Mathematical Foundations of the Calculus of Probability, by J. Neveu. Translated by Amiel Feinstein, foreword by R. Fortet. Holden-Day, Inc., San Francisco, 1965. xiii + 223 pages. $10.45.

"In comparison with the French original, this translation has benefited by the addition of a section (IV.7) on sequences of independent random variables, as well as by certain additions to the Complements and Problems", (A. Feinstein, Translator's Preface). This is an excellent book by a brilliant mathematician, expertly and beautifully translated into English. As a text it is intended for study by advanced students (corresponding roughly to the first or second year of graduate work in probability or measure theory in the United States and Canada) and as a reference work for researchers. There is no doubt that it will succeed brilliantly in fulfilling these aims.

The book develops the mathematical foundations of the theory of random processes to the point where the reader should have no difficulty in further pursuing the subject in any direction he chooses. To this end, the theory of measure and integration is developed including all the theorems for construction of a probability measure by extension from an algebra to a δ-algebra, from a compact subclass to a semi-algebra, from finite product-spaces to infinite product-spaces. General random processes are discussed and results on their separability and measurability are proved. The theory of conditional expectations, martingales and submartingales are fully and elegantly discussed. The basic convergence and continuity properties of such processes are proved. The next subject is ergodic theory and Markov processes with general state spaces. Here the general theory is followed by the ergodic theory for such processes, where mean and individual ergodic theorems are proved and applications to stopping times are given.

Approximately 100 interesting and far-ranging problems are included and thus even decision theory and sufficient statistics are treated to some extent.

Summarizing, this excellent book should serve admirably for a course in advanced probability theory, and as a textbook for a high-level course on measure theory. It is also delightful reading material for anyone interested in this field and can be also used for self-study purposes by one who has had an elementary course in probability or measure theory.

M. Csörgő, McGill University

Étude du Schéma Fluide Parfait et des Équations de Mouvement dans les Théories Pentadimensionelles de Jordan-Thiry et de Kaluza-Klein, by Aline Surin. Gauthier-Villars, Paris, 1965. 137 pages.

Five-dimensional unitary field theories date back to 1921, when Kaluza discovered that the electromagnetic potentials, and their gauge invariance, could be formally included in the metric of a five-dimensional manifold possessing a one-parametric isometry, which was to be interpreted physically such that the orbital curves of the V_5 were to be identified with the world points of the four-dimensional manifold of physical experience, i.e., of space and time. Whereas in the original proposal of Kaluza the orbital curves were assumed to be geodesics, which is tantamount to setting the metric component g_{55} constant, Jordan and Thiry developed the theory on the assumption that g_{55} was an extra field variable, which called for a physical interpretation of its own. None of the five-dimensional theories has ever matured beyond the stage of a speculation, even though such outstanding physicists as Pauli, Jordan, and Einstein himself have worked on it.

In this monograph Mrs. Surin has developed within the framework of five-dimensional field theory both the equations of perfect fluids and the Einstein-Infeld-Hoffman approach to the ponderomotive equations. Her treatment emphasizes the formal aspects and deals fully with the Cauchy problems posed. She has little to say about any physical motivation. Approximation methods are developed and discussed quite well.

The work is organized into two main parts, the first being devoted to the Jordan-Thiry theory, the second to the Kaluza-Klein theory. Within the first part individual chapters are devoted to foundations, the perfect fluids, approximate solution of the (combined) field equations, and ponderomotive theory. The second part, which is the shorter of the two, contains chapters on fundamentals, and is devoted to a discussion of the approximation procedure through the second order. The bibliography contains references both to texts and classical papers and to recent developments, with emphasis on the post-war contributions of the French school.

Peter G. Bergmann, Syracuse University

<u>The Basic Laws of Arithmetic</u>, by Gottlob Frege (Translated, edited, and introduced by Montgomery Furth). University of California Press, Berkeley, 1965. lxiii + 144 pages. $5.00.

Everyone knows how Frege's monumental and pioneering work on the deduction of arithmetic from logic had its foundations cut away by the discovery of the Russell paradox. As a result his work itself has been undervalued. Here the most fundamental parts have been selected, with an extensive historical introduction. The translation is a model of how mathematical translations should be made.

H.A. Thurston, University of British Columbia

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