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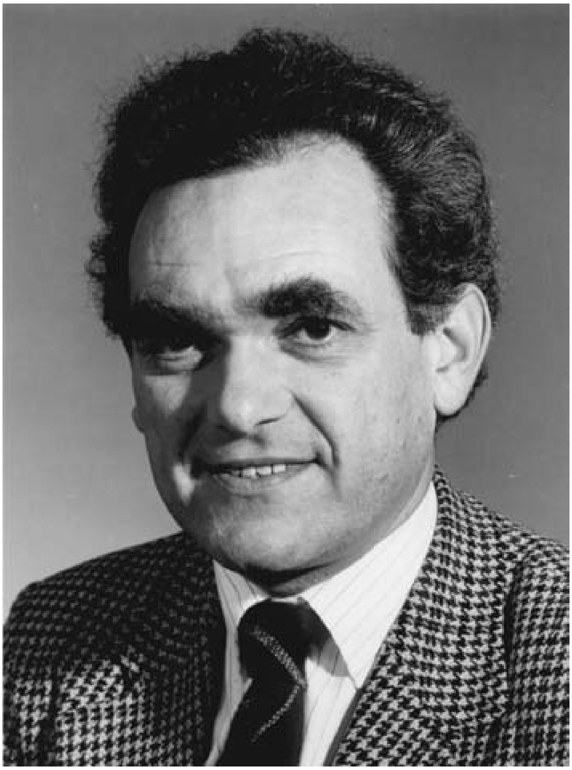
OBITUARY

BARRY EDWARD JOHNSON 1937–2002



Barry Johnson made major contributions to the theory of Banach algebras, by stimulating research on automatic continuity and cohomology in these algebras. His research on the continuous Hochschild cohomology of Banach and operator algebras led to major developments in these areas, and to a recognition of 'amenability' as more than simply a group-theoretic idea, but also one that is widely applicable in modern analysis.

1. *Education and administration*

Barry Johnson was born on 1 August 1937 in Woolwich, south-east London; the family moved when he was young to Surrey, where he attended Epsom County Grammar School for Boys. They emigrated to Hobart, Tasmania late in 1951, where Barry went to Hobart High School for the last two years of his school education. The parents and the two younger children of the family returned to England in February 1954, leaving him in Hobart in the care of a kind family. Barry's father was persuaded by the Headmaster of Hobart High School to leave him to attend the

University of Tasmania, where he was a student from March 1953 until late 1957. The degree structure was based on the old Scottish model of a three-year Ordinary degree (Mathematics with subsidiary Physics and Chemistry in Barry's case), followed by an additional year to give an Honours degree in Mathematics. Barry was an outstanding student at school and university. There was no academic tradition in the family: his father was a skilled toolmaker, and late in life a planning engineer, while his mother was a secretary at Warner Bros. While at the University of Tasmania, Barry completed his National Service during the vacations, which exempted him from National Service when he returned to England at the end of his Honours degree; he taught at a grammar school in Tamworth before going up to Cambridge. He was awarded a Rhondda Memorial Studentship, which supported him while studying for a PhD at Gonville and Caius College, Cambridge, supervised by John Williamson, starting in October 1958. Of this move to Cambridge, Barry wrote in 1978 that 'This was probably the luckiest thing that happened to me, because I was supervised by Professor J. H. Williamson who has a remarkable record as a supervisor. I met a number of research students who now have university teaching positions and also gained a lot of experience of life to transform me from a very dull bookish type to a more socially acceptable individual.' Barry also acknowledged his educational indebtedness to the Epsom and Hobart schools, and to the Department of Mathematics at the University of Tasmania. Several of Barry's early research papers evolved from his thesis, 'Centralisers in topological algebras', which was examined by J. H. Williamson and J. R. Ringrose.

After one-year lecturing posts at the Universities of California, Berkeley (1961–1962) and Yale (1962–1963), and two years at Exeter, he moved to the University of Newcastle upon Tyne in 1965. Except for periods on sabbatical leave in the USA, he remained at Newcastle upon Tyne as lecturer, reader (1968) and professor (1969). Barry enjoyed lecturing at all levels, from service teaching – even first-year agriculture students – to postgraduate students, and would initially describe himself to total outsiders as a 'teacher'. He supervised seven research students, of whom two are still currently active in mathematical research. At Newcastle upon Tyne he was at various times Head of Pure Mathematics, Head of the School of Mathematics and Dean of the Faculty of Science. He was active on many University Committees, and acted as the University representative on the Governing Body of the Newcastle Royal Grammar School for many years. On three occasions he was a visiting professor in the USA: Yale (1970–1971), UCLA and UC Santa Barbara (1990–1991), and University of California, Berkeley (1999). He was elected to the Royal Society in 1978.

Barry devoted considerable time and effort to administering mathematics, and to ensuring that its standards were maintained in the UK. He was on the Council of the London Mathematical Society, was its President for 1980–1982, and was editor of its Newsletter for four years. During the time of Barry's Presidency of the London Mathematical Society, there was controversy over the Society's position on the International Congress of Mathematicians in Warsaw while martial law was being enforced in Poland, where some well-known mathematicians were prisoners of the regime. Barry believed that the London Mathematical Society should be non-political, and that protest was an individual matter. He was a member of the Research Assessment Exercise Pure Mathematics panel in 1992, and chaired this panel in the 1996 Reseach Assessment Exercise. He served on the Science and Engineering Research Council mathematics committee, and chaired its review panel of the Isaac Newton Institute in 1993. Barry did his full share of external examining, both of PhD students and final Honours degree courses in the UK. In the late sixties, Barry Johnson made an outstanding contribution to running the North British Functional Analysis Seminar (NBFAS), which was founded by F. F. Bonsall (Edinburgh), J. R. Ringrose (Newcastle) and J. H. Williamson (York). This was the first UK inter-university seminar designed to support and encourage research in a single specialist area of mathematics across several universities. He ran the Seminar for the first nine years, and was instrumental in widening its scope from the initial three member institutions, so that there are currently twelve members, including some as far afield as Cambridge and Belfast.

Barry's first marriage in Reno in 1961 to Jennifer Munday ended in separation in 1977, though he maintained a close relationship with his daughter and two sons. Shortly after this, he met Margaret Jones and lived happily with her for the rest of his life, becoming close to her three children by her previous marriage; they married in 1991 in Santa Barbara. Barry and Margaret are fondly remembered by many mathematicians for their parties, and for the support that they provided to visitors to the Department and at conferences.

Barry was a voracious reader, reading anything and everything: histories, novels, biographies and travel books. The other important thing in his life was walking regularly. He 'walked through' many of his mathematics problems, and in any crisis would go off alone and walk. Barry also liked listening to music, and doing his own repairs and alterations to their house and their cottage on the Scottish borders. Substantial DIY and woodwork were hobbies epitomized by a perfect scale model of their own house, which he made as a dolls' house for his daughter. His work and his private life were separate, with friends in one area often unaware of his accomplishments in the other.

Barry is warmly remembered for his sharp and lively humour, involving repartee and a keen eye for the ridiculous, which made him entertaining company for family and colleagues.

Barry developed cancer in February 2000 and struggled bravely with it through a major operation, radiotherapy and chemotherapy, with his life becoming physically more difficult. His research activity continued to the end of his life. A recently completed paper [**71**] was found among his effects, and joint research was still in progress (see Section 6). He was able to attend some of the talks at an international conference held in Newcastle upon Tyne in June 2001 to mark his scientific contributions and his retirement. He clearly enjoyed seeing colleagues and old friends at the conference. He died in St Oswald's Hospice, Newcastle upon Tyne, on Sunday, 5 May 2002.

2. *Mathematical research*

Barry Johnson's deepest and most influential work has been in two areas: the automatic continuity of isomorphisms, derivations and intertwining operators on Banach algebras, and in the Hochschild cohomology of Banach algebras, C*-algebras and von Neumann algebras. The discussion below is subdivided under the headings: 'automatic continuity', 'Hochschild cohomology and amenability of Banach algebras', 'perturbations in Banach algebras', 'derivations', and 'centralizers and other research'. Of these, his work on cohomology has had the most important impact on international research.

2.1. *Automatic continuity* [**4, 6-9, 14, 15, 18, 20, 21, 23, 38, 48, 51**]

The idea behind automatic continuity is that certain natural algebraic equations satisfied by a linear operator between Banach spaces combine with the overall structure of the spaces to force the continuity of the linear operator. Johnson presumably became interested in these problems during his year in Yale (1962–1963), where C. E. Rickart had worked on the the uniqueness of norm problem; his first automatic continuity paper was submitted from Yale. Following the work of M. Eidelheit ⟨**10**⟩ on $B(H)$ in 1940 and I. Gelfand ⟨**13**⟩ on commutative Banach algebras in 1941, C. E. Rickart ⟨**29**⟩ conjectured in 1950 that there was a unique Banach algebra topology on a semisimple Banach algebra. At about the same time, I. Kaplansky had obtained many of the same results as C. E. Rickart ⟨**29**⟩, but did not publish his results. Answering this question amounts to showing that an isomorphism between semisimple Banach algebras is continuous. In 1967, Johnson solved this well-known problem, using the purely algebraic Jacobson theory of irreducible representations with delicate inductive estimates in analysis and a gliding hump argument [**15**]. This solution stimulated research in automatic continuity, which evolved rapidly for a few years after his proof, and then developed steadily thereafter. Jointly with A. M. Sinclair, he solved a conjecture of Kaplansky on the automatic continuity of derivations on semisimple Banach algebras by modifying the techniques used in the uniqueness of norm proof [**18**]. The overall strategy is similar in both theorems. Provided that there are no finite-dimensional irreducible representations of the algebra, the techniques work for purely additive derivations, yielding the information that the derivation has to be real linear. Simpler alternative proofs were subsequently given for both these theorems (see ⟨**7**, Chapter 5⟩ and ⟨**5**⟩). Before this breakthrough, Johnson published a series of results on automatic continuity, developing the theory and gaining experience with the methods. These results covered the following cases: centralisers, or multipliers, of Banach algebras [**6**], linear operators intertwining with a pair of suitable continuous linear operators [**7**], operators leaving certain translation-invariant subspaces invariant [**8**], homomorphisms of $B(X)$ for certain Banach spaces X (see [**9**]) and derivations of commutative semisimple Banach algebras [**14**]. The abstract case of intertwining operators was motivated by homomorphisms in the introduction of the paper, and was a motivation for [**21**]. Johnson returned to automatic continuity after 1969 on three occasions [**38, 47, 50**], though by then the main thrust of automatic continuity had branched away from his approach, due to research of H. G. Dales ⟨**6**⟩ and J. Esterle ⟨**11, 12**⟩ on the existence of discontinuous homomorphisms from $C(\Omega)$. For a full discussion of automatic continuity and Johnson's contributions to it, see the monograph *Banach algebras and automatic continuity*, by H. G. Dales ⟨**7**⟩.

2.2. *Hochschild cohomology and amenability of Banach algebras* [**17, 22, 24, 28, 30, 33, 36, 45, 54-56, 61-65, 67, 70-72**]

In the late sixties, cohomological ideas were starting to become important in Banach algebras and von Neumann algebras, with the few scattered results sometimes not worded in cohomological terms, and with no coherent theory. Johnson's American Mathematical Society Memoir, *Cohomology of Banach algebras* [**28**], laid the foundation of an extensive theory of (continuous) Hochschild cohomology of Banach algebras based on Hochschild cohomology for algebras ⟨**17**⟩. The Hochschild cohomology theory of Banach algebras is analogous to the classical

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algebraic Hochschild cohomology of associative algebras, with Banach modules and continuous multilinear operators in place of ring modules and standard cochains. However, many of the traditional applications of ring cohomology cannot be extended to Banach algebras without severe restrictions: examples are extensions and lifting derivations. The reasons for this are that the image of a continuous linear operator may not be closed, and that a closed linear subspace of a Banach space does not in general have a closed complement. Johnson emphasized that for a successful theory, the topological properties of the module should match the topological properties of the algebra and the averaging required in the calculations: Banach modules should be used over Banach algebras if one is averaging over compact groups; dual Banach modules should be used if one is averaging over amenable groups, and dual normal modules should be used over von Neumann algebras. The idea that an average is just a suitable fixed-point result, which goes back to von Neumann's construction of Haar measure on a locally compact group, is highlighted by the chapter title '$\mathcal{H}^1(L^1(G), X^*)$ and fixed points' in [**28**]. Averaging in cohomology is to be interpreted in this wide sense.

Here is the application of the averaging idea in its simplest form, of showing that $\mathcal{H}^1(A, X) = 0$ for a bimodule X over a complex unital algebra A, when A contains a discrete multiplicative group G that suitably generates A. This comes down to proving that for a derivation D from A to X, there is an x in X so that $D(a) = ax - xa$ for all a in A; a derivation is a linear map on A satisfying $D(ab) = aD(b) + D(a)b$ for all $a, b \in A$. Here are the constructions of x in the finite-dimensional, purely algebraic situation, and when G is a discrete amenable group. If G is a finite group and A is the linear span of G, then

$$x = |G|^{-1} \sum_{g \in G} g^{-1} D(g),$$

where $|G|$ is the number of elements in G. If G is an amenable group with invariant mean m_g on $l^\infty(G)$, if the Banach algebra A is the closed linear span of G with G norm bounded, and if X is a dual Banach A-module with predual Banach A-module X_*, then

$$\langle x, \xi \rangle = m_g(\langle g^{-1} D(g), \xi \rangle)$$

for all ξ in X_* . In cohomological terms, this final result says that for these Banach algebras, $\mathcal{H}^1(A, X) = 0$ for all dual modules X over A. Johnson used this as the definition of an *amenable Banach algebra* [**28**, p. 60].

Johnson studied the amenability of closed ideals, of quotients, and of the Banach algebra of continuous linear operators on a Banach space [**28**]. He subsequently modified the idea of a *diagonal* in the cohomology of finite-dimensional algebras to provide an *approximate diagonal* and a *virtual diagonal* in Banach algebras, and related them to amenability [**30**]. The importance of the second cohomology group $\mathcal{H}^2(A, X)$ is clear in the papers [**28, 30, 33**]. By introducing the bounded group cohomology of a discrete group and showing that the second bounded cohomology group $\mathcal{H}^2_b(\mathbb{F}_2)$ of the free group $\mathbb{F}_2$ on two generators is non-zero, he deduced that $\mathcal{H}^2(l^1(\mathbb{F}_2), l^1(\mathbb{F}_2))$ is non-zero [**28**, p. 38]. Bounded group cohomology was discovered independently by M. Gromov, and was related to geometrical problems by him and his coworkers (see ⟨**14**⟩ for relevant references and discussion). Using harmonic analysis results on the Banach algebra $C(\mathbb{Z}_n)\hat{\otimes}C(\mathbb{Z}_n)$ and a map of N. Varopoulos ⟨**35**⟩, Johnson showed that if A is an amenable infinite-dimensional abelian Banach algebra, then there is a Banach A-module X with $\mathcal{H}^2(A, X)$ not zero [**30**].

While Johnson was creating the cohomology theory of general Banach algebras, there was a quite independent development of *norm continuous* and *weakly continuous* cohomology theories of von Neumann algebras and C*-algebras by R. V. Kadison and J. R. Ringrose ⟨**20**⟩. As a result of joint consultations, Johnson made fundamental contributions to the cohomology of von Neumann algebras and C*-algebras. Firstly, the fixed-point technique of [**28**, Chapter 3] was modified to show that $\mathcal{H}^1(N,N)=0$ for all von Neumann algebras N, and to study the first cohomology from the Banach algebra $l^1(G)$ into a uniformly convex Banach module over $l^1(G)$; see [**22**]. The result that $\mathcal{H}^1(N,N)=0$ or, equivalently, that derivations on a von Neumann algebra are inner, had been proved in 1966 by S. Sakai ⟨**32**⟩, following R. V. Kadison ⟨**18**⟩. Fixed-point techniques have played a role in all cases where $\mathcal{H}^n(N,N)$ has been shown to be zero ⟨**34**⟩.

More importantly for future research, Johnson was instrumental in showing that for a dual normal module X over a von Neumann algebra N, $\mathcal{H}^n(N,X)=\mathcal{H}^n_w(N,X)$ for all positive integers n, where $\mathcal{H}^n$ denotes norm continuous cohomology and $\mathcal{H}^n_w$ denotes normal cohomology [**24**]. In normal cohomology $\mathcal{H}^n_w$, all maps are required to be multilinear normal maps; that is, they are separately continuous in the weak topologies of the algebra and module. This result, together with an important technical lemma on relative M-multimodular normal cohomology with respect to a hyperfinite von Neumann subalgebra M of N, has been used in all subsequent attempts at proving that $\mathcal{H}^n(N,N)=0$ for a von Neumann algebra N (see ⟨**2**⟩ and ⟨**34**⟩ for references).

Until 1986 the cohomology group $\mathcal{H}^n(N,N)$ was known to be zero only for N a hyperfinite von Neumann algebra, and for one non-hyperfinite group von Neumann algebra M that Johnson had constructed to satisfy ingenious relations ensuring that $\mathcal{H}^2(M,M)=0$; see [**33**]. This single example was important in indicating that $\mathcal{H}^n(N,N)$ might be zero for a larger class of von Neumann algebras than the hyperfinite ones.

While Johnson was working on his Memoir [**28**] in 1970, A. Ya. Helemskii was working on a more abstract approach to the homology of Banach algebras (see ⟨**16**⟩). Johnson wrote to Helemskii in June 1970 to ensure that they were aware of one another's research, in a letter described by Helemskii as 'kind, generous and considerate'. Johnson's judgement on the correct level of abstraction and structure of the continuous cohomology theory of Banach algebras was excellent. He based his development on Hochschild's algebraic version using explicit complexes of continuous multilinear maps, rather than on a more abstract theory based on 'Ext' and 'Tor'. His approach took a middle route between the more abstract version of Helemskii ⟨**16**⟩, and the more concrete one of Kadison and Ringrose ⟨**20**⟩.

Based on Johnson's Memoir, these ideas on amenability were used, modified and extended by subsequent mathematicians to relate amenability in different situations to other natural good properties of the algebras. Examples of this are the work by A. Connes, showing that von Neumann amenability and injectivity are equivalent for von Neumann algebras, and that amenability implies nuclearity for C*-algebras ⟨**3, 4**⟩. Also building on Johnson's work, E. G. Effros studied amenability and virtual diagonals in von Neumann algebras ⟨**9**⟩, U. Haagerup showed that nuclearity implies amenability for C*-algebras ⟨**15**⟩, S. Popa published results on the amenable embedding of one type II_1 factor in another ⟨**24**⟩, and of the amenability of a completely positive map ⟨**25**⟩. Z.-J. Ruan gave results showing that the amenability of a locally compact group G is equivalent to the operator amenability of the Fourier algebra $A(G)$ $\langle\mathbf{30}\rangle$, and studied the amenability of Kac (or Hopf–von Neumann) algebras $\langle\mathbf{31}\rangle$.

After 1976, Johnson concentrated his research in cohomology on amenability and derivation questions, rather than the higher groups that were the centre of his earlier research. He showed that the Fourier algebra $A(G)$ of a compact group G is amenable if the set $\{d_\pi : \pi \in \hat{G}\}$ is bounded, where $\hat{G}$ is the set of equivalence classes of irreducible representations and d_π is the dimension of the irreducible representation π; see [**62**, Theorem 5.3]. He deduced this from the calculation that the minimal norm of an approximate diagonal in the Fourier algebra $A(G)$ of a finite group G is

$$\sum d_\pi^3 / \sum d_\pi^2,$$

where both summations are taken over the set of all equivalence classes $\hat{G}$ of irreducible representations of G. This was the main motivation that led Z.-J. Ruan to introduce operator amenability, and to show that the operator amenability of $A(G)$ characterizes the amenability of G, as mentioned above $\langle\mathbf{30}\rangle$.

A Banach algebra A is permanently weakly amenable if, for each positive integer n, each derivation from A into its nth iterated dual A^{*n} is inner $\langle\mathbf{8}\rangle$. In two of his last papers Johnson showed that $l^1(G)$ is permanently weakly amenable if G is a free group [**66**] or G is a hyperbolic group [**71**], completing results in $\langle\mathbf{8}\rangle$. The proof in both cases involves ergodic properties of the action of the the non-amenable group on a suitable measure space, which is the hyperbolic boundary ∂G of G here. The amenable group case follows from general results of Johnson's in [**28**].

2.3. *Perturbations in Banach algebras* [**39, 41, 43, 50, 52, 53, 59, 60**]

In the mid-seventies several mathematicians studied perturbations of products, representations, and other structures of Banach algebras and operator algebras (see [**39**] and $\langle\mathbf{28}\rangle$ for references to earlier work). In late 1975 Johnson [**39**], and independently I. Raeburn and J. L. Taylor $\langle\mathbf{28}\rangle$, proved essentially the same results on perturbations of the product, and of representations, of Banach algebras under the assumptions that certain Hochschild cohomology groups over the algebra are zero. Johnson's approach is an indication of the way in which he often thought out the results from the beginning in an independent way. The Raeburn–Taylor method involves proving an infinite-dimensional implicit function theorem $\langle\mathbf{28}$, Theorem 1$\rangle$, and showing that the various spaces in the definition of the cohomology are related to derivatives of certain maps between suitable manifolds. Johnson's approach is to tackle the results directly via the Banach contraction mapping theorem and careful inductive constructions, which with hindsight is just mimicking the proof of the implicit function theorem. Here is a special case of Johnson's [**39**, Theorem 2.1] and Raeburn–Taylor's $\langle\mathbf{28}$, Theorem 3$\rangle$.

THEOREM. *Let A be a Banach algebra, and let π denote the usual product on A as a bilinear operator from $A \times A$ to A. If $\mathcal{H}^2(A,A) = \mathcal{H}^3(A,A) = 0$, then there is an $\varepsilon > 0$ such that if ρ is another associative multiplication on A with $\|\pi - \rho\| < \varepsilon$, where the norm is that of a bilinear operator, then (A,ρ) is isomorphic to (A,π) via an invertible map T on A. Further, as $\|\pi - \rho\|$ tends to zero, $\|T - I\|$ tends to zero.*

The ε depends on the norms of inverse maps in the cohomology calculations. Unique to Johnson's paper are a technical open mapping lemma [**39**, Lemma 6.1] that enables one to weaken the hypotheses needed in some of the results in both papers (see the note added in print in ⟨**28**, p. 266⟩), and several natural examples. He shows that the algebra $B(X)$ of all bounded linear operators on a Banach space X and the algebra $C(\Omega)$ of all continuous complex-valued functions on a compact metrizable space Ω have $\mathcal{H}^n(A, A) = 0$ for all positive integers n, and thus satisfy the required hypotheses.

Johnson considered a different type of perturbation result in [**50, 52**], on almost multiplicative linear functionals. Here is a brief description. Let A and B be Banach algebras, and for each continuous linear operator T from A to B, let

$$T^{\vee}(a, b) = T(ab) - T(a)T(b)$$

for all a, b in A. The pair (A, B) is said to be an AMNM (*Almost Multiplicative maps are Near Multiplicative*) pair if, for each positive ε and K, there is a positive δ such that if T is a continuous linear operator from A into B with $\|T\| < K$ and $\|T^{\vee}\| < \delta$, then there is a multiplicative linear map T' from A into B with $\|T - T'\| < \varepsilon$. The main result of [**52**] is that if A is an amenable Banach algebra and B is a Banach algebra such that as a Banach B-module B is isomorphic to $(B_*)^*$ where (B_*) is a Banach B-module, then (A, B) is an AMNM pair. Other results of this type, together with examples and counter-examples, are studied in [**50**], [**52**] and [**59**].

2.4. *Derivations* [**25, 29, 34, 40, 42, 68, 69**]

Results on derivations run throughout Johnson's research from 1969 (see [**14**]) until his last research [**69**]. The derivation results that involve automatic continuity are discussed in that section, and those that are close to general cohomological problems are discussed there. Here are some others.

In 1972, jointly with S. K. Parrott, Johnson considered two closely related problems associated with a von Neumann algebra N on a Hilbert space H; see [**29**]. Let $\mathcal{K}(H)$ denote the algebra of compact linear operators on H. Is a derivation from N into $\mathcal{K}(H)$ inner? If b is a bounded linear operator on H and $xb - bx$ is in $\mathcal{K}(H)$ for all x in N, is b in $N' + \mathcal{K}(H)$, where N' is the commutant of N? They answered 'yes' to both questions for von Neumann algebras N that do not 'contain certain intractable type II_1 factors as direct summands'. They solve the problem firstly for commutative von Neumann algebras, and then reduce their other cases to this. The full problem was solved by S. Popa ⟨**23**⟩ in 1987 (see also ⟨**26**⟩ and ⟨**27**⟩).

One of the intriguing problems that Johnson solved in 2000 concerned 'local derivations' from a C*-algebra A into a Banach A-module X. He showed that if T is a continuous linear operator from A into X such that for each a in A there is a derivation D_a from A into X with $T(a) = D_a(a)$, then T is a derivation from A into X; see [**68**]. The hard step of the proof involves the particular case where A is equal to $C_0(\mathbb{R})$ and X is equal to the dual of the Banach module $C_0(\mathbb{R})\hat{\otimes}C_0(\mathbb{R})$. In 1990, R. V. Kadison ⟨**19**⟩ had proved this result for A a von Neumann algebra and X a dual module over it.

If D is a derivation on the group algebra $L^1(G)$ of a locally compact group G, is there a bounded regular measure μ on G such that $D(x) = x * \mu - \mu * x$ for all x in $L^1(G)$? This was one of the problems that motivated Johnson to look at the amenability of Banach algebras. Johnson returned to this problem in 2001, having answered it in the affirmative for amenable, SIN and some matrix groups in [**28**, Proposition 4.1]. Using detailed properties of connected Lie groups, he showed that the answer is also 'yes' for G a connected locally compact group [**69**]. In [**56**], Johnson showed that if G is a locally compact group, then each derivation from $L^1(G)$ into $L^\infty(G)$ is inner. The proof is interesting in that the usual linear averaging argument used to show that derivations are inner is replaced by a supremum of a suitable set [**56**, Lemma], and so by a non-linear process.

2.5. *Centralizers and other research* [**1-3, 5, 10-13, 16, 19, 26, 27, 31, 32, 37, 44, 46, 47, 57, 58, 66**]

Johnson's Ph.D. thesis and several early papers were on centralisers of topological algebras [**1, 2, 10**], with a detailed study of the complications that can occur in algebras whose left and right structures are rather different. Centralisers are now called 'multipliers', due to their connections with harmonic analysis. They have also been helpful in explaining the structure of non-unital C*-algebras. Throughout Johnson's career, he returned at intervals to problems associated with the Banach algebra $L^1(G)$ and its associated multiplier algebra $M(G)$ of bounded Borel measures on the locally compact group G; see [**3, 12, 13, 47, 66**].

At the time when there seemed a faint possibility of a hierarchy of $*$-algebras C*, AW* and W* (that is, von Neumann) algebras, a class of algebras called QW* algebras was introduced in 1965 by G. A. Reid. These contained the W*-algebras, and were contained in AW*-algebras. In 1967 Johnson showed [**11**] that QW*-algebras are, in fact, AW*-algebras.

He and Simon Wassermann gave an example of the failure of the slice map criterion in C*-algebras [**44**]. His remaining research is scattered over different areas of functional analysis and Banach algebra theory.

2.6. *General impact of Barry Johnson's research*

The importance of Johnson's research may be seen by looking at the remarks, contents and lists of references of the following lecture notes and books on Banach algebras, automatic continuity and Hochschild cohomology of various kinds of Banach algebras, witten, respectively, by F. F. Bonsall and J. Duncan (1973) ⟨**1**⟩, A. M. Sinclair (1976) ⟨**33**⟩, A. Ya. Helemskii (1989) ⟨**16**⟩, T. W. Palmer (1994) ⟨**21, 22**⟩, A. M. Sinclair and R. R. Smith (1995) ⟨**34**⟩, and H. G. Dales (2001) ⟨**7**⟩. These books indirectly contain a detailed assessment of some of Johnson's research.

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References

⟨**1**⟩ F. F. Bonsall and J. Duncan, *Complete normed algebras*, Ergeb. Math. Grenzgeb. 80 (Springer, New York/Heidelberg, 1973).

⟨**2**⟩ E. Christensen, F. Pop, A. M. Sinclair and R. R. Smith, 'Hochschild cohomology of factors with property Γ', *Ann. of Math.* 158 (2003) 635–659.

⟨**3**⟩ A. Connes, 'Classification of injective factors', *Ann. of Math.* (2) 104 (1976) 73–115.

⟨**4**⟩ A. Connes, 'On the cohomology of operator algebras', *J. Funct. Anal.* 28 (1978) 248–253.

⟨**5**⟩ H. G. Dales, 'Automatic continuity: a survey', *Bull. London Math. Soc.* 10 (1978) 129–183.

⟨**6**⟩ H. G. Dales, 'A discontinuous homomorphism from $C(X)$', *Amer. J. Math.* 101 (1979) 647–734.

⟨**7**⟩ H. G. Dales, *Banach algebras and automatic continuity*, London Math. Soc. Monogr. (N.S.) 24 (Oxford Univ. Press, New York, 2000).

⟨**8**⟩ H. G. Dales, F. Ghahramani and N. Gronbaek, 'Derivations into iterated duals of Banach algebras', *Studia Math.* 128 (1998) 19–54.

⟨**9**⟩ E. G. Effros, 'Amenability and virtual diagonals for von Neumann algebras', *J. Funct. Anal.* 78 (1988) 137–153.

⟨**10**⟩ M. Eidelheit, 'On isomorphisms of rings of linear operators', *Studia Math.* 9 (1940) 97–105.

⟨**11**⟩ J. Esterle, 'Injection de semi-groupes divisibles dans des algèbres de convolution et construction d'homomorphismes discontinus de $\mathcal{C}(K)$', *Proc. London Math. Soc.* (3) 36 (1978) 59–85.

⟨**12**⟩ J. Esterle, 'Sur l'existence d'un homomorphisme discontinu de $\mathcal{C}(K)$', *Proc. London Math. Soc.* (3) 36 (1978) 46–58.

⟨**13**⟩ I. Gelfand, 'Normierte Ringe', *Rec. Math.* [*Mat. Sb.*] N.S. 9 (51) (1941) 3–24.

⟨**14**⟩ R. I. Grigorchuk, 'Some results on bounded cohomology', *Combinatorial and geometric group theory* (*Edinburgh, 1993*), London Math. Soc. Lecture Note Ser. 204 (Cambridge Univ. Press, Cambridge, 1995) 111–163.

⟨**15**⟩ U. Haagerup, 'All nuclear C*-algebras are amenable', *Invent. Math.* 74 (1983) 305–319.

⟨**16**⟩ A. Ya. Helemskii, *The homology of Banach and topological algebras*, translated from the Russian by Alan West, Math. Appl. (Soviet Series) 41 (Kluwer Acad. Publ., Dordrecht, 1989).

⟨**17**⟩ G. Hochschild, 'On the cohomology groups of an associative algebra' *Ann. of Math.* (2) 46 (1945) 58–67.

⟨**18**⟩ R. V. Kadison, 'Derivations of operator algebras', *Ann. of Math.* (2) 83 (1966) 280–293.

⟨**19**⟩ R. V. Kadison, 'Local derivations', *J. Algebra* 130 (1990) 494–509.

⟨**20**⟩ R. V. Kadison and J. R. Ringrose, 'Cohomology of operator algebras II. Extended cobounding and the hyperfinite case', *Ark. Mat.* 9 (1971) 55–63.

⟨**21**⟩ T. W. Palmer, *Banach algebras and the general theory of ∗-algebras,* vol. 1, *Algebras and Banach algebras*, Encyclopedia Math. Appl. 49 (Cambridge Univ. Press, Cambridge, 1994).

⟨**22**⟩ T. W. Palmer, *Banach algebras and the general theory of ∗-algebras,* vol. 2, *∗-algebras*, Encyclopedia Math. Appl. 79 (Cambridge Univ. Press, Cambridge, 2001).

⟨**23**⟩ S. Popa, 'The commutant modulo the set of compact operators of a von Neumann algebra', *J. Funct. Anal.* 71 (1987) 393–408.

⟨**24**⟩ S. Popa, 'Classification of amenable subfactors of type II', *Acta Math.* 172 (1994) 163–255.

⟨**25**⟩ S. Popa, 'On Connes' joint distribution trick and a notion of amenability for positive maps', *Enseign. Math.* (2) 44 (1998) 57–70.

⟨**26**⟩ S. Popa and F. Rădulescu, 'Derivations of von Neumann algebras into the compact ideal space of a semifinite algebra', *Duke Math. J.* 57 (1988) 485–518.

⟨**27**⟩ F. Rădulescu, 'Vanishing of $H^2_w(M, K(H))$ for certain finite von Neumann algebras', *Trans. Amer. Math. Soc.* 326 (1991) 569–584.

⟨**28**⟩ I. Raeburn and J. Taylor, 'Hochschild cohomology and perturbations of Banach algebras', *J. Funct. Anal.* 25 (1977) 258–266.

⟨**29**⟩ C. E. Rickart, 'The uniqueness of norm problem in Banach algebras', *Ann. of Math.* (2) 51 (1950) 615–628.

⟨**30**⟩ Z.-J. Ruan, 'The operator amenability of $A(G)$', *Amer. J. Math.* 117 (1995) 1449–1474.

⟨**31**⟩ Z.-J. Ruan, 'Amenability of Hopf von Neumann algebras and Kac algebras', *J. Funct. Anal.* 139 (1996) 466–499.

⟨**32**⟩ S. Sakai, 'Derivations of W*-algebras', *Ann. of Math.* (2) 83 (1966) 273–279.

⟨**33**⟩ A. M. Sinclair, *Automatic continuity of linear operators*, London Math. Soc. Lecture Note Ser. 21 (Cambridge Univ. Press, Cambridge 1976).
⟨**34**⟩ A. M. Sinclair and R. R. Smith, *Hochschild cohomology of von Neumann algebras*, London Math. Soc. Lecture Note Ser. 203 (Cambridge Univ. Press, Cambridge, 1995).
⟨**35**⟩ N. Th. Varopoulos, 'Tensor algebras and harmonic analysis', *Acta Math.* 119 (1967) 51–112.

Works of Barry Edward Johnson

These papers are listed in the order used in Barry Johnson's last CV, with the addition of:
[**42**], which was omitted from his CV,
[**71**], which was found as a complete unchecked typescript in his papers, and
[**72**], which was in progress when he died.

1. 'An introduction to the theory of centralisers', *Proc. London Math. Soc.* (3) 14 (1964) 299–320.
2. 'Centralisers on certain topological algebras', *J. London Math. Soc.* (2) 39 (1964) 603–614.
3. 'Isometric isomorphisms of measure algebras', *Proc. Amer. Math. Soc.* 15 (1964) 186–188.
4. 'Continuity of homomorphisms of topological algebras', *Proc. Cambridge Phil. Soc.* 60 (1964) 171–172.
5. 'An addition theorem for the associated Legendre functions', *J. Reine Angew. Math.* 218 (1965) 17–22.
6. 'Continuity of centralisers on Banach algebras', *J. London Math. Soc.* (2) 41 (1966) 639–640.
7. 'Continuity of linear operators commuting with continuous linear operators', *Trans. Amer. Math. Soc.* 128 (1967) 88–102.
8. 'Continuity of transformations which leave invariant certain translation invariant subspaces', *Pacific J. Math.* 20 (1967) 223–230.
9. 'Continuity of homomorphisms of algebras of operators', *J. London Math. Soc.* (2) 42 (1967) 537–541.
10. 'Centralisers and operators reduced by maximal ideals', *J. London Math. Soc.* (2) 43 (1968) 231–233.
11. 'AW*-algebras are QW*-algebras', *Pacific J. Math.* 23 (1967) 231–233.
12. 'The Silov boundary of $M(G)$', *Trans. Amer. Math. Soc.* 134 (1968) 289–296.
13. 'Symmetric maximal ideals in $M(G)$', *Proc. Amer. Math. Soc.* 18 (1967) 1040–1044.
14. 'Continuity of derivations on commutative algebras', *Amer. J. Math.* 91 (1969) 1–10.
15. 'The uniqueness of the (complete) norm topology', *Bull. Amer. Math. Soc.* 73 (1967) 537–539.
16. 'A commutative semi-simple annihilator Banach algebra which is not dual', *Bull. Amer. Math. Soc.* 73 (1967) 407–409.
17. 'The Wedderburn decomposition of Banach algebras with finite dimensional radical', *Amer. J. Math.* 90 (1968) 866–876.
18. (with A. M. Sinclair), 'Continuity of derivations and a problem of Kaplansky', *Amer. J. Math.* 90 (1968) 1067–1073.
19. 'Separate continuity and measurability', *Proc. Amer. Math. Soc.* 20 (1969) 420–422.
20. 'Continuity of homomorphisms of algebras of operators II', *J. London Math. Soc.* (2) 1 (1969) 81–84.
21. (with A. M. Sinclair), 'Continuity of linear operators commuting with continuous linear operators', *Trans. Amer. Math. Soc.* 146 (1969) 533–540.
22. (with J. R. Ringrose), 'Derivations of operator algebras and discrete group algebras', *Bull. London Math. Soc.* 1 (1969) 70–74.
23. 'Continuity of operators commuting with quasinilpotent operators', *Indiana Univ. Math. J.* 20 (1971) 913–915.
24. (with R. V. Kadison and J. R. Ringrose), 'Cohomology of operator algebras III, reduction to normal cohomology', *Bull. Soc. Math. France* 100 (1972) 73–96.
25. 'Norms of derivations of $\mathcal{L}(X)$', *Pacific J. Math.* 38 (1971) 465–469.
26. (with A. L. Shields), 'Hyperinvariant subspaces of operators on the space of complex sequences', *Michigan Math. J.* 19 (1972) 189–191.
27. (with R. A. Hirschfeld), 'Spectral characterisation of finite dimensional algebras', *Proc. Konikl. Nederl. Akad. Wet.*, Ser. A 75 (1975) 19–23.
28. 'Cohomology in Banach algebras', *Mem. Amer. Math. Soc.* 127 (1972) 1–96.
29. (with S. K. Parrott), 'Operators commuting with a von Neumann algebra modulo the set of compact operators', *J. Funct. Anal.* 11 (1972) 39–61.
30. 'Approximate diagonals and cohomology of certain annihilator Banach algebras', *Amer. J. Math.* 94 (1972) 685–698.
31. 'Automorphisms of commutative Banach algebras', *Proc. Amer. Math. Soc.* 40 (1973) 497–499.
32. 'Some examples in harmonic analysis', *Studia Math.* 48 (1973) 181–188.

33. 'A class of II_1 factors without property P but with zero second cohomology', *Ark. Mat.* 12 (1974) 53–159.
34. (with J. P. Williams), 'The range of a normal derivation', *Pacific J. Math.* 58 (1975) 105–122.
35. 'Banach algebras: introductory course', *Algebras in analysis* (Academic Press, London, 1975) 63–83.
36. 'Introduction to cohomology in Banach algebras', *Algebras in analysis* (Academic Press, London, 1975) 84–100.
37. (with C. V. Coffman and J. J. Schaffer), 'The structure of translation invariant memories: two addenda', *J. Differential Equations* 20 (1976) 270–281.
38. 'Norming $C(\Omega)$ and related algebras', *Trans. Amer. Math. Soc.* 220 (1976) 37–58.
39. 'Perturbations of Banach algebras', *Proc. London Math. Soc.* (3) 34 (1977) 439–458.
40. (with C. A. Akemann), 'Derivations of non-separable C*-algebras', *J. Funct. Anal.* 33 (1978) 311–331.
41. 'Stability of Banach algebras', *Proc. IV Prague Topology Symposium*, Lecture Notes in Math. 609 (Springer, Berlin/Heidelberg/New York, 1977) 109–114.
42. 'Characterization and norms of derivations on von Neumann algebras', *Algebres d'operateurs* (Sem., Les Plans-sur-Bex, 1978), Lecture Notes in Math. 725 (Springer, Berlin/Heidelberg/New York, 1979) 228–236.
43. 'A counterexample in the perturbation theory of C*-algebras', *Canad. Math. Bull.* 25 (1982) 311–316.
44. (with S. Wassermann), 'The failure of the slice map criterion', *J. Operator Theory* 9 (1982) 91–94.
45. 'Low dimensional cohomology of Banach algebras', *Operator algebras and applications*, part 2 (Kingston, Ont., 1980), Proc. Sympos. Pure Math. 38 (ed. R. V. Kadison, Amer. Math. Soc., Providence, RI, 1982) 253–259.
46. Presidential address: 'Non commutative generalisations of mathematics', *Bull. London Math. Soc.* 14 (1982) 465–471.
47. 'A proof of the translation invariant form conjecture for $L^2(G)$', *Bull. Science Math.* (2) 107 (1983) 301–310.
48. 'Continuity of homomorphisms of G modules', *Pacific J. Math.* 120 (1985) 111–121.
49. 'Perturbations of semigroups', *Quart. J. Math. Oxford* (2) 36 (1985) 315–324.
50. 'Approximately multiplicative functionals', *J. London Math. Soc.* (2) 34 (1986) 489–510.
51. 'Continuity of generalised homomorphisms"', *Bull. London Math. Soc.* 19 (1987) 67–71.
52. 'Approximately multiplicative maps between Banach algebras', *J. London Math. Soc.* (2) 37 (1988) 294–316.
53. 'Perturbations of multiplication and homomorphisms', *Deformation theory of algebras and structures and applications*, Proceedings of the NATO Advanced Study Institute (Kluwer, Dordrecht, 1988) 565–582.
54. 'Derivations from $L^1(G)$ into $L^1(G)$ and $L^\infty(G)$', *Proceedings of the Luxemburg International Symposium on Harmonic Analysis*, Lecture Notes in Math. 1359 (Springer, Berlin, 1987) 191–198.
55. 'Weak amenability of group algebras of connected complex semisimple Lie groups', *Proc. Amer. Math. Soc.* 111 (1991) 77–185.
56. 'Weak amenability of group algebras', *Bull. London Math. Soc.* 23 (1991) 281–284.
57. 'Weakly compact homomorphisms between Banach algebras', *Math. Proc. Cambridge Phil. Soc.* 112 (1992) 157–163.
58. 'Weakly compact homomorphisms from group algebras', *Proc. Amer. Math. Soc.* 119 (1993) 1249–1258.
59. 'Pseudo regular elements and the auxiliary multiplication they induce', *Pacific J. Math.* 159 (1993) 61–73.
60. 'Near inclusions for subhomogeneous C*-algebras', *Proc. London Math. Soc.* (3) 68 (1994) 399–422.
61. (with N. Grønbæk and G. A. Willis), 'Amenability of Banach algebras of compact operators', *Israel J. Math.* 87 (1994) 289–324.
62. 'Two commutative amenable non-symmetric Banach algebras'. *J. Australian Math. Soc.* (Ser. A) 59 (1995) 225–231.
63. 'Non-amenability of the Fourier algebra of a compact group', *J. London Math. Soc.* (2) 50 (1994) 361–374.
64. 'Symmetric amenability and the non-existence of Lie and Jordan derivations', *Math. Proc. Cambridge Phil. Soc.* 120 (1996) 455–473.
65. 'Higher dimensional weak amenability', *Studia Math.* 123 (1997) 117–134.
66. 'Harmonic functions on nilpotent groups' *Integral Equations Operator Theory* 40 (2001) 454–464.
67. 'Permanent weak amenability of group algebras of free groups', *Bull. London Math. Soc.* 31 (1999) 569–573.

68. 'Local derivations on C*-algebras are derivations' *Trans. Amer. Math. Soc.* 353 (2000) 313–325.
69. 'The derivation problem for group algebras of connected locally compact groups', *J. London Math. Soc.* (2) 63 (2001) 441–452.
70. (with M. C. White), 'A non-weakly amenable augmentation ideal', in preparation.
71. 'Derivations into iterated duals of group algebras and similar modules', found in Barry Johnson's files; to be submitted.
72. (with F. Gourdeau and M. C. White), 'The cyclic and simplicial cohomology of $l^1(\mathbf{N})$', in preparation.

School of Mathematics
The University of Edinburgh
King's Buildings
Mayfield Road
Edinburgh EH9 3JZ

Allan Sinclair