcations occur when the stability changes along a family of periodic orbits, for a system with two degrees of freefom. In a system with three degrees of freedom, the periodic orbits along a family may loose stability through complex instability. Contopoulos (let.Nuovo.Cim.38.257) showed that, at this point, when the bifurcation sequence terminates, a complex instability appears. Heggie (CM in press) studied the bifurcations at this transition point to complex instability and showed that what bifurcates is not a new family of periodic orbit, but a family of invariant tori. This was detected in the general planar three body problem. Contopoulos also showed that in a systems with three degrees of freedom, the bifurcation sequence terminates at an inverse bifurcation.

Integrability. The problem of integrability is important in the study of chaotic and ordered motion. Necessary conditions for the existence of algebraic first integrals in a Hamiltonian system with two degrees of freedom have been studied by Yoshida (CM 31. 363 and 381). He showed that the existence of an algebraic first integral controls the Kovalevskaya exponent characterizing a singularity of the solution and that the appearance of irrational or imaginary exponents proves the non-existence of a sufficient number of first integrals. In this way, the three body problem and the Hénon-Heiles system are proved to be non-integrable. The integrability of a Hamiltonian system with three degrees of freedom at a resonance 1:2:ω with ω:1,2,3,4 was studied by Van der Aa (CM 31.163) by making use of the Birkhoff normal form. He found that, in general, no quadratic or cubic integral exists, but asymptotic integrability occurs for special values of the parameter. A new integrable system was

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found by Mignard and Hénon (CM 33.239) in their study of the motions of a particle around a planet taking into account the solar radiation pressure.

Chaotic motion. Generation of chaotic motion and its relation to be interaction of resonances is one of the problems of non-linear dynamics. Yokoyama (CM 33.99) studied a non-linear Hamiltonian system where ordered motion reappears as the energy increases and this was explained by the criterion of the interaction of resonances. Yi-Sui Sun (CM 30.7 and 30.111) studied the appearance of ordered or chaotic motion in a three dimensional area conserving mapping. Chaotic motion in the restricted three body problem was found by Hadjidemetriou and Ichtiaroglou (IAU 74.141 and AA 131.20) for the circular case by considering a mapping on a surface of section. Milani and Nobili (AA in press) found that the chaotic regions are more important and escape orbits exist in the elliptic case.

Another method to detect chaotic motion is by the use of the Lyapounov characteristic exponents which allow a precise quantitative definition of the stochasticity of the orbit. Gonczi and Froeschlé (CM 25.271) applied this method to the restricted three-dimensional three body problem, while Huang and Innanen (33.42.001 and 37) discussed the stability of the general planar three body problem, arguing that there exist local additional integrals when the initial elements are situated inside the stable region.

II. DYNAMICAL SYSTEMS WITH THREE OR MORE DEGREES OF FREEDOM

The general behaviour of dynamical systems with three or more degrees of freedom presents properties that do not appear in systems with two degrees of freedom. However, due to the complexity of the problem, papers deal with non-linear systems which are simpler than those which appear in actual problems of Celestial Mechanics, but which show typical features present in the systems with three degrees of freedom in general. For instance, systems of three coupled oscillators near various resonances were studied by Martinet et al (CM 25.93) and Magnenet (CM 28.319). Other types of dynamical systems with three degrees of freedom were studied by Contopoulos and his colleagues (32.151.087 and CM in press).

Very few things have been done for systems with more than three degrees of freedom. Stellmacher (CM 32.23 and 247) studied analytically the existence and the stability of families of periodic orbits in a Hamiltonian system with N degrees of freedom, applying her results to 3 degrees of freedom. Hadjidemetriou and Michalodimitrakis (AA 93.204) have studied the general planar 4 body problem. Several families of periodic orbits and their stability were computed for the actual masses of Jupiter and three Galilean satellites: one periodic orbit, at resonance 1:2:4 is very close to the actual motion and is found to be stable. Grigorelis is presently computing families of periodic orbits of the general 4 body problem with equal masses.

III. PERIODIC ORBITS

In the last three years, new existence proofs for particular types of periodic orbits were given and generalizations of existing proofs were made. Much numerical computations of periodic orbits were also made recently, though the computations of families of periodic orbits of the general three body problem was not as popular as some years ago. In addition, analytic expansions using computerized litteral algebra were obtained. A review on periodic orbits by J. Hadjidemetriou has been prepared (FME in press).

Proofs of existence. Meyer (J.Diff.eq. 39.2 and CM 23.69) proved the existence of periodic orbits of the general n-body problem, starting from a periodic solution of a restricted (n-1)-body problem, generalizing older results on the three body problem. Message (CM 28.107) proved the existence of periodic orbits of the third sort in the general problem of three bodies. Kammeyer (CM 30.329) proved the existence

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of symmetric periodic orbits in the rectilinear three body problem with the middle mass larger than the others. Belburno (CM 25.195 and 397) proved the existence of periodic orbits in the restricted three-dimensional 3-body problem by continuation of collision orbits.

Analytical methods. Several investigators developed methods to construct analytically a periodic orbit. Presler and Broucke (Comp.M.Apl. 7.451 and 473) and Davoust (CM 31.241 and 293) used the Linstedt method. Wiesel (CM 23.231) gave a formal solution for the motion near a periodic solution, extending Floquet's theory. Stellmacher (CM 28.351) gave an algorithm to construct a nearly circular periodic orbit for the motion of a satellite around an oblate planet. Ding and Tong (37.042.100) generalized the Poincaré theorem of analytic continuation of periodic orbits to the case of multiple parameters and showed that the importance of the oblateness of the primary cannot be neglected in the restricted three body problem.

Numerical methods. Many families of periodic orbits in the general three body problem were computed by Broucke et al (CM 24.63), Markellos (CM 25.3) and Delibaltas (CM 29.191) or in the restricted 3 dimensional problem by Robin (CM 23.97) and Taylor (CM 29.51 and 75). Brown's conjecture concerning the termination of the long period family around L_4 was disproved numirically by Henrard (CM 31.115) and analytically by Garfinkel (SZB in press). Other studies around Lagrangian points were made by Doubochine (CM 33.21) and Gomez and Noguera (CM in press). In addition, periodic orbits of particular interest to space flight were computed by several investigators.

IV. STABILITY

Various concepts and definitions of stability have been proposed, ranging from the stability in the neighborhood of a periodic orbit or an equilibrium position to a global view. Both empirical and rigorous approaches were used. A review of the stability concepts was given by Szebehely (SZB in press).

Stability near a periodic solution. Yoshida (CM 32.73) showed that, in certain cases, the characteristic exponents can be found, and thus the linear stability can be established. Sknol (CM 33.159) studied the motion around an equilibrium point in an autonomous Hamiltonian system with two degrees of freedom near the main resonances. The regions of non-linear stability around the equilibrium points L_4 and L_5 in the restricted three body problem were studied by Szebehely and McKenzie (CM 23.131). The relation between resonance and instability in a planetary system with two or more planets has been studied by Hadjidemetriou (CM 27.305) and the mechanism by which the instability is generated was found.

Global stability. The classical Hill stability appears in the circular restricted three body problem for large values of the Jacobi constant. The Hill type stability is an extension of this kind of stability to the general three body problem. If the product of the energy by the square of the angular momentum C is below some negative function of the masses, the possible domain of evolution in the phase space is divided into three disconnected parts and the motion remains for ever in one of them. There exists then a binary that the third body can neither approach nor disrupt (however, this kind of stability does not prevent the escape of the third mass, but motions of exchange type cannot exist). Many studies have been done in the last ten years on this problem. After the last of a series of papers by Szebehely (CM 22.7) more precise results have been obtained by Markellos and Roy (CM 23.269) who showed that Hill's stability in the circular case is valid for direct satellites but underestimates the actual possible maximum distances from the planet by at least a factor 2. Valsecchi et al (CM 32.217) extended this numerically to elliptic orbits and foundsignificant differences with the circular case. C. Huang extented the notion to non spherical bodies (33.042.070), while Marchal and Bozis (CM 26.311) have generalized the criterion to zero and positive energy. They found the forbidden zones and also that stable systems with negative energy exist. They form a hierarchical configuration (close binary and a distant body) that is not disrupted. It appears thenthat most of the triple stellar systems are Hill stable. On the contrary, Ding and Huang (31.042.120), studying the general three-body, showed that most natural satellites are Hill unstable. Nugeyre and Bouvier (CM 25.51) have also studied hierarchical structures in stellar systems and showed that the choice of suitable coordinates results in simpler formalism for stability.

Much work has recently been done by Roy and his collaborators on the stability of hierarchical systems. Roy (31.042.053, IAU 74.277 and SZB in press), Walker and Roy (CM 24.195, 29.117 and 29.267), Walker (CM 20.149 and 215) and Milani and Nobili (CM 31.213 and 241; BER 219) have applied the empirical parameter method to the problem of the stability of N body hierarchical dynamical systems and are using it to approach the stability of the solar system. Szebehely and Whipple (CM 32.137 and BER 195) studied the stability of the restricted problem of n massive primaries and ν small bodies, by making use of the integrals of motion. A similar method is also used by Szebehely to study the stability of binary asteroids.

V. THE INVERSE PROBLEM

This problem arises when a set of orbits is given and one wants to determine a force field or the potential. The problem was proposed by Szebehely in 1974 who reduced it to be solution of partial differential equations that he established. Several people have worked recently on various aspects of this problem using Cartesian coordinates with two or three degrees of freedom in an inertial or a rotating frame or using generalized coordinates. Szebehely and Broucke (CM 24.23) studied the determination of the potential field from a given family of periodic orbits in a rotating frame. Bozis, in a series of papers (CM 28.367, 29.329, 31.43 and 129, AA 134.360) and Xanthopoulos and Bozis (AA 122.251, IAU 74.253) studied the problem of determining the potential (or force field if the system is not conservative) from a given mono-parametric or two parametric family of orbits in the plane or in three dimensions. Erdi (CM 28.209) generalized Szebehely's equations to three dimensional orbits. Varadi and Erdi (CM 39.395) considered a two parametric family in three dimensions and the corresponding system of partial differential equations for the determination of the potential was studied by methods of differential geometry. Melis and Piras (CM 32.87) studied the determination of the potential generating a given family of orbits in the N dimensional configuration space of a holonomic system. Puel (CM 32.319) showed that Szebehely's equation of the inverse problem is equivalent to a multiple variational problem deduced from Maupertuis' principle. Similarly, Gonzalez-Gascon et al. (CM 33.85) studied the connection of this equation with the equations of Dainelly and Whittaker on similar problems.

E - QUALITATIVE ANALYSIS IN CELESTIAL MECHANICS

(Main contributor : C. Marchal)

In Celestial Mechanics, as in many other domains, the quantitative and the qualitative analyses are complementary. The quantitative analysis gives excellent informations of the future or the past of some particular solutions of interest but is usually unable to give general informations and its accuracy generally decreases and becomes null for large times. On the contrary, the qualitative analysis gives partial, but rigorously demonstrated properties, that are valid for very long periods of time, and even often for infinite time.

The qualitative analysis especially deals with the problems of integrals of motion, symmetries, periodic orbits, final evolutions, structure of a set of solutions, analysis and regularization of singularities, escaping motions, bounded motions, oscillotary motions, asymptotic motions, etc... It has been initiated by

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Poincaré, Sundman, Chazy, Khilmi, Sitnikov, Merman, Alekseev, etc... The Kolmogorov-Arnold-Moser theorem can be considered as the result of a qualitative analysis.

The fantastic progress of computers has led to many improvements in the quantitative analysis of the n-body problem. These improvements have disclosed the extreme complexity of the set of different solutions and have given new orientations and a new impetus to the qualitative analysis, especially in the domains of final evolutions and tests of escape. Let us consider the different questions successively.

I. FINAL EVOLUTIONS IN THE N-BODY PROBLEM

The existence of five main types of final evolution are now well established (see Alekseev 30.042.079, Marchal and Saari, J. of diff.eq. 20, p 150).

1. The triple or multiple collision at some time t_0. When t goes to t_0, all n bodies go to a definite final position at a bounded distance, and the bodies going to the collision approach a "central configuration" (Marchal 31.042.056). This type of final evolution is exceptional and has a measure zero in the phase space. Several studies of particular cases of multiple collisions have recently been published. Eschbach (CM 27.157) has analysed the n-tuple collisions in the n-body problem and discussed various cases in function of the energy h of the system. Lacomba and Simó have constructed the linear and rectangular degenerate cases of the four body problem leading to quadruple collisions (CM 28.49).

2. The infinite expansion in a bounded interval of time. This type of final evolution requires $n \geqq 4$ and very strong oscillations. An example was given by Mather and McGehee in 1974. It is conjectured that this type of expansion is infinitely rare.

3. The super-hyperbolic expansion. It is analogous to the previous type of motion and is problably also infinitely rare.

4. The hyperbolic expansion. It is very usual type and corresponds to most of the phase space. The limit of the ratio of the largest mutual distance R to the time t is bounded and non-zero. Masses may cluster in sub-systems in which the energy and the angular momentum have bounded limits.

5. The parabolic or sub-parabolic expansion. All mutual distances remain of the order of $t^{2/3}$ at most and the energy integral h in the axes of the center of masses is negative or zero. If h=0, the expansion is parabolic and non-infinitesimal masses approach a central configuration. If h<0, many possibilities exist: parabolic expansion, sub-parabolic expansion, bounded motions, formation of subsystem (clusters), periodic, asymptotic, oscillating, quasi-periodic, chaotic motions, etc... It is known that this type corresponds to a set of positive measure, but a complete analysis of these cases still remains to be done. Some partial studies were recently done by Lacomba and Simó (CM 28.37) who assigned a boundary (total collision) manifold to each energy surface. In the case h<0, there exists an infinity manifold components which reflect the complexity of the situation encountered when the total energy is negative.

II. REGULARIZATION OF THE SINGULARITIES OF THE N-BODY PROBLEM

The n-body problem has two types of singularities: the collision of two or several bodies and the infinite expansion in a bounded interval of time (see case I.2 above). The regularization of singularities answers to the question: "How to extend naturally a solution after a singularity?". It has a theoretical interest and is also useful for numerical integrations of nearly singular solutions. Two types of regularization exist (McGehee, in Dynamical Systems, theory and Applications, J. Moser ed., p 550, 1975), the analytical (or Siegel's) and the topological or by continuity (Easton's regularization). They are independent and may be different as shown by McGehee in 1975. It is well known that the collision of two bodies is always regularizable in both senses. The infinite expansion is an essential singularity and is never regularizable. It has been found that the possible cases of regularization (Marchal 31.042.056 and Richa, Thèse de 3e cycle, Univ. de Paris 6, "Cas de régularisation des singularités du problème des n corps") are very rare: they may appear only in the analysis of rectilinear motions. In addition, all but perhaps one masses are infinitesimal masses and the latter have particular mass ratio (Simó CM 21.25). However, in other cases of triple collisions, regularized coordinates on plane collision manifold may be found (Waldogel, CM 28.69) and permit numerical studies of the collision.

III. FINAL EVOLUTIONS IN THE THREE BODY PROBLEMS

Ten final evolutions of three non-infinitesimal masses have been identified:

1. The triple collision requiring a zero angular momentum.

2.3.4. The hyperbolic, the hyperbolic-parabolic types that require a positive energy h and the hyperbolic elliptic type for which h may be zero or negative. They present all an hyperbolic expansion.

5. The tri-parabolic expansion with $h=0$.

6. The parabolic elliptic expansion with $h<0$.

7. The bounded type: all distances and velocities remain finite and $h<0$.

8. The Khilmi and Sitnikov oscillating type: $h<0$ and the velocities remain bounded while the largest mutual distance is such that $\limsup R=+\infty$ and $\liminf R=<+\infty$.

9. The second oscillating type: $h<0$. The mutual distances remain bounded, but the largest velocity is not bounded. An infinite number of arbitrarily close approaches occurs.

10. The third oscillating type: it is a combination of types 8 and 9.

Several sub-types cal also be identified.

Presently, it is known that cases 1,3,5 and 6 are infinitely rare. It is conjectured that this is true for cases 8 and 10. The others correspond to sets of positive measure in the phase space. An important consequence of the positive measure of type 9 which presents an infinite number of arbitrarily close passages is that many more collisions can happen in this indirect way than in the direct mode (Marchal 25.151.093). This type of motion could be found in new born triple stellar systems, but not in old triple systems: the latter would already have undergone a collision.

For any non-zero values of the mass ratios, all combinations of original (decreasing time) evolution and final (increasing time) evolution are possible, provided that they correspond to the same sign of energy integral and that some evolutions beginning and/or ending by the type 1 be excluded. The cases corresponding to sets of positive measure in phase space are any combination of types 2 and 4 (with or without exchange) and also the evolutions $7\to7$, $9\to9$ and perhaps $8\to8$, $10\to10$. New criteria for hyperbolic and hyperbolic-elliptic motions were proposed by Merman (31.042.029) and considerable successive progresses have appeared recently in the domain of the tests of escape (Zare, 1981 (CM 24.345), Laskar, 1982 (unpublished), Laskar and Marchal, 1984 (CM 32.15), Marchal, Yoshida and Sun-Yi-Sui, 1984 (CM 33.193). It is to be noted that the smallest mass is the most unstable and the most subject to be ejected and the present tests seen to approach very near to true limit of escape. However, no general test of bounded motion is known and seems to be difficult to obtain: because of the conservation of measure in phase space, their limit would be a set of solutions. In any case, it appears, from the above mentionned results, that the bounded solutions are much more rare than it was generally expected.

IV. STRUCTURE OF A SET OF SOLUTIONS IN THE THREE BODY PROBLEM - CONJECTURES

The numerous studies made on the general structure of a set of solutions have led to distingich three types or orbits:

A. Periodic and quasi-periodic orbits,
B. Semi-ergodic or chaotic orbits,
C. Open orbits leading to an escape or a collision (evolution 1 to 6 of section III).

There are also some exceptional orbits such as asymptotic orbits. The structure of the phase space is usually presented through some simple area preserving mappings (see also the section on periodic orbits and stability) where similar behaviour exists (Easton 31.021.020). The ergodic throrem being extended to open systems, it can be shown that almost all orbits remain in the same class during the evolution for increasing and decreasing time (Marchal, J. of Diff.Eq. 23, p 387). However, only the orbits of class A and B have the usual strong stability properties. It seems that, as in most time independent Hamiltonian problems, orbits of class A are the backbone of the set of solutions: the chaotic orbits fill densely the bounded holes left by the periodic and quasi-periodic orbits, while the open orbits are found in the unbounded holes. However, it seems that bounded holes of chaotic motions are exceptional and correspond to exceptional cases like the restricted problem. Generally, holes appear to be unbounded and almost all apparently chaotic orbits end by an escape. This "Arnold diffusion conjecture" would mean that the escape orbits of type C are dense everywhere in phase space. This conjecture is to be put together with the still not demonstrated Poincaré conjecture that the periodic orbits are dense in the set of bounded orbits and also the conjecture on the structure of solutions (for non infinitesimal masses, apart from a set of measure zero in phase space, all bounded and oscillating orbits belong to the Arnold torus and are either periodic or quasi-periodic: other orbits have least one escape for both $t \to + \infty$). These three conjectures are in the center of the progress in the analysis of the three body problem. The last conjecture is not true in the restricted three body problem: this can be interpreted as the situation in which one of the masses is infinitesimal may not be representative of the complete problem. Recently, a weakened version of Poincaré's conjecture was proved by Gomez and Llibre (CM 24.325).

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