Mathematical Notes.

A Review of Elementary Mathematics and Science.

PUBLISHED BY

THE EDINBURGH MATHEMATICAL SOCIETY.

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No. 1. **April 1909.**

Graphical Treatment of Geometrical Progression.—The idea of the sum to infinity of a geometrical progression admits of a simple but very effective illustration already very much in vogue among teachers.

We take a line OA two units long, and from it cut off in succession segments

$$OX_1 = 1,\ X_1X_2 = \tfrac{1}{2},\ X_2X_3 = \tfrac{1}{4},\ X_3X_4 = \tfrac{1}{8},\ \text{etc.}$$

The beginner sees vividly

(i) that no matter how many segments we cut off, there is always some part of the original line left; so that the sum of any number of terms of the G.P.

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \dots$$

is less than 2;

(ii) that if we mark any point y between O and A, as near to A as we please, then after a sufficient number of cuts the point of section falls between y and A; so that the sum of a sufficient number of terms of the G.P.

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \dots$$

falls short of 2 by as little as we please.

The purpose of this note is to point out an almost equally luminous illustration of the general case when the common ratio is any proper fraction, positive or negative.

The construction is conveniently explained in terms of coordinates, referred to axes Ox, Oy.

The G.P. being

$$a + ar + ar^2 + \dots$$

let P_1 be the point (a, ar) and X_1P_1 the ordinate of this point.

Draw X_1A parallel to the bisector of the angle xOy (the line through X_1 of gradient 1), and let X_1A meet OP_1 at A.

Between X_1A and OA draw the lines

$$P_1Q_1,\ Q_1P_2,\ P_2Q_2,\ Q_2P_3,\ P_3Q_3,\ \dots$$

parallel to Ox and Oy alternately.

Since the gradient of X_1A is 1, and that of OA is r, we have

$$P_1Q_1 = X_1P_1$$
$$= ar,$$

and
$$Q_1P_2 = r \cdot P_1Q_1$$
$$= ar^2,$$

and similarly
$$P_2Q_2 = ar^2,$$
$$Q_2P_3 = P_3Q_3 = ar^3,$$

and so on.

Let the feet of the ordinates of P_2, P_3, ..., A be X_2, X_3, ..., B.

We see that s_n or $a + ar + \dots + ar^{n-1}$ is represented by OX_n.

It is also clear that the "path" $OX_1P_1Q_1P_2\dots$ converges to the apex A if r is numerically less than 1, and that in this case the "sum to infinity" is OB.

Now
$$BA = r \cdot OB,$$
or
$$OB - a = r \cdot OB,$$
so that
$$s_\infty = \frac{a}{1-r}.$$

(The construction will give s_n likewise.

For
$$X_nQ_{n-1} = OX_n - OX_1$$
$$= s_n - a,$$
$$Q_{n-1}P_n = ar^n,$$
so that
$$s_n - a + ar^n = r \cdot s_n$$
and
$$s_n = a\frac{1-r^n}{1-r}\Big).$$

Suppose parallels drawn to the axis of y,
in Fig. 1, PX between O and AB;
in Fig. 2, PX and P_1X_1 at equal distances on the two sides of AB.

It may be taken as evident that a point traversing the "path" will, after a certain definite number of the horizontal and vertical steps have been accomplished, remain always between PX and AB in the one case, between PX and P_1X_1 in the other.

We have thus a proof by intuition that if we fix on any number ϵ, however small, we can choose the integer n so that the sum of n or any greater number of terms of the G.P. (with r numerically less than 1) differs from $a/(1-r)$ by less than ϵ.

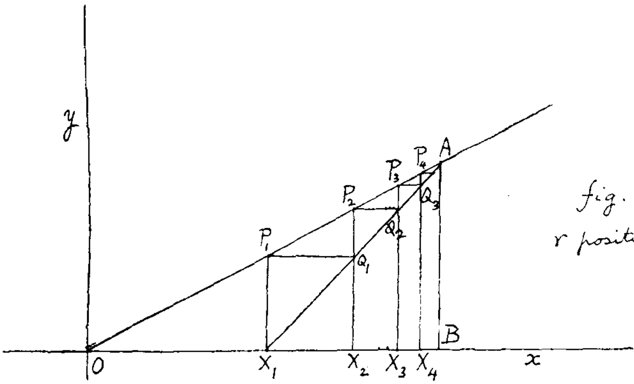

fig. 1, r positive.

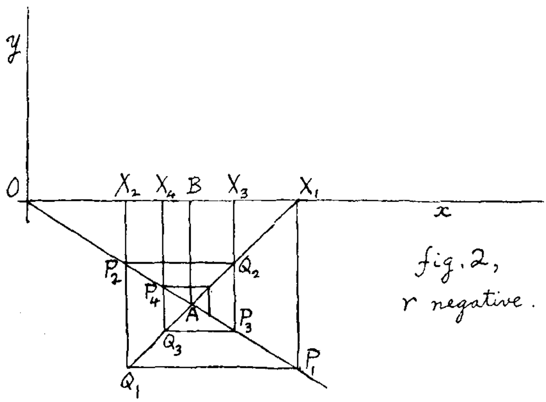

fig. 2, r negative.

JOHN DOUGALL.

Oral and Written Work in Arithmetic.—The following notes deal mainly with the Arithmetic of the primary school, but the subject under consideration is only a single aspect of a very wide question.

It is a common remark that Oral (or Mental) and Written Arithmetic should be as nearly as possible identical; that pupils ought to be able to do mentally with small numbers whatever they

https://doi.org/10.1017/S1757748900000372 Published online by Cambridge University Press