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MORAVA MODULES AND THE $K(n)$-LOCAL PICARD GROUP

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The chromatic approach to homotopy theory naturally leads to the study of the $K(n)$-local stable homotopy category [3, Section 7]. In this thesis we study this category in three different ways.

The first is to closely study the category of Morava modules, as defined in [2]. It turns out that this category is equivalent to the category of complete $E^{\vee}_{*}E$-comodules. Using this, we develop a theory of (relative) homological algebra for Morava modules. We use this to give an explicit identification of the E_2-term of the $K(n)$-local E_n-based Adams spectral sequence (see [1, Appendix A]. This turns out to be related to work of Goerss *et al.* [2], as constructed as part of their resolution of the $K(2)$-local sphere at the prime 3.

The second part is computational in nature; we show that for a large class of groups the Tate spectrum E^{tG}_{p-1} always vanishes, where $G \subset \mathbb{G}_{p-1}$ is a closed subgroup of the Morava stabiliser group. Such a result was previously known to be true $K(p-1)$-locally, but we show that it holds even before this. We use this to deduce some self-duality results for the $K(p-1)$-local Spanier–Whitehead dual of the homotopy fixed point spectrum E^{hG}_{p-1}.

In the final chapter we study the $K(n)$-local Picard group. In particular, we show that, when p is an odd prime, the subgroup κ_n of elements such that $E^{\vee}_{*}X \simeq E_*$ as continuous modules over the Morava stabiliser group is always a p-group, and decomposes as a direct product of cyclic groups. Then, specialising to the case $n = p-1$, we discuss the decomposition of the group of exotic elements, by studying the map from the Picard group of the $K(n)$-local category to the Picard group of E^{hG}_n-modules. We finish by explaining the connection to Gross–Hopkins duality, and

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outline an approach to constructing elements of κ_n when $n > 2$. In fact, this method already allows us to (independently) construct elements of κ_2 that are nonzero in I_2, the Gross–Hopkins dual of the sphere.

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