# ON THE EXCITATION OF SOLAR FIVE-MINUTE OSCILLATIONS

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ABSTRACT.   A simple prescription for the dynamics of convection
perturbed by stellar pulsation is used in an estimation of the growth
rates of solar five-minute modes.  Convection appears to enhance the
excitation of the modes, and the maximum in the growth rate versus
frequency found previously when oscillatory convective perturbations
were ignored is still present.

## 1.   INTRODUCTION

The principal cause of the excitation of the solar five-minute
oscillations of the sun is still an unresolved issue.  Either the modes
are intrinsically overstable, or they are excited by nonlinear
interactions with other motions;  the possibilities are reviewed in
these proceedings by Chitre.  Whatever the process, however, it is
essential to our understanding of the phenomenon, and to any
quantitative estimate of the amplitudes, that the intrinsic growth
rates be known.
     Growth rates of five-minute oscillations of realistic model solar
envelopes have been estimated before.  Ulrich (1970)  studied
high-degree modes, as did Ando and Osaki (1975, 1977) who used the
Eddington approximation to model radiative transfer but ignored the
modulation of the heat and momentum fluxes due to turbulence.  They
found instability throughout much of the k-ω plane where
oscillations had been observed.  Subsequently, Gough (1980) found
instability of radial modes from calculations that took account of the
modulation of the convective heat flux and Reynolds stresses by a
mixing-length approach (Baker and Gough, 1979), but which treated
radiative transfer in a crude way.  However, Berthomieu et al. (1980),
using a similar prescription, found nonradial p modes with l=200 and
600 to be stable.  Antia et al. (1981, 1982) have shed light on that
result:  using a prescription for convective fluxes based on diffusion
formulae, they too found high-degree nonradial modes to be stable when
the turbulent Prandtl number is unity (which appears to be the value
that most nearly corresponds to formulae used by Berthomieu et al.),
but when smaller values, chosen to produce linearized modes of

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convective instability that seemed to accord best with observation, were used, they found that convection actually contributed to the instability of the modes.

The computations of Antia et al. (1982) were an attempt to model the oscillations realistically, for they employed the Eddington approximation to radiative transfer to estimate heat-flux perturbations and in addition took some account of the fluctuations in the convective fluxes. However, their treatment of convection was quite rudimentary. This paper is a step towards rectifying that situation by what we hope is an improvement to the treatment of the dynamics of the convection.

## 2.  TIME-DEPENDENT CONVECTION FOR NONRADIAL OSCILLATIONS

We have adopted a simple prescription of convection based on the idea of diffusive mixing, yet which we hope embodies much of the dynamics of apparently more sophisticated approaches. It is in a sense an amalgam of the ideas embodied in the discussions of Unno (1967, 1977; see also Unno et al., 1979), Gough (1977) and Antia et al. (1982).

In a spherically symmetrical nonpulsating star the convective heat flux $F_c$ , for example, is in the radial direction, and according to local mixing-length prescriptions its magnitude $F_c$ may be written $F_c = K_t\beta$ , where $\beta$ is the magnitude of the superadiabatic temperature gradient $\beta$ and $K_t (\beta)$ is a turbulent conductivity which depends on $\beta$ and, of course, the local mean state of the fluid. We assume that this equation can be generalized to the vector equation $F_c = K_t (\beta)\beta$ when spherical symmetry is broken; thus we presume that the turbulent conductivity tensor is isotropic.

We assume this approximation to be valid for pulsating stars too, and set $F_c = F_{co} + F_{c1}$ , where the suffix 0 refers to the static state and the suffix 1 to the Lagrangian perturbation. We can now write

$$F_{c1} = K_{t1}\beta_o + K_{to}\beta_1. \tag{1}$$

Since $K_{t1}$ is a scalar, it may be determined by identifying the vertical component of $F_{c1}$ with the expression obtained from a theory of convection in radially pulsating stars. The turbulent viscous stress tensor can be calculated in the same spirit, though we have not carried that through in the calculations reported here.

Our generating convection prescription, valid for radial pulsations, is a hybrid of the approaches of Unno (1967) and Gough (1977). We set

$$\frac{F_{c1}}{F_{co}} = \frac{\rho_1}{\rho_o} + \frac{C_{p1}}{C_{po}} + \frac{W_1}{W_o} + \frac{\theta_1}{\theta_o} \equiv \frac{K_{t1}}{K_{to}} + \frac{\beta_1}{\beta_o} \tag{2}$$

where $\rho$ and $c_p$ are density and specific heat at constant pressure, and W and $\theta$ the amplitudes of the vertical component of velocity and the temperature fluctuation in a dominant convective eddy, which are computed from their approximate equations of motion [Equations (4.1) and (4.2) from Gough (1977)]. Perturbing the equations of motion is straightforward except for the mixing-length l and the eddy anisotropy parameter $\Phi$, which is the ratio of the trace of the Reynolds stress

tensor to its $(r,r)$ component (where r denotes the radial direction). For these we adopt Unno's (1977) simple accounting for eddy creation and annihilation, yielding

$$\frac{l_1}{l_0} = \frac{1}{1+i\sigma}\left[\frac{H_1}{H_0} - i\sigma\left(\frac{2r_1}{r_0} + \frac{\rho_1}{\rho_0}\right)\right] \tag{3}$$

$$\frac{\Phi_1}{\Phi_0} = \frac{2i\sigma(\Phi_0-1)}{(1+i\sigma)\Phi_0}\left(\frac{3r_1}{r_0} + \frac{\rho_1}{\rho_0}\right) \tag{4}$$

where $\sigma$ is the ratio of the pulsation frequency $\omega$ to the convective growth rate, H is the pressure scale height and r is the Lagrangian radius co-ordinate. These expressions take due account of the differential compression and dilation throughout the star; aside from the sign convention used for frequency, Equation (3) would reduce to Unno's equation if pulsations were homologous.

## 3. ACOUSTIC OSCILLATIONS

Aside from the treatment of convection, p-mode oscillations of a solar model were computed in the manner described by Antia et al. (1982) using a turbulent viscosity $\nu_t = W_0 l_0/(1+\tilde{\sigma}^2)$ , where $\bar{\tilde{\sigma}} = \omega l_0/\pi W_0$. In table I we present frequencies and stability coefficients for a selection of modes with degrees $l = 1$ and $l = 100$. More extensive results are illustrated on a k-$\omega$ diagram by Chitre (these proceedings). Nearly all the modes are unstable, with the maximum growth rates occurring somewhat below 4mHz. Included in the table are growth rates computed with the perturbation $\underset{\sim}{F}_{c1}$ suppressed. There is a substantial reduction in growth rate, suggesting in common with the previous discussion by Antia et al. (1982) that the modulation of the convective heat flux can destabilize the oscillations.

TABLE I  Stability coefficients of some five-minute modes

| | $l=1$ | | | $l=100$ | |
|---|---|---|---|---|---|
| $\omega/2\pi$ | $\eta_c$ | $\eta_r$ | $\omega/2\pi$ | $\eta_c$ | $\eta_r$ |
| 1.74 | 0.07 | 0.01 | 1.83 | 0.18 | .0.02 |
| 2.42 | 0.48 | 0.04 | 2.68 | 1.22 | 0.18 |
| 3.52 | 1.17 | 0.29 | 3.42 | 1.96 | 0.47 |
| 3.94 | 1.26 | 0.26 | 3.87 | 1.94 | 0.44 |
| 4.35 | 1.00 | -0.04 | 4.30 | 1.55 | 0.03 |
| 4.63 | 0.47 | -0.55 | | | |
| 4.77 | -0.10 | -0.97 | 4.72 | 1.66 | -1.18 |

The stability coefficients $\eta$ are the ratios of the growth rate to the frequency, with the convention that $\eta > 0$ implies overstability. The suffices c and r refer to calculations in which the fluctuations in the convective heat flux were included and ignored respectively.

## 4. DISCUSSION

The results presented here provide additional evidence that solar five-minute oscillations might be overstable.  It must be appreciated, however, that there are several obvious deficiencies in our calculations.  Aside from the uncertainties inhering in the basic mixing-length prescription, which at present we are unable to assess, we have not yet incorporated fluctuations in the turbulent viscosity and we have ignored the pressure component in the Reynolds stress. Moreover, our formalism is local, and therefore fails to account for the dynamical communication across large eddies which leads, in particular, to artificial rapid spatial oscillations in the thermal component of the oscillation eigenfunctions (e.g. Baker and Gough, 1979; Gonczi and Osaki, 1980).  Those oscillations have not been adequately resolved in the greatest depths of the convection zone in most of the computations we have carried out; however, a highly resolved test computation of a particular case hardly changed the growth rate, confirming suspicions that there is considerable cancellation in their integrated effect.
   Another potentially serious source of error concerns the treatment of radiative transfer.  Christensen-Dalsgaard and Frandsen (1983) have shown how to modify the Eddington approximation to provide oscillations in tolerable agreement with those obtained from more accurate treatments of the transfer equation.  They also demonstrated that the common practice of ignoring the deviation of the mean radiative intensity from the Planck function at the top of the convection zone of the static model, which applies to our work, supplies an artificial source of mode excitation.  We plan to report in the future on some of these matters.

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