7

Supersymmetry breaking

The fundamental relation

$$[Q, P^\mu] = 0$$

of the supersymmetry algebra tells us that if supersymmetry is exact, every bosonic state other than the vacuum state must have a corresponding fermionic partner with exactly the same energy, assuming that we can identify P^0 with the Hamiltonian. To see this, we simply note that if $|\mathcal{B}\rangle$ is a bosonic eigenstate of the Hamiltonian with eigenvalue E_B, we must have,

$$P^0(Q|\mathcal{B}\rangle) = QP^0|\mathcal{B}\rangle = E_B Q|\mathcal{B}\rangle,$$

so that $|\mathcal{F}\rangle \equiv Q|\mathcal{B}\rangle$ is a fermionic eigenstate of this same Hamiltonian, with the same energy E_B. Thus the only bosonic states which are *not* paired with a fermionic state are those that are annihilated by Q. States with non-vanishing four-momenta transform non-trivially under supersymmetry (and so, are not annihilated by Q), and the only candidate for an unpaired bosonic state is the vacuum state. For massive single particle states in the rest frame, this implies that in a supersymmetric theory bosons and fermions must come in mass-degenerate pairs.

This is, of course, experimentally excluded since we know, for instance, that there is no integer spin charged particle with the same mass as that of the electron. Supersymmetry must, therefore, be a broken symmetry. While we cannot exclude the possibility that SUSY is explicitly broken by soft terms, it is much more appealing to consider that, like electroweak gauge symmetry, SUSY is broken spontaneously.

As with bosonic symmetries, if the generator Q of a supersymmetry transformation does not annihilate the vacuum, then supersymmetry is spontaneously broken. In correspondence with Eq. (1.5), we then write the condition for supersymmetry *not* to be spontaneously broken as

$$\langle 0|\delta\mathcal{O}|0\rangle \equiv \mathrm{i}\langle 0|[\bar{\alpha}Q, \mathcal{O}]|0\rangle = 0, \quad (7.1)$$

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where, in field theory, the dynamical variable $\mathcal{O}$ is a field operator and $\delta\mathcal{O}$ is its variation under a supersymmetry transformation with a Grassmann parameter α. If we find a field operator such that its *variation* is non-zero in the ground state, then supersymmetry will be spontaneously broken. Just as familiar gauge symmetries may be broken by vacuum expectation values (VEVs) of either elementary or composite field operators, $\mathcal{O}$ may be either elementary or composite. In order for Poincaré invariance not to be spontaneously broken, $\delta\mathcal{O}$ must be a spinless operator. Since SUSY connects fields whose spins differ by 1/2, $\mathcal{O}$ must thus be a spinorial operator.

7.1 SUSY breaking by elementary fields

Up to this point, we have two classes of fields in a SUSY theory: the chiral scalar superfield and the curl superfield (or equivalently the gauge potential superfield). To identify potential order parameters for SUSY breaking, let us look at the transformation of their spinor components.

The variation of the spinor component of the chiral scalar superfield is

$$\delta\psi_{\rm L} = -\sqrt{2}\mathcal{F}\alpha_{\rm L} + \sqrt{2}\partial\!\!\!/\mathcal{S}\alpha_{\rm R}$$

while for that of a gauge superfield we have the variation of the spinor component as

$$\delta\lambda_A = -{\rm i}\gamma_5\alpha\mathcal{D}_A + \frac{1}{4}[\gamma_\nu, \gamma_\mu]F_A^{\mu\nu}\alpha,$$

(we may equivalently discuss the gauge potential superfield $\hat{\Phi} \ni (V^\mu, \lambda, \mathcal{D})$ with similar results). Since Poincaré invariance requires,

$$\langle 0|\partial_\mu\mathcal{S}|0\rangle = \langle 0|F^{\mu\nu}|0\rangle = 0,$$

the condition for SUSY to be spontaneously broken is,

$$\langle 0|\mathcal{F}_i|0\rangle \neq 0 \quad \text{or} \quad \langle 0|\mathcal{D}_A|0\rangle \neq 0 \tag{7.2}$$

for some fields. We will refer to these two possibilities as F-type or D-type SUSY breaking.

Since the auxiliary fields are given by (6.43a) and (6.43b) of the last chapter, we conclude that supersymmetry is spontaneously broken if the system of equations,

$$\left(\frac{\partial\hat{f}}{\partial\mathcal{S}_i}\right)_{\hat{\mathcal{S}}=\mathcal{S}} = 0 \quad (F\text{-type}) \tag{7.3a}$$

or

$$g \sum_i \mathcal{S}_i^\dagger t_A \mathcal{S}_i + \xi_A = 0 \quad (D\text{-type}) \tag{7.3b}$$

does not have any solutions. Otherwise, supersymmetry is unbroken. In this chapter we will examine several toy models to illustrate both the F- and D-type SUSY breaking mechanisms.

Two comments are in order.

- The master formula (6.44) for the Lagrangian for SUSY gauge theories contains the terms,

$$\mathcal{L} \ni -V_{\text{scalar}} \equiv -\frac{1}{2}\sum_A \mathcal{D}_A \mathcal{D}_A - \sum_i |\mathcal{F}_i|^2.$$

  Thus, if any of the auxiliary fields develop a VEV, then so will the scalar potential.
- If $Q|0\rangle \neq 0$, then the state has infinite norm. This is because

$$\| \; Q|0\rangle \; \|^2 = \int \mathrm{d}^3x \langle 0| j^{0\dagger}(x) Q|0\rangle$$

  (where $j^\mu(x)$ is the spinorial Noether current corresponding to the super-charge Q) diverges in a translationally invariant theory unless Q annihilates the vacuum. This is exactly as for the case of spontaneous breaking of bosonic symmetries. It is often loosely stated that spontaneous SUSY breaking is signalled by the VEV of the Hamiltonian. This is not the case since if the Hamiltonian density develops a constant VEV, its integral does not exist. In fact, just as in the familiar case of ordinary symmetries where the charges do not exist when the symmetry is spontaneously broken, the generators of the super-algebra do not exist if supersymmetry is spontaneously broken; the charge and current densities are, however, well defined and it is only these that we need for most manipulations in field theory.

7.2 F-type SUSY breaking: the O'Raifeartaigh model

A simple supersymmetric model exhibiting spontaneous breaking of supersymmetry was written down by O'Raifeartaigh in 1975.[1] It contains three chiral scalar superfields $\hat{X} \ni (X, \psi_X, \mathcal{F}_X)$, $\hat{Y} \ni (Y, \psi_Y, \mathcal{F}_Y)$, and $\hat{Z} \ni (Z, \psi_Z, \mathcal{F}_Z)$ interacting via the superpotential,

$$\hat{f}(\hat{X}, \hat{Y}, \hat{Z}) = \lambda(\hat{X}^2 - \mu^2)\hat{Y} + m\hat{X}\hat{Z}, \tag{7.4}$$

[1] L. O'Raifeartaigh, *Nucl. Phys.* **B96**, 331 (1975).

with m and λ as real parameters. Since $\mathcal{F}_i = -\mathrm{i}(\partial \hat{f}/\partial \hat{\mathcal{S}}_i)^\dagger_{\hat{\mathcal{S}}=\mathcal{S}}$, we have

$$\mathrm{i}\mathcal{F}_X = 2\lambda Y^\dagger X^\dagger + mZ^\dagger, \tag{7.5a}$$

$$\mathrm{i}\mathcal{F}_Y = \lambda(X^{\dagger 2} - \mu^2), \quad \text{and} \tag{7.5b}$$

$$\mathrm{i}\mathcal{F}_Z = mX^\dagger. \tag{7.5c}$$

Note that both $\langle \mathcal{F}_Y \rangle$ and $\langle \mathcal{F}_Z \rangle$ cannot simultaneously be zero. Hence, supersymmetry must be broken.

The scalar potential of this model is,

$$V(X, Y, Z) = \sum_i |\mathcal{F}_i|^2 = |2\lambda XY + mZ|^2 + \lambda^2|X^2 - \mu^2|^2 + m^2|X|^2. \tag{7.6}$$

Notice that the potential is a sum of non-negative terms. This is a general feature of theories with global supersymmetry. Indeed, we see from the master formula (6.44) that the D- and F-term contributions to the scalar potential are separately non-negative.

To find the minimum of this potential, observe that the first term can be made to be zero no matter what $\langle X \rangle$ and $\langle Y \rangle$ are since $\langle Z \rangle$ is chosen to cancel it. The vacuum state is, therefore, infinitely degenerate. The direction (in field space) along which the first term vanishes is referred to as an F-flat direction (since the value of the potential is flat along this direction). Flat directions frequently occur in supersymmetric models. We should add here that the flatness of the (tree-level) potential is generally removed when quantum corrections are taken into account.

Returning to the potential of the O'Raifeartaigh model, the minimum thus depends only on the last two terms of (7.6) that define the self-couplings V_X for the X field. We break the complex field X into real and imaginary parts $X = \frac{X_{\mathrm{R}} + \mathrm{i}X_{\mathrm{I}}}{\sqrt{2}}$, so that

$$\begin{aligned} V_X &= \lambda^2|X^2 - \mu^2|^2 + m^2|X|^2 \\ &= \frac{\lambda^2}{4}(X_{\mathrm{R}}^2 + X_{\mathrm{I}}^2)^2 + \frac{1}{2}(m^2 - 2\lambda^2\mu^2)X_{\mathrm{R}}^2 \\ &\quad + \frac{1}{2}(m^2 + 2\lambda^2\mu^2)X_{\mathrm{I}}^2 + \lambda^2\mu^4. \end{aligned} \tag{7.7}$$

We will examine two cases for V_X, illustrated in Fig. 7.1.

Case A: If $m^2 > 2\lambda^2\mu^2$, the minimum of V_X is clearly at $\langle X \rangle = 0$. In this case, $\langle Z \rangle = 0$ but $\langle Y \rangle$ is undetermined, and $V_{\min} = \lambda^2\mu^4$. Y is a flat direction of the scalar potential.

Case B: If $m^2 < 2\lambda^2\mu^2$, $\langle X_{\mathrm{R}} \rangle \neq 0$ but $\langle X_{\mathrm{I}} \rangle = 0$ since the coefficient of X_{I}^2 is positive. The minimum will occur at $\langle X_{\mathrm{R}} \rangle^2 = 2\mu^2 - m^2/\lambda^2$ and $\langle Z \rangle = -\frac{2\lambda}{m}\sqrt{\mu^2 - \frac{m^2}{2\lambda^2}}\langle Y \rangle$. At the minimum, $V_{\min} = -\frac{\lambda^2}{4}(2\mu^2 - m^2/\lambda^2)^2 + \mu^4\lambda^2$. Note that the minimum does *not* occur at $\langle X \rangle^2 = \mu^2$.

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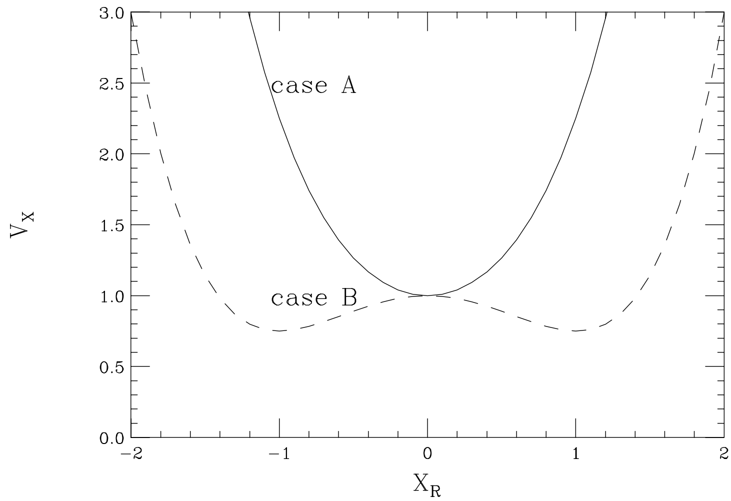

Figure 7.1 Scalar potential in the O'Raifeartaigh model for case A with $m/\mu = 2$ and $\lambda = 1$, and case B with $m/\mu = \lambda = 1$. In both cases, $X_{\mathrm{I}} = 0$.

7.2.1 Mass spectrum: Case A

Next we will construct the mass matrix for the scalar fields of case A in the O'Raifeartaigh model. As usual, we first shift the fields by their VEVs, and then rewrite the scalar potential in terms of the shifted field $Y_S = Y - \langle Y \rangle$ together with X and Z to obtain,

$$V = |2\lambda X(Y_S + \langle Y \rangle) + mZ|^2 + \lambda^2 |X^2 - \mu^2|^2 + m^2 |X|^2. \tag{7.8}$$

There are no bilinear terms in the field Y_S which must, therefore, be massless. Let us write $X = \frac{X_{\mathrm{R}} + \mathrm{i}X_{\mathrm{I}}}{\sqrt{2}}$ and $Z = \frac{Z_{\mathrm{R}} + \mathrm{i}Z_{\mathrm{I}}}{\sqrt{2}}$. We can now work out the scalar mass squared matrix for the four real fields. In the basis $(X_{\mathrm{R}},\ Z_{\mathrm{R}},\ X_{\mathrm{I}},\ Z_{\mathrm{I}})$ it is given by,

$$\begin{pmatrix} m^2 + 4\lambda^2\langle Y \rangle^2 - 2\lambda^2\mu^2 & 2\lambda m\langle Y \rangle & 0 & 0 \\ 2\lambda m\langle Y \rangle & m^2 & 0 & 0 \\ 0 & 0 & m^2 + 4\lambda^2\langle Y \rangle^2 + 2\lambda^2\mu^2 & 2\lambda m\langle Y \rangle \\ 0 & 0 & 2\lambda m\langle Y \rangle & m^2 \end{pmatrix} \tag{7.9}$$

where we have taken $\langle Y \rangle$ to be real. If $\langle Y \rangle \neq 0$, then X and Z mix. Regardless of the mixing, however, the trace of the matrix gives,

$$\sum_{\text{bosons}} \mathcal{M}^2 = 2m^2 + 2(m^2 + 4\lambda^2\langle Y \rangle^2). \tag{7.10}$$

To find the fermion masses, we must examine fermionic bilinear terms that can be derived from the superpotential. These terms will arise from second derivatives

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of the superpotential, evaluated at the VEV of the scalar fields. Since only Y has a VEV, it is clear that just $\partial^2 \hat{f}/\partial \hat{X}^2$ or $\partial^2 \hat{f}/\partial \hat{X}\partial \hat{Z}$ yield non-zero contributions. In particular, ψ_Y is a massless fermion. For the remaining fermions, in the basis of $(\psi_X,\ \psi_Z)$, we have the mass matrix,

$$\mathcal{M}_{\text{fermion}} = \begin{pmatrix} 2\lambda\langle Y\rangle & m \\ m & 0 \end{pmatrix} \tag{7.11}$$

so that

$$\mathcal{M}_{\text{fermion}}\mathcal{M}^{\dagger}_{\text{fermion}} = \begin{pmatrix} 4\lambda^2\langle Y\rangle^2 + m^2 & 2\lambda\langle Y\rangle m \\ 2\lambda\langle Y\rangle m & m^2 \end{pmatrix}. \tag{7.12}$$

We thus see that,

$$\sum_{\text{fermions}} \mathcal{M}^2 = m^2 + (m^2 + 4\lambda^2\langle Y\rangle^2). \tag{7.13}$$

The *supertrace* is defined as

$$STr\mathcal{M}^2 = \sum_{\text{particles}} (-1)^{2J}(2J+1)m_J^2, \tag{7.14}$$

where the sum is over all particles in the theory, J is the spin and m_J is the mass. In any model where supersymmetry is unbroken, the degeneracy of the fermion and boson masses, together with the equality of bosonic and fermionic degrees of freedom obviously means that the supertrace must vanish. In the case under study, summing over all bosons and fermions, we obtain

$$STr\mathcal{M}^2 = 0 \tag{7.15}$$

even though supersymmetry is spontaneously broken. We will shortly see that (at tree level) the supertrace is always zero for theories with only chiral scalar superfields, even if supersymmetry is spontaneously broken.

Exercise *Work out the mass spectrum of the model. Show that aside from the complex massless boson field Y and the massless fermion ψ_Y, the remaining boson squared masses are,*

$$m^2 + 2\lambda^2\langle Y\rangle^2 - \lambda^2\mu^2 \pm \left[(m^2 + 2\lambda^2\langle Y\rangle^2 - \lambda^2\mu^2)^2 - m^2(m^2 - 2\lambda^2\mu^2)\right]^{\frac{1}{2}},$$

$$m^2 + 2\lambda^2\langle Y\rangle^2 + \lambda^2\mu^2 \pm \left[(m^2 + 2\lambda^2\langle Y\rangle^2 + \lambda^2\mu^2)^2 - m^2(m^2 + 2\lambda^2\mu^2)\right]^{\frac{1}{2}},$$

while the remaining fermion masses are,

$$\sqrt{\lambda^2\langle Y\rangle^2 + m^2} \pm \lambda\langle Y\rangle.$$

You can now confirm that the supertrace formula is satisfied. Notice also that if $V_{\min} = \lambda^2\mu^4$ vanishes, supersymmetry is restored in the spectrum.

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The fact that there is a massless fermion in the spectrum is a general feature in theories with spontaneous supersymmetry breaking. This fermion is called the goldstino. It is the analogue of the Goldstone boson that arises when global bosonic symmetries are spontaneously broken. We will not present a general argument that shows that spontaneous SUSY breaking always results in a goldstino. The proof parallels that given by Goldstone, Salam, and Weinberg for the Goldstone theorem.[2] The goldstino is a spin $\frac{1}{2}$ fermion because the SUSY generator itself carries spin $\frac{1}{2}$. It is the fermionic partner of the auxiliary field that develops a SUSY breaking VEV.

Notice that aside from the $\pm 2\lambda^2\mu^2$ terms in the diagonal X_R and X_I entries in the mass matrix for scalars, the mass matrices look supersymmetric. In other words, but for these terms, the mass matrices (7.9) and (7.12) would be those of a theory with unbroken SUSY. This should not be surprising because, at tree level, the order parameter $\mathcal{F}_Y$ for SUSY breaking (and hence also the goldstino) couples to only the X field, as is evident from the form of the superpotential.

It is easy to see from the result of the exercise above that the heaviest boson is heavier than the heaviest fermionic state. But because the sums over the squared masses of the bosons and fermions are the same, this also means that the lightest of the massive bosons must be lighter than the lightest of the massive fermions. In other words, the spontaneous breaking of supersymmetry results in fermion masses that are bracketed between the boson masses. The "SUSY breaking" $2\lambda^2\mu^2$ contribution does not enter the fermion masses, but splits the boson masses about their would-be value (equal to the fermion masses) in the absence of SUSY breaking.[3] Since this pattern of mass splittings has its origin in the vanishing of the supertrace – a general feature in models with global supersymmetry broken spontaneously by F-terms – it is very difficult to use this mechanism to get realistic models with global supersymmetry broken at the TeV scale: these models typically give a spin zero superpartner lighter than all the fermions, and so run into conflict with experiment.

7.2.2 Mass spectrum: Case B

We can now similarly work out the mass spectrum for Case B, where the minimum occurs at $\langle X_R\rangle^2 = 2\mu^2 - m^2/\lambda^2$, $\langle X_I\rangle = 0$, with a flat direction along $\langle Z\rangle = -\frac{2\lambda}{m}\sqrt{\mu^2 - \frac{m^2}{2\lambda^2}}\langle Y\rangle$. For the most part, we will leave it to the reader to work out the details for this case. From the scalar potential, it is straightforward to work

[2] J. Goldstone, A. Salam and S. Weinberg, *Phys. Rev.* **127**, 965 (1962), Sec. III.

[3] It is reasonable that spontaneous SUSY breaking yields new contributions to boson masses but not to fermions. We saw in Chapter 3 that a SUSY breaking mass for a fermion in a chiral scalar multiplet would have been a hard breaking of supersymmetry.

out the scalar mass matrix. The bosonic contribution to the supertrace is as given in the exercise below.

Exercise *For case B, calculate the bilinear terms in the scalar potential of the shifted fields and show that*

$$\sum_{\text{bosons}} \mathcal{M}^2 = 4\lambda^2(\langle X_R\rangle^2 + \langle Y_R\rangle^2 + \langle Y_I\rangle^2 + 2\mu^2). \tag{7.16}$$

(Remember that $\langle X_R\rangle \neq 0$, while $\langle X_I\rangle = 0$.)

To identify the goldstino, we recall that the auxiliary fields obtain VEVs

$$\mathrm{i}\langle \mathcal{F}_X\rangle = 0, \tag{7.17a}$$

$$\mathrm{i}\langle \mathcal{F}_Y\rangle = \lambda(\frac{1}{2}\langle X_R\rangle^2 - \mu^2), \tag{7.17b}$$

$$\mathrm{i}\langle \mathcal{F}_Z\rangle = m\frac{\langle X_R\rangle}{\sqrt{2}}. \tag{7.17c}$$

We can then work with orthogonal linear combinations of the superfields $\hat{Y}$ and $\hat{Z}$, so that the auxiliary component of just one of the linear combinations develops a VEV:

$$\hat{P} = \frac{\frac{m\langle X_R\rangle}{\sqrt{2}}\hat{Y} - \lambda(\frac{1}{2}\langle X_R\rangle^2 - \mu^2)\hat{Z}}{\sqrt{\frac{1}{2}m^2\langle X_R\rangle^2 + \lambda^2(\frac{1}{2}\langle X_R\rangle^2 - \mu^2)^2}} \equiv \hat{Y}\cos\theta - \hat{Z}\sin\theta \tag{7.18a}$$

and

$$\hat{Q} = \frac{\lambda(\frac{1}{2}\langle X_R\rangle^2 - \mu^2)\hat{Y} + \frac{m\langle X_R\rangle}{\sqrt{2}}\hat{Z}}{\sqrt{\frac{1}{2}m^2\langle X_R\rangle^2 + \lambda^2(\frac{1}{2}\langle X_R\rangle^2 - \mu^2)^2}} \equiv \hat{Y}\sin\theta + \hat{Z}\cos\theta. \tag{7.18b}$$

In this case, we have $\langle \mathcal{F}_P\rangle = 0$ and $\langle \mathcal{F}_Q\rangle \neq 0$. We then expect that ψ_Q, the fermionic component of $\hat{Q}$, will be the massless goldstino field. To establish this, as well as to obtain the fermionic contribution to the supertrace, we write the superpotential in terms of $\hat{X}$, $\hat{P}$, and $\hat{Q}$,

$$\hat{f}(\hat{X}, \hat{P}, \hat{Q}) = \lambda(\hat{X}^2 - \mu^2)(\hat{P}\cos\theta + \hat{Q}\sin\theta) + m\hat{X}(\hat{Q}\cos\theta - \hat{P}\sin\theta). \tag{7.19}$$

We see from (6.44) that ψ_Q is massless since there is neither a diagonal mass for it (no $\hat{Q}^2$ term in the superpotential) nor a bilinear mixing with either ψ_P or ψ_X.

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Exercise *That there is no mixing of ψ_Q with ψ_P is obvious from the superpotential. Verify that the ψ_X–ψ_Q mixing term also vanishes.*

In the (ψ_X, ψ_P) basis, the non-vanishing fermion mass submatrix can be written as,

$$\mathcal{M}_{\text{fermion}} = \mathcal{M}\frac{1-\gamma_5}{2} + \mathcal{M}^\dagger \frac{1+\gamma_5}{2}, \tag{7.20a}$$

with,

$$\mathcal{M} = \begin{pmatrix} 2\lambda \frac{\langle Y_\mathrm{R}\rangle + \mathrm{i}\langle Y_\mathrm{I}\rangle}{\sqrt{2}} & \sqrt{2}\lambda\cos\theta\langle X_\mathrm{R}\rangle - m\sin\theta \\ \sqrt{2}\lambda\cos\theta\langle X_\mathrm{R}\rangle - m\sin\theta & 0 \end{pmatrix}. \tag{7.20b}$$

Except when $\langle Y_\mathrm{I}\rangle = 0$, the fermion “mass matrix” is γ_5-dependent. The reader who is not familiar with how to deal with this is referred to the technical note at the end of this chapter.[4] There, we show that the squared masses of the fermions are given by the eigenvalues of the matrix $\mathcal{M}^\dagger\mathcal{M}$.

Exercise *By explicitly computing the sum of the squared masses for the fermions in case B, verify that the supertrace once again vanishes.*

7.3 *D*-type SUSY breaking

As an illustration of SUSY breaking by D-terms, we consider a simple model with just one chiral superfield coupled to a $U(1)$ gauge field. We include a Fayet–Iliopoulos (FI) D-term. Gauge symmetry precludes any superpotential interactions.

The scalar potential for this model is just

$$V = \frac{1}{2}\mathcal{D}^2 = \frac{1}{2}(g\mathcal{S}^\dagger\mathcal{S} + \xi)^2. \tag{7.21}$$

The minimum of the potential occurs at

$$(a)\quad \langle\mathcal{S}^\dagger\mathcal{S}\rangle = 0 \ \text{ if } \xi > 0,$$
$$(b)\quad \langle\mathcal{S}^\dagger\mathcal{S}\rangle = -\frac{\xi}{g} = \frac{|\xi|}{g} \ \text{ if } \xi < 0.$$

In case (a), SUSY is spontaneously broken because the D-term acquires a vacuum expectation value. The gauge symmetry remains unbroken because $\langle\mathcal{S}^\dagger\mathcal{S}\rangle = 0$. In case (b), the $U(1)$ gauge symmetry is spontaneously broken but SUSY remains intact.

[4] The matrices M and N of the note can be identified as $\frac{\mathcal{M}+\mathcal{M}^\dagger}{2} = M$ and $\frac{\mathcal{M}^\dagger-\mathcal{M}}{2} = \mathrm{i}N$.

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7.3.1 Case A

In this case, the auxiliary field $\mathcal{D} = g\mathcal{S}^\dagger\mathcal{S} + \xi$ acquires a VEV $\langle\mathcal{D}\rangle = \xi$. Because there is no superpotential, the chiral fermion ψ remains massless. The FI term causes a mass splitting with the scalar $\mathcal{S}$, which acquires a mass $\sqrt{g\xi}$.

The $U(1)$ gauge boson and gaugino are massless at tree level. The gaugino, which is the partner of the $\mathcal{D}$ field that acquires a VEV, plays the role of the goldstino. In this toy theory, since the complex field $\mathcal{S}$ is the only state to acquire mass, the supertrace is just

$$STr\mathcal{M}^2 = 2g\xi$$

in accord with the general sum rule Eq. (7.35) discussed later in this chapter. Notice that, unlike the O'Raifeartaigh model, this model does not suffer from the problem of "scalars lighter than fermions."

7.3.2 Case B

In this case, gauge symmetry is spontaneously broken because $\mathcal{S}$ acquires a VEV, but SUSY remains intact. Let us work out the spectrum of the model to see explicitly how this works.

Bosons

The relevant piece of the Lagrangian for vector bosons is

$$\mathcal{L} \ni \left[(\partial_\mu\mathcal{S})^\dagger - \mathrm{i}gV_\mu\mathcal{S}^\dagger\right][\partial^\mu\mathcal{S} + \mathrm{i}gV^\mu\mathcal{S}] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{7.22}$$

We note that by redefining the phase of $\mathcal{S}$, $\langle\mathcal{S}\rangle$ can be chosen to be real without loss of generality. As usual, we then shift $\mathcal{S} \to \mathcal{S} + \langle\mathcal{S}\rangle$ and then re-write the Lagrangian in terms of $\mathcal{S} = \frac{\mathcal{S}_\mathrm{R}+\mathrm{i}\mathcal{S}_\mathrm{I}}{\sqrt{2}}$ (here, $\mathcal{S}_\mathrm{R}$ and $\mathcal{S}_\mathrm{I}$ are fluctuations about the vacuum) to obtain

$$\mathcal{L} \ni \frac{1}{2}(\partial_\mu\mathcal{S}_\mathrm{R})^2 + \frac{1}{2}(\partial_\mu\mathcal{S}_\mathrm{I})^2 + \sqrt{2}g\langle\mathcal{S}\rangle V_\mu\partial^\mu\mathcal{S}_\mathrm{I} + g^2\langle\mathcal{S}\rangle^2 V_\mu V^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The field $\mathcal{S}_\mathrm{I}$ can be absorbed by a local gauge transformation by an amount $\frac{-\mathcal{S}_\mathrm{I}}{\sqrt{2}g\langle\mathcal{S}\rangle}$. This piece of the Lagrangian becomes

$$\mathcal{L} \ni \frac{1}{2}(\partial_\mu\mathcal{S}_\mathrm{R})^2 + g^2\langle\mathcal{S}\rangle^2 V_\mu V^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{7.23}$$

The vector boson has developed a mass $m_V^2 = 2g^2\langle\mathcal{S}\rangle^2 = 2g|\xi|$, while the $\mathcal{S}_\mathrm{I}$ field has disappeared, being eaten by the vector boson field. This is, of course, the familiar Higgs mechanism. A real scalar $\mathcal{S}_\mathrm{R}$ remains. Its mass can be obtained from the scalar potential (7.21).

Exercise *Show that the Higgs field $\mathcal{S}_R$ of our toy model has the same mass as the vector boson.*

Fermions

Since there is no superpotential, bilinear terms for fermions come only from gaugino chiral fermion mixing induced by a VEV of the scalar field. The relevant term is,

$$\mathcal{L} \ni -\sqrt{2}\langle \mathcal{S} \rangle g \bar{\lambda} \frac{1-\gamma_5}{2} \psi + \quad \text{h.c.}$$

We see that there are two degenerate Majorana fermions that can be combined into a single Dirac fermion $\chi_D = \frac{1-\gamma_5}{2}\psi + \frac{1+\gamma_5}{2}\lambda$, with a mass again equal to that of the bosons.

Thus, in case B, the physical particles are one massive spin 1 boson and a spin 0 boson, each with mass $\sqrt{2g|\xi|}$ and one Dirac fermion with the same mass. The spectrum is clearly supersymmetric, but the original gauge symmetry is hidden.[5]

7.4 Composite goldstinos

We have considered examples where the goldstino is an elementary field that occurs in the Lagrangian. This need not always be the case. The goldstino may be a composite fermion just as the composite pion may be regarded as a (pseudo)-Goldstone boson.

This is realized if chiral fermions condense. If $\hat{\mathcal{S}}$ and $\hat{\mathcal{S}}'$ are two left-chiral superfields,

$$\hat{\mathcal{S}} = \mathcal{S}(\hat{x}) + \mathrm{i}\sqrt{2}\bar{\theta}\psi_L + \mathrm{i}\bar{\theta}\theta_L \mathcal{F}(\hat{x}) \quad \text{and}$$
$$\hat{\mathcal{S}}' = \mathcal{S}'(\hat{x}) + \mathrm{i}\sqrt{2}\bar{\theta}\psi'_L + \mathrm{i}\bar{\theta}\theta_L \mathcal{F}'(\hat{x}),$$

then the composite field $\hat{\mathcal{S}}\hat{\mathcal{S}}'$ is given by,

$$\hat{\mathcal{S}}\hat{\mathcal{S}}' = \mathcal{S}\mathcal{S}'(\hat{x}) + \mathrm{i}\sqrt{2}\bar{\theta}(\mathcal{S}'\psi_L + \mathcal{S}\psi'_L)(\hat{x}) - 2\bar{\theta}\psi_L\bar{\theta}\psi'_L(\hat{x}) + \mathrm{i}\bar{\theta}\theta_L(\mathcal{F}\mathcal{S}' + \mathcal{S}\mathcal{F}')(\hat{x}).$$

Using the fact that $-2\bar{\theta}\psi_L\bar{\theta}\psi'_L = \bar{\theta}\theta_L\bar{\psi}'\psi_L$, we have

$$\hat{\mathcal{S}}\hat{\mathcal{S}}' = \mathcal{S}\mathcal{S}' + \mathrm{i}\sqrt{2}\bar{\theta}(\mathcal{S}\psi'_L + \mathcal{S}'\psi_L) + \mathrm{i}\bar{\theta}\theta_L(\mathcal{S}\mathcal{F}' + \mathcal{F}\mathcal{S}' - \mathrm{i}\bar{\psi}'\psi_L). \tag{7.24}$$

[5] This spectrum corresponds to that of the $j = 1/2$ supermultiplet discussed toward the end of Section 4.4.

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Thus, the auxiliary component of the composite superfield contains the product of the fermion components of the elementary superfields.

This led theorists in the 1980s to consider SUSY technicolor-like models where $\hat{\mathcal{S}}$ and $\hat{\mathcal{S}}'$ were taken to be $\mathbf{n}$ and $\mathbf{n}^*$ representations of a confining group $SU(n)$.[6] It was assumed that the chiral fermions would condense forming an $SU(n)$ singlet condensate. If such a condensate forms, then SUSY is dynamically broken by the F-term of the composite superfield. The goldstino field would then be a composite object, the fermionic component $(\mathcal{S}'\psi_{\mathrm{L}} + \mathcal{S}\psi'_{\mathrm{L}})$ of a composite superfield. Whether or not such a condensate forms is a dynamical question, and is more difficult to address.

7.5 Gaugino condensation

If we allow non-renormalizable interactions (as we must if we want to include gravity in our effective low energy theory), some of our considerations have to be suitably generalized. Of importance to us here is the fact that instead of starting with just $\overline{\hat{W}^c_A}\hat{W}_A$ whose $\bar{\theta}\theta_{\mathrm{L}}$ component led to the kinetic term for gauge fields and gauginos, we could have started with

$$f_{AB}(\hat{\mathcal{S}}_{\mathrm{L}i})\overline{\hat{W}^c_A}\hat{W}_B$$

whose $\bar{\theta}\theta_{\mathrm{L}}$ component also leads to a SUSY invariant action. To maintain gauge invariance, we must require that the dimensionless function f_{AB} transform as a representation contained in the symmetric product of two adjoints. The function f_{AB} is known as the *gauge kinetic function*. For a renormalizable gauge theory, we must have $f_{AB} = \delta_{AB}$, but otherwise more general forms are possible.

We will explore some important implications of a non-trivial gauge kinetic function in later chapters. For the present purposes, we only note that in supergravity models the expression (6.43a) for the auxiliary component of chiral superfields is modified: in particular, it picks up a term proportional to,

$$\left.\frac{\partial f_{AB}}{\partial \hat{\mathcal{S}}_{\mathrm{L}i}}\right|_{\hat{\mathcal{S}}=\mathcal{S}} \bar{\lambda}_A\lambda_B.$$

Thus if there are new strong gauge interactions that result in a non-vanishing condensate $\langle\bar{\lambda}_A\lambda_A\rangle$ of gauginos, supersymmetry may be dynamically broken.[7] Gaugino condensation is considered by many authors as a promising way of breaking supersymmmetry.

[6] M. Dine, W. Fischler and M. Srednicki, *Nucl. Phys.* **B189**, 575 (1981); S. Dimopoulos and S. Raby, *Nucl. Phys.* **B192**, 353 (1981).
[7] S. Ferrara, L. Girardello and H. P. Nilles, *Phys. Lett.* **B125**, 457 (1983).

7.6 Goldstino interactions

It is well known that at low energy, the couplings of Goldstone bosons to other particles are fixed only by symmetry considerations, and do not depend on the details of the model. One might similarly expect that the low energy interactions of the goldstino with other multiplets are similarly model-independent. To understand how this comes about, we begin by recalling that (3.13b) tells us that each chiral multiplet contributes

$$j^\mu = \partial\!\!\!/(-\mathrm{i}A\gamma_5 - B)\gamma^\mu\psi + (G\gamma_5 + \mathrm{i}F)\gamma^\mu\psi,$$

to the supercurrent. We may thus write the supercurrent as,

$$j^\mu = \partial\!\!\!/(-\mathrm{i}A_g\gamma_5 - B_g)\gamma^\mu\psi_g + (G_g\gamma_5 + \mathrm{i}F_g)\gamma^\mu\psi_g + j^{\mu,\mathrm{rest}}, \tag{7.25a}$$

where the subscript g refers to the fields in the goldstino multiplet, and $j^{\mu,\mathrm{rest}}$ includes contributions to the supercurrent from all other supermultiplets. If SUSY is spontaneously broken by the vacuum expectation value of the complex auxiliary field

$$\langle\mathcal{F}\rangle = \left\langle\frac{F_g + \mathrm{i}G_g}{\sqrt{2}}\right\rangle,$$

that we take to be real, the supercurrent acquires a term *linear* in the goldstino field, and can be written as,

$$j^\mu = \mathrm{i}\sqrt{2}\langle\mathcal{F}\rangle\gamma^\mu\psi_g + \partial\!\!\!/(-\mathrm{i}A_g\gamma_5 - B_g)\gamma^\mu\psi_g + (G_g\gamma_5 + \mathrm{i}F_g)\gamma^\mu\psi_g + j^{\mu,\mathrm{rest}}, \tag{7.25b}$$

where F_g and G_g now denote the shifted fields.

Conservation of the supercurrent then implies that

$$0 = \partial_\mu j^\mu = \mathrm{i}\sqrt{2}\langle\mathcal{F}\rangle\partial\!\!\!/\psi_g + \partial_\mu j^{\mu,\mathrm{rest}} + \cdots \tag{7.26}$$

where the ellipsis denotes bilinear (or higher, after the auxiliary fields F_g and G_g are eliminated) terms in fields from the goldstino supermultiplet. This is the equation of motion for the goldstino. It may be obtained from the phenomenological Lagrangian density,

$$\begin{aligned}\mathcal{L}_{\mathrm{goldstino}} &= \frac{\mathrm{i}}{2}\bar{\psi}_g\partial\!\!\!/\psi_g + \left[\frac{1}{2\sqrt{2}\langle\mathcal{F}\rangle}\bar{\psi}_g\partial_\mu j^{\mu,\mathrm{rest}} + \quad \mathrm{h.c.}\right] + \cdots \\ &= \frac{\mathrm{i}}{2}\bar{\psi}_g\partial\!\!\!/\psi_g + \left[\frac{-1}{2\sqrt{2}\langle\mathcal{F}\rangle}\left(\partial_\mu\bar{\psi}_g\right)j^{\mu,\mathrm{rest}} + \quad \mathrm{h.c.}\right] + \cdots,\end{aligned} \tag{7.27}$$

where in the last step, we have omitted a term that is a total derivative. Again, the ellipsis denotes couplings of the goldstino to its superpartner which are not relevant to our discussion. By using the explicit form of the supercurrent (3.13b) it is now

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straightforward to work out the couplings of the goldstino with fields in *other* supermultiplets. The first term in the supercurrent that originates in the kinetic energy piece of the Lagrangian gives rise to a *model-independent* interaction of the goldstino with the scalar and fermion members of the chiral multiplet with components $(\mathcal{S}, \psi)$. We will leave it to the reader to work out that

$$\mathcal{L}_{\text{goldstino}} = \frac{\mathrm{i}}{2}\bar{\psi}_g \partial\!\!\!/ \psi_g - \frac{\mathrm{i}}{\langle \mathcal{F} \rangle} \left(\partial_\mu \bar{\psi}_g\right) \left[\partial\!\!\!/ \mathcal{S} \gamma^\mu \frac{1+\gamma_5}{2} \psi - \partial_\mu \mathcal{S}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} \psi \right] + \cdots , \tag{7.28}$$

where the ellipsis denotes other interactions of the goldstino. There is one such term for each chiral multiplet. In a gauge theory, gauginos and gauge bosons also contribute to the supercurrent, and there will be an analogous gauge boson–gaugino–goldstino interaction. We will return to these goldstino couplings when we consider decays of supersymmetric particles into gravitinos (the superpartners of the graviton) in Chapter 13.

7.7 A mass sum rule

In previous sections, we alluded to the fact that the superparticle spectrum is significantly constrained even when SUSY is spontaneously broken. For instance, for F-type breaking, we saw that $STr\mathcal{M}^2 = 0$, which implied that at least one of the scalar components of chiral scalar superfields must be lighter than any of the fermions. For D-type breaking, we saw that this was not always the case. These features are not particular to the specific model that we considered. To see this, we can compute the squared masses of each particle using the Lagrangian density in (6.44), and hence the respective contributions to the supertrace.

Tree-level masses are defined by the coefficients of bilinear terms in fields expanded about the minimum of the scalar potential. The mass sum rule we obtain holds somewhat more generally, in that it holds for "masses" defined about any scalar field configuration, not just a local extremum. These "masses" are, of course, field-dependent. The immediate payoff is that we can immediately infer that our STr formula is also valid for the special case of spontaneously broken gauge symmetries where we are computing the coefficients of field bilinears about a non-trivial classical minimum.

7.7.1 Scalar contributions

The scalar potential of a supersymmetric theory has the form,

$$V(\mathcal{S}, \mathcal{S}^*) = \sum_i \left| \frac{\partial \hat{f}}{\partial \hat{\mathcal{S}}_i} \right|^2_{\hat{\mathcal{S}} = \mathcal{S}} + \frac{1}{2} \sum_A \left(\sum_i \mathcal{S}_i^\dagger g t_A \mathcal{S}_i + \xi_A \right)^2 , \tag{7.29}$$

where we suppress the index α in (6.44).

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Potential mass terms include terms like $\mathcal{S}^\dagger\mathcal{S}$ as well as terms like $\mathcal{S}^2 + \text{h.c.}$ We will let the reader check (see exercise below) that while the latter terms may affect the individual masses, they never contribute to the trace of the scalar mass matrix. Hence the scalar boson contribution to the supertrace can be obtained from,

$$m_{ij}^2 = \frac{\partial^2 V}{\partial \mathcal{S}_i^\dagger \partial \mathcal{S}_j}.$$

We can then write this as,

$$STr\mathcal{M}^2_{\text{scalars}} = 2\sum_{i,j}\left(\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\left(\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)^*_{\hat{\mathcal{S}}=\mathcal{S}} + 2\sum_A \mathcal{D}_A Tr(gt_A) + 2\sum_{i,A} g^2\mathcal{S}_i^\dagger t_A t_A \mathcal{S}_i, \tag{7.30}$$

where the 2 comes from the fact that each complex scalar is really two degrees of freedom, and $\mathcal{D}_A$ is a shorthand for $\sum_i \mathcal{S}_i^\dagger g t_A \mathcal{S}_i + \xi_A$.

7.7.2 Vector contributions

The vector boson mass matrix arises from the kinetic energy terms for scalars:

$$\mathcal{L} \ni (D_\mu\mathcal{S}_i)^\dagger D^\mu\mathcal{S}_i = (\partial_\mu\mathcal{S}_i + \mathrm{i}gt_A V_{\mu A}\mathcal{S}_i)^\dagger(\partial^\mu\mathcal{S}_i + \mathrm{i}gt_B V_B^\mu\mathcal{S}_i).$$

Here i labels different chiral scalar multiplets. Every multiplet that transforms non-trivially under the gauge group contributes to the "field-dependent" vector mass matrix. Of course, the tree-level physical masses will get contributions from only those multiplets that develop a gauge symmetry breaking VEV. The vector contribution to the supertrace is,

$$STr\mathcal{M}^2_{\text{vectors}} = 2\times 3\times g^2\sum_{A,i}(\mathcal{S}_i^\dagger t_A)(t_A\mathcal{S}_i), \tag{7.31}$$

where the factor of 2 arises because the vector fields are real, and the 3 comes from the three degrees of freedom for each massive spin 1 field (the factor $2J+1$ in the definition of the STr).

7.7.3 Fermion contributions

The technical note on γ_5-dependent fermion mass matrices at the end of this chapter shows that if the fermion bilinears in the Lagrangian density for Majorana fermions

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is given by,

$$\begin{aligned}\mathcal{L} \ni -\frac{1}{2}\bar{\chi}_a\mathcal{M}_{ab}\frac{1-\gamma_5}{2}\chi_b + \quad \text{h.c.} \\ = -\frac{1}{2}\bar{\chi}_{\rm R}\mathcal{M}\chi_{\rm L} - \frac{1}{2}\bar{\chi}_{\rm L}\mathcal{M}^\dagger\chi_{\rm R},\end{aligned} \tag{7.32}$$

then the squared masses of the fermions are given by the eigenvalues of the matrix $\mathcal{M}\mathcal{M}^\dagger$.

In our master formula (6.44), fermion bilinears arise from the superpotential interactions in the last line, or from mixing between gauginos and chiral fermions in line 2. The relevant terms can be written as,

$$\mathcal{L} \ni -\frac{1}{2}\begin{pmatrix}\bar{\lambda}_A & \bar{\psi}_i\end{pmatrix}\begin{pmatrix} 0 & \sqrt{2}g(\mathcal{S}^\dagger t_A)_j \\ \sqrt{2}g(\mathcal{S}^\dagger t_B)_i & \left(\frac{\partial^2 \hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\end{pmatrix}\frac{1-\gamma_5}{2}\begin{pmatrix}\lambda_B \\ \psi_j\end{pmatrix} + \quad \text{h.c.} \tag{7.33}$$

We can now easily obtain the fermionic contribution to the supertrace,

$$\begin{aligned} STr\mathcal{M}^2_{\text{fermions}} = (-1)\times 2 \times \Bigg[\sum_{i,A} 4g^2(\mathcal{S}^\dagger t_A)_i(t_A\mathcal{S})_i \\ + \sum_{i,j}\left(\frac{\partial^2\hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}}\left(\frac{\partial^2\hat{f}}{\partial\hat{\mathcal{S}}_i\partial\hat{\mathcal{S}}_j}\right)^*_{\hat{\mathcal{S}}=\mathcal{S}}\Bigg],\end{aligned} \tag{7.34}$$

where the 2 comes from the two spin degrees of freedom for the Majorana fermions.

We can now combine the contributions (7.30), (7.31), and (7.34) to obtain the tree-level mass sum rule for globally supersymmetric models with spontaneously broken supersymmetry,

$$STr\mathcal{M}^2 = 2\sum_A \mathcal{D}_A \text{Tr}(gt_A). \tag{7.35}$$

Here, the trace refers to a sum over *complex* fields in the chiral supermultiplets. Nowhere in its derivation did we assume that we are at an extremum of the scalar potential. If we now evaluate this at a non-trivial classical minimum, the masses entering on the left-hand side are simply the tree-level masses in the theory, while the right-hand side is the D term whose VEV is one of the order parameters for SUSY breaking.

We now understand why the supertrace vanished for both cases in the O'Raifeartaigh model but not for the model with D-type SUSY breaking. We also note that the right-hand side of (7.35) vanishes if the gauge group is simple. For a model such as the MSSM, with a $U(1)$ hypercharge symmetry, the right-hand side will vanish since the representations are chosen to be anomaly free, i.e. the sum of all the $U(1)_Y$ charges cancels. The problem of light scalars then re-surfaces even in models with D-type SUSY breaking.[8]

Exercise *By decomposing the complex scalar fields into their real and imaginary parts, show that terms such as* $m'^2_{ij}\mathcal{S}_i\mathcal{S}_j + \text{h.c.}$ *cannot contribute to the supertrace.*

Hint: Aside from "off-diagonal" terms involving products of real and imaginary parts of fields which cannot contribute to the trace, show that these terms can be written as $\frac{1}{2}(m'^2_{ij} + m'^{2*}_{ij})(\mathcal{S}_{\mathrm{R}i}\mathcal{S}_{\mathrm{R}j} - \mathcal{S}_{\mathrm{I}i}\mathcal{S}_{\mathrm{I}j})$*, so that the real and imaginary components make equal and opposite contributions to the trace.*

7.8 Explicit supersymmetry breaking

There is as yet no compelling theory of SUSY breaking. We have alluded to potential phenomenological problems that arise if global supersymmetry is spontaneously broken at the TeV scale. Indeed, the strategy most common to model-building today is to assume that supersymmetry is broken in a sector of a theory that is essentially decoupled from our world of quarks, leptons, and gauge and Higgs bosons (and their superpartners). The effects of SUSY breaking in this "hidden sector" are then communicated to our world by messenger interactions. The low energy phenomenology that results is qualitatively dependent on what these messenger interactions are, but we will return to this in later chapters.

It is fair to say that we have not yet discovered the dynamics which causes the breaking of supersymmetry. Hopefully, when this dynamics is discovered, we will find that supersymmetry, like gauge symmetry, is spontaneously broken. The spontaneous breaking of supersymmetry does not alter the supersymmetry relationship between various (tree level) dimensionless couplings in the Lagrangian density in Eq. (6.44).[9] For instance, (tree level) chiral fermion–scalar–gaugino interactions are *fixed* by the usual gauge coupling. As we saw in Chapter 3, altering these would be a hard breaking of supersymmetry in that it would result in a re-appearance of quadratic divergences that we have worked so hard to eliminate. Spontaneous breaking of supersymmetry does not lead to new quadratic divergences.

In the absence of knowledge about SUSY breaking dynamics, the best that we can do is to parametrize the effects of SUSY breaking by adding to the Lagrangian all possible SUSY breaking terms, consistent with all desired (unbroken) symmetries at the SUSY breaking scale, that do not lead to the re-appearance of quadratic

[8] In realistic models because electric charge is strictly conserved and particles with different charges cannot mix, the supertrace vanishes separately in each charge sector. In other words, there should be an up-type scalar quark lighter than the up quark, a down-type scalar quark lighter than the down quark, and an integrally charged scalar lighter than an electron!

[9] Spontaneous SUSY breaking means $\langle\mathcal{F}\rangle$ or $\langle\mathcal{D}\rangle \neq 0$, but the dimensionless (gauge or superpotential) couplings come from the unshifted parts of $\mathcal{F}$ and $\mathcal{D}$.

divergences. In Chapter 3, we referred to such operators as *soft* SUSY breaking operators, and gave examples of these in the context of the Wess–Zumino model.

Girardello and Grisaru have classified the forms of the soft breaking operators in a general theory.[10] They have shown that to all orders in perturbation theory,

- linear terms in the scalar field $\mathcal{S}_i$ (relevant only for singlets of all symmetries),
- scalar masses,
- and bilinear or trilinear operators of the form $\mathcal{S}_i\mathcal{S}_j$ or $\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k$ (where $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j$ and $\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k$ occur in the superpotential),
- and finally, in gauge theories, gaugino masses, one for each factor of the gauge group,

break supersymmetry softly. In general, masses of fermions in chiral supermultiplets, chiral fermion–gaugino mixing masses (these are relevant only if there are chiral supermultiplets in the adjoint representation of the gauge group), and trilinear scalar interactions involving $\mathcal{S}_i$ and $\mathcal{S}_j^\dagger$ are hard. Finally, all dimension four SUSY breaking couplings are hard.

It is not hard to understand why the dimensionful mass terms and trilinear interactions listed above lead to softly broken supersymmetry. If SUSY is explicitly broken by an operator with a coupling $M_{\rm SUSY}$ with dimension of mass, a quadratic divergence in any operator would have a coefficient proportional to $M_{\rm SUSY}\Lambda^2$, where Λ is the ultra-violet cut-off. Only dimension one operators (i.e. operators linear in a spin-zero field) can have such a coefficient. Thus, in a theory in which there are no scalars that are singlets of all the symmetries, *all* dimensionful, renormalizable SUSY breaking operators are soft. In theories with singlets, we get further restrictions by studying the quadratic divergences in tadpole graphs, and the further restrictions listed by Girardello and Grisaru apply.

Spontaneous breaking of supersymmetry leads to soft SUSY breaking operators. We illustrate this with examples of scalar and gaugino mass terms, as well as trilinear SUSY breaking scalar interactions. These operators, as we will see, play an important role in realistic model building. If there is a left-chiral superfield $\hat{U}$ whose F-term develops a SUSY breaking VEV, then the terms,

$$\frac{1}{M^2}\int \mathrm{d}^4\theta\,\hat{U}\hat{U}^\dagger\hat{\mathcal{S}}_{\rm L}\hat{\mathcal{S}}_{\rm L}^\dagger = \frac{|\langle F\rangle|^2}{M^2}\mathcal{S}^\dagger\mathcal{S} + \cdots\,, \tag{7.36a}$$

$$\frac{1}{M}\int \mathrm{d}^2\theta_{\rm L}\hat{U}\,\overline{\hat{W}_A^c}\,\hat{W}_A = \frac{\langle F\rangle}{M}\bar{\lambda}_A\lambda_A + \cdots\,, \tag{7.36b}$$

and

$$\frac{1}{M}\int \mathrm{d}^2\theta_{\rm L}\hat{U}\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k = \frac{\langle F\rangle}{M}\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k + \cdots \tag{7.36c}$$

[10] L. Girardello and M. Grisaru, *Nucl. Phys.* **B194**, 65 (1982).

are, respectively, mass terms for chiral supermultiplet scalars, gauginos, and trilinear interactions of chiral supermultiplet scalars in an effective theory (below the scale M) with spontaneously broken supersymmetry.

In the literature one sometimes sees soft terms of an explicitly broken SUSY theory written in this way. Then $\hat{U}$, which has only a non-vanishing F-component (equal to the SUSY breaking parameter), is not a dynamical superfield. This is then a technical device, and $\hat{U}$ is referred to as a *spurion*.

We summarize this section by listing all SUSY breaking operators consistent with the absence of quadratic divergences in any renormalizable theory. These are:

1. Linear, bilinear, and trilinear scalar self-interactions analytic in the complex scalar field, consistent with gauge and other symmetries. Linear terms are obviously absent if there are no singlet superfields. It is customary to write the bilinear and trilinear soft breaking interactions as
$$B_{ij}\mu_{ij}\mathcal{S}_i\mathcal{S}_j \quad \text{and} \quad A_{ijk}f_{ijk}\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k$$
where the terms $\mu_{ij}\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j$ and $f_{ijk}\hat{\mathcal{S}}_i\hat{\mathcal{S}}_j\hat{\mathcal{S}}_k$ occur in the superpotential. It should, however, be kept in mind that such soft SUSY breaking terms are possible even if the corresponding terms have been set to zero in the superpotential.
2. Scalar mass terms, and
3. gaugino mass terms.

The soft SUSY breaking Lagrangian may thus be written as

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &\sum_i C_i\mathcal{S}_i + \sum_{i,j} B_{ij}\mu_{ij}\mathcal{S}_i\mathcal{S}_j + \sum_{i,j,k} A_{ijk}f_{ijk}\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k + \quad \text{h.c.}\\ &-\sum_{i,j}\mathcal{S}_i^\dagger m_{ij}^2\mathcal{S}_j - \frac{1}{2}\sum_{A,\alpha} M_{A\alpha}\bar{\lambda}_{A\alpha}\lambda_{A\alpha} - \frac{\mathrm{i}}{2}\sum_{A,\alpha} M'_{A\alpha}\bar{\lambda}_{A\alpha}\gamma_5\lambda_{A\alpha},\end{aligned} \tag{7.37}$$

where α runs over the different factors of the gauge group. We note that there are two types of gaugino bilinears that we have introduced above. The first of these is what the reader will recognize as a usual mass term for gauginos. The second term is a CP-odd "mass term" that is not precluded unless we further assume that the SUSY breaking sector does not contain additional sources of CP violation. In models without gauge singlet superfields (of which the MSSM, discussed in the next chapter, is an example), additional terms may be allowed. These include,

4. mixing mass terms between gauginos and fermion members of chiral supermultiplets in an adjoint representation, and
5. trilinear scalar interactions of the form $\mathcal{S}_i\mathcal{S}_j\mathcal{S}_k^*$.

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Mass terms for fermions in a chiral supermultiplet are redundant since these can be reabsorbed into the bilinear terms in the superpotential, together with appropriate redefinition of soft SUSY breaking masses and couplings in the scalar sector.

It is important to stress that the introduction of explicit SUSY breaking terms into the Lagrangian is a *parametrization* of our ignorance about the dynamics of SUSY breaking. An understanding of SUSY breaking (which will hopefully be obtained in the future) should lead to $\mathcal{L}_{\rm soft}$, but with the various soft SUSY breaking parameters being *determined* in terms of the (presumably much fewer) fundamental parameters of a more complete theory.

7.9 A technical aside: γ_5-dependent fermion mass matrices

We see from the last line of the master formula (6.44) that the bilinear terms in (Majorana) fermion fields will, in general, be γ_5-dependent. This is what we encountered in our discussion of Case B of the O'Raifeartaigh model. Similar terms can also arise from mixing between the gauginos and "matter" fermions when the scalar fields on the second line of (6.44) acquire complex VEVs. We thus have to understand how to obtain the fermion mass spectrum from these γ_5-dependent fermion mass matrices.

We write the fermion bilinear terms in the Lagrangian density as,

$$-\mathcal{L} = \frac{1}{2}\bar{\mathcal{N}}_i[M_{ij} + {\rm i}\gamma_5 N_{ij}]\mathcal{N}_j, \tag{7.38}$$

(summation is implied) where $\mathcal{N}_i$ are Majorana spinors and i is a label that distinguishes different particle types. Hermiticity of $\mathcal{L}$ requires that M and N are Hermitian matrices. Since the $\mathcal{N}_i$ are Majorana spinors, M and N also have to be symmetric (and hence real) matrices since $\bar{\mathcal{N}}_i\Gamma\mathcal{N}_j = \bar{\mathcal{N}}_j\Gamma\mathcal{N}_i$ for $\Gamma = I$ or γ_5. This will be crucial later. The Lagrangian density can be written by separating the left- and right-chiral parts of the spinors as,

$$-2\mathcal{L} = \bar{\mathcal{N}}_{{\rm L}i}[M_{ij} + {\rm i}N_{ij}]\mathcal{N}_{{\rm R}j} + \bar{\mathcal{N}}_{{\rm R}i}[M_{ij} - {\rm i}N_{ij}]\mathcal{N}_{{\rm L}j}.$$

We can always find unitary matrices U and V such that $V^\dagger[M + {\rm i}N]U = D$, and $U^\dagger[M - {\rm i}N]V = D^\dagger$, where D is a diagonal (but not necessarily real) matrix. V is the unitary matrix that diagonalizes the Hermitian matrix $[M + {\rm i}N][M - {\rm i}N]$ to give,

$$V^\dagger[M + {\rm i}N][M - {\rm i}N]V = DD^\dagger.$$

U is the corresponding matrix for $[M - {\rm i}N][M + {\rm i}N]$ which is also Hermitian.

It is important to note that $[M + {\rm i}N][M - {\rm i}N]$ and $[M - {\rm i}N][M + {\rm i}N]$ have the same (real and positive) eigenvalues. Furthermore, since M and N are real

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matrices, if the column X is an eigenvector of $[M + \mathrm{i}N][M - \mathrm{i}N]$, then X^* is an eigenvector of $[M - \mathrm{i}N][M + \mathrm{i}N]$ with the same eigenvalue. As a result, we can choose $V = U^*$. This guarantees that the spinor ψ' defined by,

$$\mathcal{N}_{\mathrm{L}} = V\psi'_{\mathrm{L}}, \quad \mathcal{N}_{\mathrm{R}} = U\psi'_{\mathrm{R}} = V^*\psi'_{\mathrm{R}}$$

is Majorana when the original spinor $\mathcal{N}$ is Majorana. Writing the Lagrangian density (7.38) in terms of ψ' we obtain (with matrix multiplication implied),

$$-\mathcal{L} = \frac{1}{2}[\bar{\psi}'_{\mathrm{L}} D \psi'_{\mathrm{R}} + \bar{\psi}'_{\mathrm{R}} D^\dagger \psi'_{\mathrm{L}}], \tag{7.39}$$

which (though it still contains a γ_5-dependent mass term) is now diagonal in particle type. We can now get rid of this γ_5 dependence in the fermion bilinears by performing chiral rotations,

$$\psi'_{\mathrm{L}j} = \mathrm{e}^{-\mathrm{i}\phi_{\mathrm{L}j}}\psi_{\mathrm{L}j}, \quad \psi'_{\mathrm{R}j} = \mathrm{e}^{-\mathrm{i}\phi_{\mathrm{R}j}}\psi_{\mathrm{R}j},$$

(no summation over j). These transformations leave the kinetic terms invariant. If we write the elements of the diagonal matrix D by

$$D_i = m_i \mathrm{e}^{\mathrm{i}a_i},$$

with m_i and a_i as real numbers, the γ_5 dependence in (7.39) is removed if we choose,

$$a_i + \phi_{\mathrm{L}i} - \phi_{\mathrm{R}i} = 0.$$

This, of course, fixes only the difference $\phi_{\mathrm{L}i} - \phi_{\mathrm{R}i}$, but not the two separately. In order to maintain the Majorana character of ψ_i, we should also choose,

$$\phi_{\mathrm{L}i} = -\phi_{\mathrm{R}i}.$$

We can now write (7.39) in terms of ψ to obtain,

$$-\mathcal{L} = \sum_i \frac{m_i}{2}[\bar{\psi}_{\mathrm{L}i}\psi_{\mathrm{R}i} + \bar{\psi}_{\mathrm{R}i}\psi_{\mathrm{L}i}] = \sum_i \frac{m_i}{2}\bar{\psi}_i\psi_i. \tag{7.40}$$

We see that the m_i are then the positive masses for the fermions. A straightforward way to obtain these is to note that the m_i^2 are the eigenvalues of $D^\dagger D$ which, of course, coincide with eigenvalues of $(M - \mathrm{i}N)(M + \mathrm{i}N)$ (or of $(M + \mathrm{i}N)(M - \mathrm{i}N)$).

We should remember several things from this discussion.

- "Fermion masses" (by this we mean the coefficient of $\bar{\psi}\psi$ in the Lagrangian density) are not physical objects. It is only the squares of these that give the squared masses of the fermions. A special case of this that we will have frequent occasion to use is when a fermion mass has the "wrong sign". In this case, the reader can easily check that the transformation, $\psi \to \gamma_5\psi$ fixes this sign.

This does not, however, preserve the Majorana nature of ψ. If ψ is Majorana, the appropriate transformation should be $\psi \to i\gamma_5\psi$. Both these transformations preserve the kinetic terms, but may introduce additional γ_5 matrices as well as i's in interaction terms. These are important as they lead to physically observable changes in amplitudes.

- For a system of fermions, the physical masses are given not by the eigenvalues of the "fermion mass matrix" (which need not even be Hermitian). Instead, the eigenvalues of the mass matrix times its Hermitian adjoint are the squares of the fermion masses.
- We can eliminate γ_5 dependence in fermion bilinears by separately rotating the left- and right-chiral components. Care must be taken, however, if we are dealing with Majorana fermions, to preserve their Majorana character. Such γ_5-dependent mass terms, which are not precluded by Poincáre invariance, frequently signal CP violation.[11] In two-component notation, the analogue of these is phases in masses for spinor fields.

[11] Recall that $\bar{\psi}\gamma_5\psi$ is odd under CP.