CAN STELLAR MASS BE MEASURED BY ASTEROSEISMOLOGY?

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ABSTRACT. Theoretical calculations show that the detailed pattern of frequencies from stellar oscillations can in principal produce data that can determine stellar masses independent of other input data.

The determination of stellar properties through the analysis of stellar oscillations analogous to the solar 5-minute oscillations relies on the comparison of various aspects of the oscillation spectrum to theory. The spectral features which can be used will naturally depend on the quality of the data. Each of the modes of oscillation observed in the integrated starlight can be classified according to the degree ℓ of the spherical harmonic $Y^m_\ell(\theta,\phi)$ which describes the pattern of the motion over the stellar surface and the radial order n which gives the number of nodes in the radial direction. I denote the frequencies by $\nu_{\ell,n}$. At the crudest level the data will only contain an indication of the regular spacing between modes. Each spectral peak will consist alternately of the nearly degenerate pairs: $(\nu_{0,n+1}, \nu_{2,n})$ and $(\nu_{1,n+1}, \nu_{3,n})$. The primary measurement from this data is $\Delta\nu = \nu_{\ell,n+1} - \nu_{\ell,n}$. As is known from pulsation theory, this spacing depends on the average density of the star $\bar\rho=3M/(4\pi R^3)$. Quantitatively, Ulrich (1986) has shown that $\Delta\nu \cong q\ (\bar\rho/\bar\rho_\odot)^{1/2}$ where $\bar\rho_\odot$ is the average solar density and the constant q can be chosen equal to 133.8 μHz to accurately represent the calculations for a reasonable range of mass and age. **Fortunately q is not sensitive to ℓ/H, X or Z; increases of 0.1, 0.01 and 0.001 in ℓ/H, X and Z lead to changes in q of - 0.05, +0.44 and - 0.08 μHz respectively.**

If either the mass or radius are known from some other data then the q relation can be used to get the other quantity. Variants of this technique have been used by Guenther and Demarque (1985) and Demarque, Guenther and Van Altena (1985) in comparing their models to the observations of oscillations in ε Eri by Noyes et al (1984) and observations of α Cen and Procyon by Gelly et al (1986). If for example the mass of the star is known then the radius can be derived with a relative uncertainty equal to one third the relative uncertainty in the mass. Radii derived this way may be more accurate than could be found from other means. Ages can be estimated from this method if the composition is known since a given evolutionary track will pass through

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the correct radius at only one age. Unfortunately, a critical parameter -the helium abundance - is poorly known and its uncertainty renders the comparison imprecise.

The age determination method described by Ulrich (1986) is independent of the match to the stellar radius and largely independent of the uncertainties in the helium abundance. This method requires better frequency resolution than the methods based on q since it depends on the resolution of the degeneracy between the $\ell = 0$ and 2 and the $\ell = 1$ and 3. In addition, the stellar mass enters the determination weakly. For example if the quantity $\delta_{13} = (\nu_{1,n+1} - \nu_{3,n})/\Delta\nu$ has the value 0.13 then one interpretation has the stellar mass equal to $0.8M_{\odot}$ and an age equal to 1.8×10^9 yr while another interpretation has the mass equal to $1.2M_{\odot}$ and an age equal to 6.6×10^9 yr. Although these extreme interpretations require distinctly differing spectral characteristics, at a reduced level some ambiguity will remain.

One of the remarkable features of the Asteroseismology method is the potential richness of the data. Although the quantities $\Delta\nu$, δ_{02} and δ_{13} are useable in a direct fashion and can be interpreted in a straightforward manner by using the asymptotic theory by Tassoul (1980), the data clearly contains additional information about the star. This additional information will have to be found in deviations from the asympototic theory such as the curvature observed in echelle diagrams like those published by Grec, Fossat and Pomerantz (1980), Harvey and Duvall (1984) and Henning and Scherrer (1986). Unfortunately, the task of interpreting the deviations is complicated by the fact that current solar models do not agree with the observations.

In spite of the uncertainty due to the discrepancy between the models and the observations, it is worthwhile to determine what properties of the frequency spectrum might be sensitive to the stellar masses and ages. The asteroseismic problem is more restricted than the helioseismic problem since the data are limited to just the $\ell = 0$ to 3 modes. Consequently, the ν dependence in the asteroseismic problem becomes the primary source of information. Using a simple power series representation of the frequencies similar to that adopted by Ulrich (1986) I write

$$\nu = \nu_0 + \Delta\nu(n - n_0) + b_2(n - n_0)^2 + b_3(n - n_0)^3 \qquad (1)$$

where n_0 is an arbitrary zero point that I have taken as 18 for $\ell = 0$ and 1 and as 17 for $\ell = 2$ and 3. The subscript ℓ has been suppressed in equation (1). A representation like that in equation (1) is advantageous since application to observed spectra should be possible by means of a simple least square fit to the data. A key issue is the range in n that should be included in the least square fit. Another question is the number of terms to be included beyond b_3 and whether b_3 itself should be retained. Tradeoffs over these issues are obviously necessary since a wider range in n will possibly require more terms and better data. The higher values of n cause a theoretical problem because the chromospheric mode can interact with the interior mode and perturb the interior frequencies. In fact this effect may be present in the observations which show that for the sun the higher n modes have a broader eigenfrequencies than the lower n modes. The S shaped curve noted in the observations by Henning and Scherrer (1986) and predicted theoretically may be a function of the detailed stellar structure. In order to study that property of the models and avoid the chromospheric perturbations, I have taken the range in n to be from 11 to 22 and have used just the four terms shown in equation (1).

The coefficients b_2 and b_3 resulting from the least square fits to the theoretical frequencies computed for the models discussed by Ulrich (1986) are shown in figure 1. The value of ℓ used for the figure was 0 although a very similar pattern was found for all ℓ values. These coefficients are divided by $\Delta\nu$ in order to make the results dimensionless. The pattern shown in figure 1 especially for $b_2/\Delta\nu$ is quite encouraging. If theory and observation could be adequately developed, this type of result could easily remove the ambiguity in stellar mass. Unfortunately when the procedure is applied to the solar data from Henning and Scherrer (1986) the resulting value of $10^3 b_2/\Delta\nu$ is -0.5 to - 0.6 instead of the value of +1.2 required by figure 1. It is unclear whether this substantial discrepancy is a result of a deficiency in the model or an observational problem with identifying frequencies when they become weak.

In summary, it appears possible for asteroseismic data to provide adequate information to permit the determination of stellar masses without reference to other sources of data. Unfortunately, the method also appears to be sensitive to details of the model calculation so that calibration with primary mass determinations will almost certainly be required.

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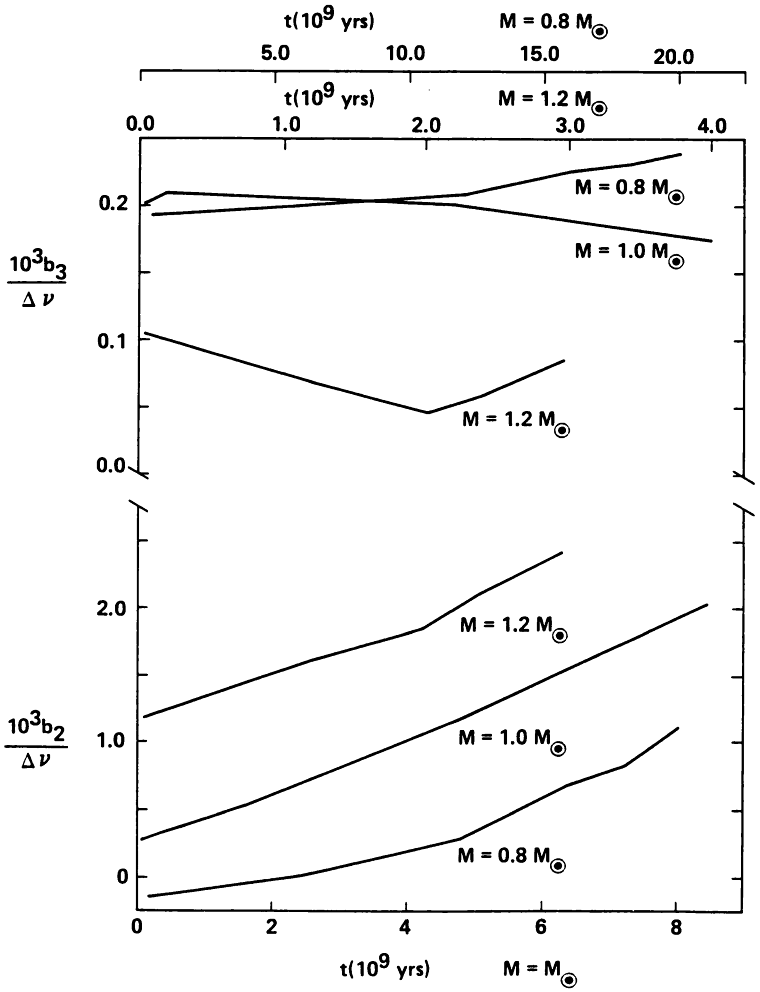

Figure 1 Fitting coefficients for the dependence of $\ell = 0$ modes of oscillation on time for three different evolutionary sequences of mass 0.8, 1.0 and 1.2 $M_\odot$. Only those modes with $11 < n < 22$ were included in the least square fit. The time scales for the three models were adjusted so that the initial change of central hydrogen abundance with position along the x axis of the plot is the same. Actual times for the three models are shown along the top and bottom axes.