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A MATHEMATICAL ANALYSIS OF WIND EFFECTS ON A LONG-JUMPER

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Abstract

A perturbation model is used to predict the distance jumped by a long-jumper for a range of tailwinds and headwinds. The zeroth-order approximation is based on gravity being the only force present, the effects of drag and lift only being included in the first-order corrections. The difference in predicted distances produced by the zeroth and first-order approximations is less than 2% for headwinds or tailwinds up to $4\,\mathrm{ms}^{-1}$. Most increases or decreases due to wind are caused by changes in the run-up speed, and consequently the take-off angle and speed.

1. Introduction

Wind affects the performance of long-jumpers. The International Athletics Union acknowledges this by imposing a special rule relating to wind during long-jump performances. The average wind component parallel to the track is measured near the jumping pit during an interval encompassing the run-up and the jump. If this measurement exceeds $2\,\mathrm{ms}^{-1}$, no record-breaking jump is recognised.

Undoubtedly the wind will affect the aerodynamic forces on the jumper such as drag, lift and sideways forces, but as pointed out by Frohlich [5] there is also an effect on the take-off conditions. A number of authors (Brearley [2], Burghes et al. [3], Ward-Smith [11], [13], [14], Frohlich [5], de Mestre [4]) considered the effects of density change on Bob Beamon's world record jump which has now lasted more than 20 years without being surpassed. From these analyses it appears that in the absence of wind the drag effect during a

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long-jump would reduce the jump by no more than 1%. No analysis on long jumps has included other aerodynamic forces such as lift.

Previous calculations of wind effects on long-jumpers have been restricted to numerical solutions of the governing equations. This paper modifies the perturbation solution in de Mestre [4] for a long-jumper influenced by gravity and drag to include lift corrections, and then extends this work to analyse the effects of wind. Representative calculations for a long-jumper assisted by a tailwind or jumping into a headwind are presented.

2. Wind-free analysis

In the absence of wind, the velocity of the jumper relative to the ground is the same as the velocity relative to the air. The long-jumper is modelled as a projectile acted on by constant gravity plus the two components of the total aerodynamic force (drag and lift), which at jumpers' speeds are usually assumed to be proportional to the square of the air speed. With the origin at the position of the jumper's centre of mass at take-off, the $\overline{x}$ direction chosen as parallel to the run-up track and the $\overline{y}$ direction chosen as vertically upwards, the governing equation of motion is

$$m\frac{d^2\overline{\mathbf{r}}}{d\overline{t}^2} = -mg\hat{\mathbf{k}} - \frac{1}{2}\rho SC_D\overline{v}^2\hat{\boldsymbol{\tau}} + \frac{1}{2}\rho SC_L\overline{v}^2\hat{\mathbf{n}}. \tag{1}$$

Here ρ denotes the density of the air, S is a typical cross-sectional area of the jumper, m denotes the jumper's mass, $\overline{\mathbf{r}}$ denotes the position vector of the jumper at any time $\overline{t}$ during the jump, $\overline{\mathbf{v}}$ denotes the corresponding velocity vector, while C_D and C_L denote the drag and lift coefficients respectively. The unit vectors $\hat{\mathbf{k}}$, $\hat{\boldsymbol{\tau}}$ and $\hat{\mathbf{n}}$ are respectively in the directions vertically upwards, parallel to the jumper's velocity, and perpendicular to the jumper's velocity but lying in the vertical plane through the athlete's centre of mass.

To include the aerodynamic forces more precisely would require knowledge of the drag and lift coefficients at each stage of the motion of the jumper through the air. In addition the typical area S is usually chosen as the projected area of the jumper in a plane normal to the jumper's velocity. This also changes during the long jump from a maximum value in the take-off position to a much smaller value just before landing. In this paper the usual assumption is made that SC_D and SC_L are each some average constant for the duration of each long jump.

Equation (1) relates only to the projectile motion part of the long jump—the aerial phase. The distance calculated for the jump consists of the aerial-phase distance ($\overline{x}_F$) travelled forward by the centre of mass from take-off to landing plus the distance of the centre of mass ahead of the take-off board at take-off and the distance of the centre of mass behind the landing point at landing.

The initial conditions for the projectile motion are taken as

$$\left.\begin{aligned} \frac{d\bar{\mathbf{r}}}{d\bar{t}} &= \mathbf{V} = [V\cos\alpha,\ V\sin\alpha] \\ \bar{\mathbf{r}} &= \mathbf{0}, \end{aligned}\right\} \tag{2}$$

when $\bar{t} = 0$. The problem can be solved more generally by including a lateral component U in the velocity of the jumper at take-off. However, there is no advantage athletically for a jumper to do this, as the jumping distance is always measured in the $\bar{x}$ direction; likewise there is no mathematical advantage, as the algebra is increased in complexity although the analysis is essentially the same. Therefore, although this analysis could be extended to include cross-winds and sideways aerodynamic forces, the advantages in information gained are marginal.

The solution of (1) subject to conditions (2) produces $\bar{\mathbf{r}}(\bar{t})$ from which $\bar{x}$ can be determined when $\bar{y} = -h$, the distance of fall of the centre of mass from take-off to landing.

The variables are nondimensionalised using

$$\bar{\mathbf{v}} = \mathbf{v}V, \qquad \bar{t} = tV/g, \qquad \bar{\mathbf{r}} = \mathbf{r}V^2/g, \tag{3}$$

and so the dimensionless forms of equations (1) and (2) are

$$\left.\begin{aligned} \ddot{\mathbf{r}} &= -\hat{\mathbf{k}} - \varepsilon_D v^2\hat{\boldsymbol{\tau}} + \varepsilon_L v^2\hat{\mathbf{n}} \\ \text{with } \dot{\mathbf{r}} &= [\cos\alpha,\ \sin\alpha], \quad \mathbf{r} = 0 \text{ when } t = 0 \end{aligned}\right\} \tag{4}$$

Here $\varepsilon_D = \rho S C_D V^2/(2mg)$, $\varepsilon_L = \varepsilon_D C_L/C_D$, and a dot denotes differentiation with respect to t. In terms of components in the x, y directions the equations (4) are

$$\left.\begin{aligned} \ddot{x} &= -\varepsilon_D\dot{x}v - \varepsilon_L\dot{y}v \\ \ddot{y} &= -1 - \varepsilon_D\dot{y}v + \varepsilon_L\dot{x}v \\ \text{with } \dot{x} &= \cos\alpha,\ \dot{y} = \sin\alpha,\ x = y = 0 \text{ when } t = 0. \end{aligned}\right\} \tag{5}$$

For most projectile problems this set of equations would usually have to be solved numerically, but for the long jump modelled using constant gravity and variable drag, de Mestre [4] obtained a perturbation solution based on $\varepsilon_D \ll 1$. Since ε_L is also small compared with unity the analysis is extended here to include lift effects. In Section 4 it will be shown that this no-wind analysis is extremely useful in obtaining the distance jumped when a wind is blowing. Thus with

$$\mathbf{r} = \mathbf{r}_0 + \varepsilon_D\mathbf{r}_1 + O(\varepsilon_D^2), \tag{6}$$

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the zeroth-order equations become

$$\ddot{x}_0 = 0, \qquad \ddot{y}_0 = -1,$$

with $\dot{x}_0 = \cos\alpha$, $\dot{y}_0 = \sin\alpha$, $x_0 = y_0 = 0$ when $t = 0$. The solutions are

$$x_0 = t\cos\alpha, \qquad y_0 = t\sin\alpha - \tfrac{1}{2}t^2, \tag{7}$$

and hence

$$\begin{aligned} v_0 &= \sqrt{\dot{x}_0^2 + \dot{y}_0^2} \\ &= \sqrt{\cos^2\alpha + (\sin\alpha - t)^2} \end{aligned} \tag{8}$$

With $\beta = C_L/C_D$ the first-order equations are therefore

$$\begin{aligned} \ddot{x}_1 &= -v_0\cos\alpha + \beta(t - \sin\alpha)v_0, \\ \ddot{y}_1 &= (t - \sin\alpha)v_0 + \beta v_0\cos\alpha \end{aligned}$$

with $\dot{x}_1 = \dot{y}_1 = x_1 = y_1 = 0$ when $t = 0$, where the results (7) and (8) have been used. The corresponding solutions are

$$\left.\begin{aligned} x_1 &= \beta g_1 - f_1, \\ y_1 &= \beta f_1 + g_1, \end{aligned}\right\} \tag{9}$$

where

$$\begin{aligned} g_1(t) = {} & \frac{1}{12}(t - \sin\alpha)v_0^3 + \frac{1}{12}\sin\alpha(\sin^2\alpha + \cos^2\alpha)^{\frac{3}{2}} - \frac{1}{3}t(\sin^2\alpha + \cos^2\alpha)^{\frac{3}{2}} \\ & + \frac{1}{8}\cos^2\alpha(t - \sin\alpha)v_0 + \frac{1}{8}\cos^2\alpha\sin\alpha(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} \\ & + \frac{1}{8}\cos^4\alpha\,\ell n|t - \sin\alpha + v_0| - \frac{1}{8}\cos^4\alpha\,\ell n|(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} - \sin\alpha| \end{aligned}$$

and

$$\begin{aligned} f_1(t) = {} & \frac{1}{2}t\{\cos\alpha\sin\alpha(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} \\ & - \cos^3\alpha\,\ell n|(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} - \sin\alpha|\} \\ & + \frac{1}{6}\cos\alpha\{v_0^3 - (\sin^2\alpha + \cos^2\alpha)^{\frac{3}{2}}\} \\ & + \frac{1}{2}\cos^3\alpha\{(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} - v_0\} \\ & + \frac{1}{2}\cos^3\alpha(t - \sin\alpha)\,\ell n|t - \sin\alpha + v_0| \\ & + \frac{1}{2}\cos^3\alpha\sin\alpha\,\ell n|(\sin^2\alpha + \cos^2\alpha)^{\frac{1}{2}} - \sin\alpha|. \end{aligned}$$

The expressions for f_1 and g_1 could be further simplified, but they are left in this form for ease of transference to equivalent results in Section 4.

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The time of flight (t_F) for the jumper is obtained by solving $\bar{y} = -h$, which in nondimensional form becomes

$$-hg/V^2 = t\sin\alpha - t^2/2 + \varepsilon_D y_1(t) + 0(\varepsilon_D^2). \quad (10)$$

Therefore writing

$$t_F = t_{0F} + \varepsilon_D t_{1F} + 0(\varepsilon_D^2)$$

and substituting into (10) yields

$$t_F = t_{0F} + \varepsilon_D y_1(t_{0F})/\sqrt{\sin^2\alpha + 2hg/V^2} + 0(\varepsilon_D^2) \quad (11)$$

where

$$t_{0F} = \sin\alpha + \sqrt{\sin^2\alpha + 2hg/V^2}.$$

The corresponding dimensionless horizontal distance is

$$\begin{aligned} x_F = {} & \cos\alpha\left\{\sin\alpha + \sqrt{\sin^2\alpha + 2hg/V^2}\right\} \\ & + \varepsilon_D\left\{y_1(t_{0F})\cos\alpha/\sqrt{\sin^2\alpha + 2hg/V^2} + x_1(t_{0F})\right\} + O(\varepsilon_D^2). \end{aligned} \quad (12)$$

The foregoing analysis applies to any long jump where $\varepsilon_D \ll 1$. Ward-Smith [12], [13] considered representative characteristics for a typical long jump. These included $g = 9.81\,\mathrm{ms}^{-2}$, $\rho = 1.20\mathrm{kg}m^{-3}, m = 70\mathrm{kg}, h = 0.5\mathrm{m}$ and $SC_D = 0.36$. He also included horizontal and vertical take-off velocity components as $10.1\,\mathrm{ms}^{-1}$ and $2.87\,\mathrm{ms}^{-1}$ respectively. These produce a take-off speed of $10.5\,\mathrm{ms}^{-1}$ which is much too high, since energy is lost at the take-off board and no jumper is capable of speeds before take-off approaching $11\,\mathrm{ms}^{-1}$. Consider therefore as typical take-off values $V = 8.74\,\mathrm{ms}^{-1}$, $\alpha = 20^\circ$ based on a series of measurements made by Lafortune [8] at the Australian Institute of Sport using force plates.

The value for SC_D has been estimated as 0.36 from measured values on sprinters, cyclists and speed-skaters quoted in Ward-Smith [11]. A value for SC_L also needs to be estimated. The lift to drag ratio (C_L/C_D) for ski-jumpers has been well documented (Krylov and Remizov [7], Ward-Smith and Clements [9]), and can be as high as 0.25, but it varies with angle of incidence and would never be this large for long-jumpers. To obtain some idea of the effect of lift, a representative value 0.04 is chosen for SC_L, yielding $\beta = 1/9$.

The calculated value for ε_D is 0.024 which is certainly small enough for the perturbation expansion to be valid. With the above representative values, the projectile part of the jump yields $\bar{t}_F = 0.747\,\mathrm{s}$ and $\bar{x}_F = 6.07\,\mathrm{m}$. Without drag and lift, the time of flight is $0.746\,\mathrm{s}$ and the value of $\bar{x}_F$ is $6.13\,\mathrm{m}$. When only lift is ignored, the calculations yield $\bar{t}_F = 0.745\,\mathrm{s}$ and

$\overline{x}_F = 6.06\,\text{m}$. The inclusion of drag reduces the time of flight by an almost negligible amount, while the range is reduced by 1%. On the other hand the inclusion of lift with drag gives a marginally larger flight time than in the gravity-only case, but adds only a negligibly small amount to the range for the drag case. Therefore for a no-wind analysis it seems sufficient for most purposes to use a gravity-only model.

3. Run-up and wind

When the wind blows, not only is the projectile part of the long-jump affected, but also the run-up, and hence the take-off speed and angle. Two authors have considered the variations to the speed due to wind but no one has considered the angle variations. Ward-Smith [13] obtains the influence of wind on maximum sprinting speed by numerically solving a second-order differential equation, based on the application of the first law of thermodynamics to the acceleration phase of sprinting. On the other hand, Frohlich [5] considers the power lost or gained to the wind and has to solve the cubic equation

$$10u_0^2(u_0 - u_1) = (u_1 - W_x)^2 u_1 - u_0^3$$

where u_0 is the athlete's speed just before take-off when no wind is blowing, and u_1 is the athlete's speed just before take-off when a wind W_x is blowing parallel to the track. A positive W_x denotes a tailwind and a negative W_x a headwind.

Measurements by Hay and Miller [6] show that the velocity varies dramatically over the last few strides. Over this interval the jumpers must be preparing for take-off and continually adjusting the length of their strides so that they do not take-off beyond the take-off board and register a foul jump. Measurements by Bartlet [1] and also Lafortune [8] show that there is a small downward component to the jumper's velocity just before take-off. Because of these extra complexities it was decided to simplify the analysis, and so the technique due to Frohlich [5] is used to produce the approach speed variations due to the wind given in Table 1.

However Frohlich [5], and also Ward-Smith [13], fail to take account of the decrease in speed during take-off. From the previously mentioned experiments conducted by Bartlet [1] and Lafortune [8] a representative ratio of speed just after take-off to speed just before take-off is 0.92 ± 0.04. Therefore, with $V = 0.92u_0$ it is seen that $V = 8.74\,\text{ms}^{-1}$ corresponds to an approach speed $u_0 = 9.5\,\text{ms}^{-1}$, which is a typical run-up speed without wind. Values of V corresponding to different wind strengths and directions are also included in Table 1.

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TABLE 1. Approach speed, take-off speed, take-off angle for different winds.

$W_x(\mathrm{ms}^{-1})$	-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$
$u_1(\mathrm{ms}^{-1})$	8.84	9.01	9.18	9.34	9.50	9.65	9.79	9.92	10.04
$V(\mathrm{ms}^{-1})$	8.13	8.29	8.44	8.60	8.74	8.88	9.01	9.13	9.24
$a(°)$	21.57	21.14	20.73	20.35	20.00	19.68	19.38	19.11	18.88

To determine explicitly the change in take-off angle α for different take-off speeds V, it would be necessary to conduct experiments on long-jumpers keeping all other conditions fixed except for wind strengths. Then the take-off speed and take-off angle could be measured, and a graph used to determine a relationship between these two characteristics. In the absence of such experiments it seems reasonable to postulate that if the wind is blowing in the $\bar{x}$-direction only, then the vertical component of impulse from the spring in the athlete's take-off foot is independent of the wind strength. This assumption says that $V \sin \alpha$ is a constant for the same athlete under different wind conditions.

Qualitative support for this assumption is provided by a comparison of the long jump and high jump take-off angles and speeds. In the long jump the take-off speed is near sprinting speed and the take-off angle is near 20°, whereas in the high jump the take-off speed is low and the take-off angle is near 60°. Using this assumption the values of α are calculated for the values of V due to different wind strengths, and are also included in Table 1.

Wind analysis

When a wind $\bar{\mathbf{w}}$ is blowing, the air speed of any projectile is given by

$$\bar{\mathbf{v}}^* = \bar{\mathbf{v}} - \bar{\mathbf{w}}. \tag{13}$$

The drag and lift effects will depend on $\bar{\mathbf{v}}^*$, and only on $\bar{\mathbf{v}}$ in the absence of wind. Therefore, with the addition of wind, the basic equation (1) for the projectile part of the long-jumper's motion becomes

$$m\frac{d^2\bar{\mathbf{r}}}{d\bar{t}^2} = -mg\hat{\mathbf{k}} - \frac{1}{2}\rho S C_D |\bar{\mathbf{v}}^*|^2 \hat{\boldsymbol{\tau}}^* + \frac{1}{2}\rho S C_L |\bar{\mathbf{v}}^*|^2 \hat{\mathbf{n}}^*, \tag{14}$$

where $\hat{\boldsymbol{\tau}}^*$ is a unit vector in the direction of $\bar{\mathbf{v}}^*$, and $\hat{\mathbf{n}}^*$ is a unit vector perpendicular to $\hat{\boldsymbol{\tau}}^*$ and lying in the vertical plane. The initial conditions (2) remain the same as before. Ward-Smith [12], [13], [14], and Frohlich [5] solved (14) numerically for $C_L = 0$, because they claimed that with wind present no analytical solution is possible.

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Noone and Mazumdar [9] produced an analytical solution of an approximate form of (14), again with $C_L = 0$, but they did not use this solution in any of their calculations. They simply reverted to a Runge-Kutta numerical solution of the original equations to draw their conclusions.

An analytical perturbation solution of (14) will now be obtained, and used directly to obtain estimates for the effects of wind.

Now if (13) is integrated, then the transformation

$$\bar{\mathbf{r}} = \bar{\mathbf{r}}^* + \int_0^{\bar{t}} \bar{\mathbf{w}}\, d\bar{t}, \tag{15}$$

where $\bar{\mathbf{v}}^* = d\bar{\mathbf{r}}^*/d\bar{t}$, shifts the origin to a co-ordinate system fixed in the moving air mass. If the variables $\bar{\mathbf{r}}^*$, $\bar{\mathbf{v}}^*$, $\bar{t}$, $\bar{\mathbf{w}}$ are nondimensionalised according to (3) then the transformed variations of equations (14) and (2) become

$$\left.\begin{aligned} \ddot{\mathbf{r}}^* &= -\hat{\mathbf{k}} - \varepsilon_D {v^*}^2 \hat{\boldsymbol{\tau}}^* + \varepsilon_L^* {v^*}^2 \hat{\mathbf{n}}^* - \frac{d\mathbf{w}}{dt} \\ \text{with } \mathbf{v}^* &= [\cos\alpha - W_x/V,\ \sin\alpha - W_y/V], \quad \text{and } \mathbf{r}^* = \mathbf{0} \end{aligned}\right\} \tag{16}$$

when $t = 0$, and the initial wind velocity components are given by $\bar{\mathbf{w}}(0) = [W_x, W_y]$.

When (4) and (16) are compared it is seen that they are essentially identical except for the term $\frac{d\mathbf{w}}{dt}$, the rate of change in wind velocity during the jump. A perturbation expansion is again used when $\mathbf{w}$ is known for all t, then $\mathbf{r}^*$ and hence $\mathbf{r}$ can be calculated. Although the technique is equally applicable for varying wind the simplest illustration of wind effect is to assume that the wind acts parallel to the run-up track and is constant for the whole time-of-flight interval of the long jump; that is to take $\mathbf{w} = [W_x, 0]$. Then the transformation (15) becomes

$$\bar{\mathbf{r}} = \bar{\mathbf{r}}^* + [W_x, 0]\bar{t}$$

and (16) reduces to

$$\left.\begin{aligned} \ddot{\mathbf{r}}^* &= -\hat{\mathbf{k}} - \varepsilon_D {v^*}^2 \hat{\boldsymbol{\tau}}^* + \varepsilon_L {v^*}^2 \hat{\mathbf{n}}^* \\ \text{with } \mathbf{v}^* &= [\cos\alpha - W_x/V,\ \sin\alpha], \quad \text{and } \mathbf{r}^* = \mathbf{0} \end{aligned}\right\} \tag{17}$$

when $t = 0$.

Equation (17) can be solved by perturbation expansions and yields x^*, y^* and t^* similar to results (7)–(12) but with $\cos\alpha$ replaced by $\cos\alpha - W_x/V$ everywhere.

Thus corresponding to (11) the time of flight with wind is

$$t_F = t_{0F} + \varepsilon_D y_1^*(t_{0F})/\sqrt{\sin^2 + 2hg/V^2} + 0(\varepsilon_D^2) \tag{18}$$

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where it is noted that

$$t_{0F} = \sin\alpha + \sqrt{\sin^2\alpha + 2hg/V^2}$$

is independent of the wind strength. Moreover the zeroth-order dimensionalised time

$$\bar{t}_{0F} = \left\{V\sin\alpha + \sqrt{V^2\sin^2\alpha + 2gh}\right\}/g$$

is constant for a particular jumper under different wind conditions because of the postulate concerning the constancy of $V\sin\alpha$. Therefore, the time of flight corrections for the presence of wind only emerge in the first-order approximations.

The expression for the distance jumped due to the projectile part of the long jump is

$$\bar{x}_F = \frac{V}{g}\Big[Vt_{0F}\cos\alpha + \varepsilon_D\{y_1^*(t_{0F})(V\cos\alpha - W_x)/\sqrt{\sin^2\alpha + 2gh/V^2} + Vx_1^*(t_{0F}) + y_1^*(t_{0F})W_x\sqrt{\sin^2\alpha + 2gh/V^2}\} + 0(\varepsilon_D^2)\Big]. \quad (19)$$

Again it is noted that the zeroth-order approximation is independent of the wind. This was to be expected since the wind only affects the drag and lift during the projectile part of the motion, and these are first-order effects for the typical parameters of a long jump. However, for a particular jumper the zeroth-order jump distance varies with change in wind strength because $V^2\cos\alpha$ varies.

For the representative characteristics in Section 2 applied to expressions (18) and (19) the results are calculated and displayed in Table 2. Also included for comparison are the jump distances for gravity plus drag, in which lift is neglected.

The addition of drag to the analysis reduces the time of flight for all values of the wind considered, but only by an amount of the order of 0.1%. The addition of lift reverses this trend, so that for all headwinds and the lesser tailwinds the model predicts that the jumper will be held up in the air just slightly longer than when drag and lift are neglected.

On examination of this aerial phase of the projectile jump distance, it is noted that the addition of lift has very little effect overall and only increases the distances slightly for the stronger headwinds and lower tailwinds. The inclusion of drag reduces the gravity-only distances by 11 cm for the strongest headwind considered down to 2 cm for the strongest tailwind. Since jumpers are mainly interested in the tailwind cases (because these give longer jumps) it appears that even for the presence of winds, a gravity-only analysis will suffice for the aerial-phase calculations.

TABLE 2. Calculated wind effects on time of flight and projectile jump distances.

	W (ms^{-1})	Time of Flight (s)			Projectile Jump Distance (m)		
		Gravity	Gravity + Drag	Gravity + Drag +Lift	Gravity	Gravity + Drag	Gravity + Drag +Lift
HEADWIND	−4	0.746	0.745	0.748	5.64	5.51	5.54
	−3	0.746	0.745	0.747	5.77	5.66	5.68
	−2	0.746	0.745	0.747	5.89	5.80	5.82
	−1	0.746	0.745	0.747	6.01	5.93	5.95
	0	0.746	0.745	0.747	6.13	6.06	6.07
TAILWIND	+1	0.746	0.745	0.747	6.24	6.18	6.19
	+2	0.746	0.745	0.746	6.34	6.30	6.30
	+3	0.746	0.745	0.746	6.44	6.40	6.40
	+4	0.746	0.745	0.746	6.52	6.50	6.50

Table 2 shows that the wind has a significant effect mainly because of changes in take-off values. The difference in jump length from a $4\,\text{ms}^{-1}$ tailwind to a $4\,\text{ms}^{-1}$ headwind can be almost 1 metre. The model predicts an increase in jump distance of 23 cm from a jump on a still day to a jump by the same athlete with a $2\,\text{ms}^{-1}$ tailwind (the allowable limit for records). This compares favourably with the 27 cm predicted by Ward-Smith [14] for a faster take-off speed.

FIGURE 3. Increment in jump distance due to wind effect on run-up.

$W(\text{ms}^{-1})$	−4	−3	−2	−1	0	+1	+2	+3	+4
Distance (m) with $V = 8.74$, $\alpha = 20°$	6.02	6.03	6.05	6.06	6.07	6.08	6.09	6.10	6.11
Run-up increment (m)	−0.48	−0.35	−0.23	−0.11	0	+0.11	+0.21	+0.30	+0.39

Finally, calculations were performed to determine the aerial phase distance if the take-off parameters had not been affected by the wind, and had remained at $V = 8.74$, $\alpha = 20$. Subtracting these from the corresponding distances with the correct take-off values gives an estimate of the increments due to wind effects on the run-up and take-off characteristics. These are shown in Table 3.

It seems clear from the mathematical analysis, as it has been to most jumpers for many years, that long-jumpers should generally forget about breaking records (limited to tailwinds less than $2\,\text{ms}^{-1}$) and jump when the tailwind is strongest.

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