

STATISTICAL MECHANICS AND EQUILIBRIUM SEQUENCES OF ELLIPTICALS

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ABSTRACT. Elliptical galaxies are expected to have undergone incomplete violent relaxation. Here incomplete relaxation is regarded as a process producing a metastable, long-lived state which is dynamically stabilized by the approximate conservation of one global quantity in addition to the total energy and number of particles.

Since the early work of Lynden-Bell (1967) elliptical galaxies have been considered as the final product of incomplete violent relaxation (May and van Albada 1984), nevertheless a satisfactory treatment of incomplete relaxation is still lacking. Recently Tremaine, Henon and Lynden-Bell (1986) have addressed the problem of constraining the relaxation processes of ellipticals using statistical arguments without making use of a maximum entropy principle. Here we study incomplete relaxation from a more conventional point of view, i.e. using the classical (Boltzmann) expression for the entropy, but trying to take into proper account the collisionless nature of the system by specifying the conservation of one additional global quantity  $Q$ . Then the distribution function is obtained by extremizing the entropy of the system following the concepts and the procedure outlined by Lynden-Bell (1967) and Shu (1978). If we argue that the additional quantity is of the form  $Q = \sum_{\text{part}} q$ , i.e. the equal weight sum of one particle contributions, with  $q$  being proportional to powers of the particle angular momentum  $J$  and energy  $E$ , then we must have

$$q = J^{\alpha_1} (-E)^{-3\alpha_1/4} \tag{1}$$

in order to meet the requirements of the dynamical selection criterion formulated by us elsewhere (see Bertin and Stiavelli 1984, and this Symposium). In fact, by extremizing the entropy subject to the conservation of  $Q$  we obtain the following family of distribution functions

$$f^{(\alpha_1)} = A \exp \left[ -aE - c J^{\alpha_1} / (-E)^{3\alpha_1/4} \right] \tag{2}$$

all satisfying our dynamical selection criterion. At this stage the

value of  $\alpha_1$  is arbitrary. It is found that in the vicinity of  $\alpha_1 = 1$  these distributions are characterized by realistic mass distributions (see Fig. 1). Then the conservation of  $Q$  is tested by an extensive study of numerical experiments of dissipationless galaxy formation. Partial conservation during, and especially after the collapse, is found (see Fig. 2).

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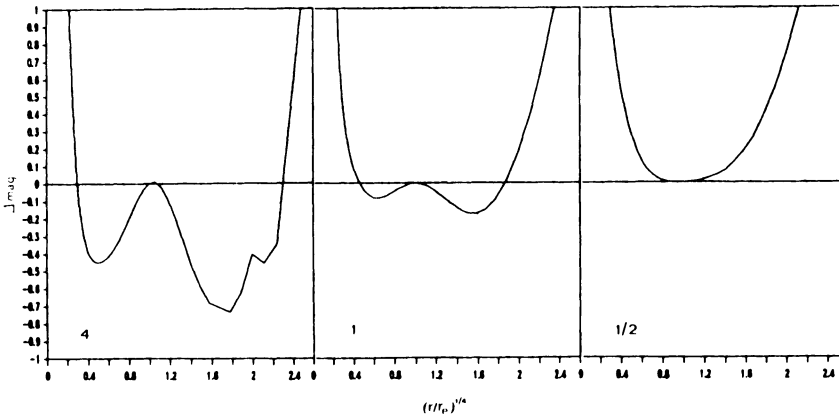


Figure 1. Residuals from the  $R^{1/4}$  law in magnitudes for the equilibrium models specified by (2) with  $\Psi = 18$  ( $\alpha_1 = 4, 1, 1/2$ ).

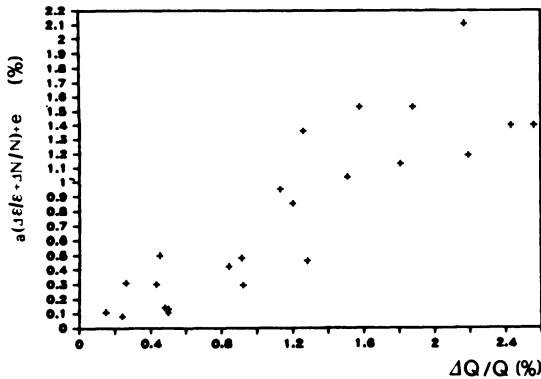


Figure 2. For a set of  $N$ -body dissipationless collapses a combination of various factors (total energy and particle number non-conservation and ellipticity) is found to correlate with the non-conservation of  $Q$  (see (1) with  $\alpha_1 = 1/2$ ), expressed in percent/crossing time.