ALPHA-QUENCHED ALPHA-LAMBDA DYNAMOS AND THE EXCITATION OF NONAXISYMMETRIC MAGNETIC FIELDS

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Abstract. We present calculations showing how stable nonaxisymmetric magnetic fields may be excited in an alpha-quenched mean field dynamo in a deep spherical shell. The large scale velocity fields (differential rotation, meridional circulation) are determined by solving the axisymmetric Navier-Stokes equation, neglecting the Lorentz force but including a term parameterizing the turbulent Reynolds stresses.

Key words: Lambda Effect – Nonlinear Dynamos – Nonaxisymmetric Fields – Active Giants

1. Introduction

Recently observational evidence has accumulated that some late type active giant stars with deep outer convection zones possess large scale, long lived, nonaxisymmetric magnetic fields (see, for example, the discussion in Moss et al., 1991). Given the small radiative core and large effective magnetic diffusivity in such stars, it is very probable that such nonaxisymmetric fields are dynamo generated. However, for simple and arbitrary distributions of α - effect and angular velocity Ω, linear mean field dynamo theory predicts that axisymmetric modes are first excited as the relevant dynamo number is increased. (Allowing the α tensor to be strongly anisotropic may alter the situation, see Rüdiger and Elstner (1992), but we will not consider that possibility here.) Special spatial distributions of α and Ω can be found for which this is not true, provided that the differential rotation is not too strong, although it appears difficult to arrange for nonaxisymmetric modes to be significantly more easy to excite. Strong differential rotation inhibits nonaxisymmetric field generation, particularly in spherical geometry.

Linear theory is in general an unreliable guide to behaviour in a strongly nonlinear regime (e.g. Brandenburg et al., 1989). Nevertheless nonlinear 'α - quenched' models with stable nonaxisymmetric fields have only been found for rather special spatial distributions of α and Ω (Rädler et al., 1990; Moss et al., 1991; Moss, 1991).

Of course, in a more realistic model, the fluid velocities would not be prescribed arbitrarily, but would form part of the solution of a self consistent problem in which the Navier-Stokes equation, including the Lorentz force, is simultaneously solved to give the large scale fluid velocities that appear explicitly in the dynamo equation. It is quite impossible at present to solve the full dynamical and thermodynamical equations for a complete stellar convection zone. Mean field theory can be employed to parameterize the effects of the small scale turbulent motions in the Navier-Stokes equation, by including a term representing the turbulent Reynolds stresses (e.g. Rüdiger, 1989). The remaining problem, of solving the mean field dynamo and Navier-Stokes equations, is nonetheless formidable.

A step towards self consistency can be made as follows. Solve the axisymmetric hydrodynamical problem, without Lorentz forces, but including the parts of the turbulent Reynolds stresses that directly drive a differential rotation (i.e. the 'Λ -

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effect', see Rüdiger, 1989). This hydrodynamic solution provides an axisymmetric azimuthal velocity (differential rotation) and a meridional circulation. Now solve the kinematic mean field dynamo equation with these large scale velocity fields present. When the dynamo is excited the field can be limited at finite amplitude by introducing a parametrization of the feedback of the magnetic field onto the turbulence (e.g. α - quenching).

We describe some such calculations, and show that, contrary to the results from simple linear models and also from kinematic nonlinear studies (Rädler et al., 1990; Moss et al., 1991), stable nonaxisymmetric field structures can be generated without choosing *a priori* special spatial profiles for α or Ω.

2. The Model

We solve the axisymmetric incompressible mean field Navier Stokes equation in a spherical shell, $0.1R \leq r \leq R$ (r, θ, ϕ are spherical polar coordinates). In the inertial frame it takes the form

$$\rho Du_i/Dt = -\partial P/\partial x_i - \partial(\rho Q_{ij})/\partial x_j, \tag{1}$$

where

$$Q_{ij} = \Lambda_{ijk}\Omega_k - N_{ijkl}\partial u_k/\partial x_l. \tag{2}$$

The reduced pressure, P, includes the gravitational term. The second term on the right hand side of (2) gives rise to the 'turbulent viscosity', ν_T. Under the simplest assumptions about the anisotropy of the turbulence caused by gravity,

$$\Lambda_{ijk} = \Lambda_V(\epsilon_{ikp}g_j + \epsilon_{jkp}g_i)g_p, \tag{3}$$

where g is the unit vector parallel to gravity. When the turbulent correlation time is less than the rotation period, we can write $\Lambda_V = \nu_T V^0 f(r)$, where V^0 is of order unity. See Rüdiger (1989) and Brandenburg et al. (1991) for further details. We take $f(r)$ to be unity in $r > 0.3R$ and to go smoothly to zero at $r = 0.1R$. Boundary conditions are that the fluid be stress free with zero radial velocity component at $r = 0.1R$, and $r = R$.

Solution of (1) yields azimuthal and meridional velocities, $v(r,\theta)\hat{\phi}$ and $\mathbf{u}_m(r,\theta)$. We then solve the mean field nonaxisymmetric dynamo equation

$$\partial\mathbf{B}/\partial t = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha\mathbf{B} - \eta_T\nabla \times \mathbf{B}), \tag{4}$$

with $\mathbf{u} = v\hat{\phi} + \mathbf{u}_m$, and taking $\alpha = \alpha_0 f(r)\cos\theta/(1 + \mathbf{B}(r,\theta,\phi,\tau)^2)$, where α_0 is a constant. Boundary conditions on the magnetic field are that it fit to a vacuum field in $r > R$ and that the region $r < 0.1R$ is a perfect electrical conductor. Our nondimensional time, τ, is measured in units of R^2/η_T.

We prescribe the angular momentum of the shell, and this defines the corresponding angular velocity Ω_0 of uniform rotation. The Taylor number $Ta = (2\Omega_0 R^2/\nu_T)^2$ and V^0 are the governing parameters for the hydrodynamic problem. Solving equation (4) introduces a further parameter, $C_\alpha = \alpha_0 R/\eta_T$. The numerical code is a modification of that described in Moss et al. (1991). A NI $\times$ NJ grid is used over $0.1R \leq r \leq R$, $0 \leq \theta \leq \pi$, and a modal expansion in Fourier components inϕ. We take NI=31 or 51, NJ=61 or 101, and four ϕ modes were found to be adequate.

Solutions of the hydrodynamic problem (equations (1), (2)) can be characterized by the parameters $C_\omega = (\Omega(R, \pi/2) - \Omega(0.1R, \pi/2))R^2/\eta_T$ and $C_m = UR/\eta_T$, where U is the mean velocity defined by equating $0.5\rho U^2$ with the mean kinetic energy density of the meridional motions. We describe the gross features of our dynamo solutions by the parameters $M(\tau) = E_{\rm nax}/E_{\rm tot}$ and $P(\tau) = (E^{(S)} - E^{(A)})/(E^{(S)} + E^{(A)})$, where $E_{\rm nax}, E_{\rm tot}, E^{(S)}, E^{(A)}$ are the energies in the nonaxisymmetric, total, equatorially symmetric and antisymmetric magnetic fields, respectively.

3. Results

Solutions of the hydrodynamical problem (equations (1) and (2)) are described in Barker (1993). The meridional flow consists of a single cell in each hemisphere and, with $V^0 > 0$, the flow at the surface is equatorward and the angular velocity increases outwards. Streamlines of the meridional flow and angular velocity contours are shown in Fig. 1, and C_ω and C_m values are given in Table 1. Note that the definitions of C_ω and C_m, although fairly conventional, are not equivalent and that, for example, when $Ta = 10^5$ the ratio of the kinetic energy of the differential rotation to that of the meridional circulation is about 180.

TABLE I

Parameters of the hydrodynamical models and summary of the dynamo calculations.

Ta	C_ω	C_m	Nature of dynamo solutions
4	1.0	0.003	Slow 'diagonal' evolution towards A0 solution
10^4	50.0	0.9	Almost 'diagonal' evolution towards A0 solution
10^5	150.0	1.3	Stable solution with M = 1 and P $\approx$ 1
10^6	440.0	1.7	Rapid evolution to stable S0 solution

We then integrate equation (4) for initial conditions P(0), M(0), and $4 \leq Ta \leq 10^6$, with $\eta_T = \nu_T$, $V^0 = +1$, taking $\mathbf{u}$ from these hydrodynamical solutions. For these values of Ta the S0, A0, S1 and A1 modes are excited at C_α values of order 10, with the axisymmetric modes more readily excited when Ta is greater than a few times 10^4. For smaller Ta, the situation is complicated (see Barker, 1993). In the following we have taken $C_\alpha = 15$, a clearly but not highly supercritical value.

For Ta=4 the evolution from an arbitrary initial state is very similar to that of the α^2 dynamos studied by Rädler et al. (1990) and Moss et al. (1991). After initial relaxation the system evolves along the diagonal in the (P,M) plane joining (1,1) and (-1,0), towards the A0 configuration on a timescale of many diffusion times. When Ta=10^4, evolution in the (P,M) plane is again towards the A0 configuration, but follows the diagonal rather less closely.

In contrast, when Ta=10^5, evolution from an arbitrary initial state (with P$\approx$0.5, M=0.5) is to M=1, on a comparatively rapid timescale. When M $\approx$ 1, P oscillates

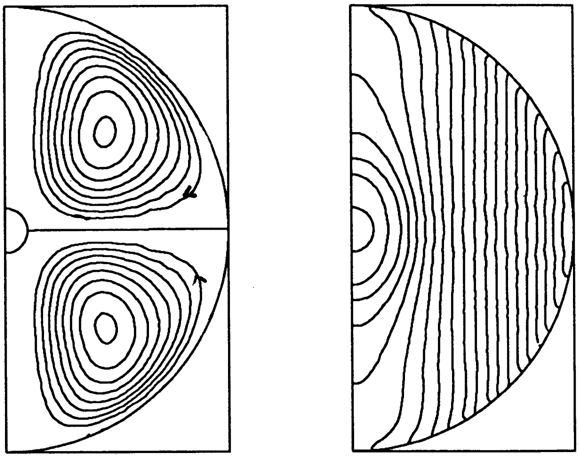

Fig. 1. Meridional streamlines (left) and contours of constant angular velocity (right) for the model with Ta = 10^5.

at low amplitude and the maximum and minimum values are increasing very slowly (this behaviour occurs for P > ca. 0.96). We did not follow the solution long enough to decide conclusively whether the final configuration was a limit cycle with P $\approx$ 1 or steady with P = 1 precisely. Contours of radial field strength in the surface $r = R$ are shown in Fig. 2. We also perturbed the pure parity dynamo solutions with M=0, P=±1 and, by following the subsequent evolution, showed that both the A0 (P= -1, M=0) and the S0 (P=+1, M=0) solutions are unstable.

When Ta=10^6, evolution from the initial mixed parity, mixed symmetry, state to the stable S0 configuration occurs on a timescale much shorter than a diffusion time. The dynamo solutions are summarized in Table 1.

4. Conclusions

We have taken an initial step towards constructing dynamically self consistent, nonaxisymmetric mean field dynamos without constrainst on symmetry properties. Our large scale velocity fields are not completely arbitrary, but arise from a dynamical model, albeit without feedback from Lorentz forces. We find that for certain Taylor numbers, corresponding to a modest degree of differential rotation, these velocity fields together with a conventional α-effect and α-quenching, can generate stable nonaxisymmetric fields. For larger values of Ta (and of differential rotation) our results are consistent with the idea that strong differential rotation inhibits nonaxisymmetric field generation and that only axisymmetric solutions are then stable.

Meridional circulation may play a minor role - experiments in which it was artificially turned off do give a slight reduction in the strength of nonaxisymmetric fields. This was also noted for linear models by Barker (1993). The sense and strength of the circulation and the excitation of a nonaxisymmetric field with $P \approx 1$ is in gen-

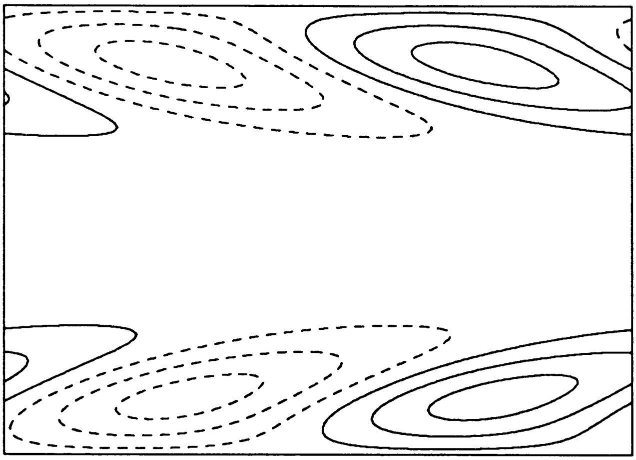

Fig. 2. Contours in the surface r=R of equal radial field strength when Ta=10^5. The top and bottom of the figure correspond to the north and south pole, respectively, and longitude ϕ runs horizontally from 0 to 2ϕ. Broken curves denote negative values.

eral terms consistent with the Gailitis dynamo (Gailitis, 1970; Moss, 1990), but the effect here is certainly small. Barker (1993) demonstrated the importance of the latitudinal dependence of the angular velocity for nonaxisymmetric field excitation.

Our results are somewhat at variance with those from conventional linear and nonlinear $\alpha^2\omega$ dynamo models. These studies have shown that nonlinear models can generate stable nonaxisymmetric fields, but they then require careful choice of α and $\Omega(\mathbf{r})$. We have not attempted a full exploration of parameter space (e.g. also varying C_α and V^0), but are content to emphasize that we do not choose α and $\Omega(\mathbf{r})$ in a completely *ad hoc* manner, but rather allow Ω to arise from a dynamical model that also gives a meridional circulation. The next step is clearly to include the Lorentz force in the Navier-Stokes equation (1). Our preliminary results suggest that, for appropriate parameter values, stable nonaxisymmetric fields continue to be excited.

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