

# REVIEW ARTICLES

*Jacques Nicolle*

## ON SYMMETRY

Having observed manifestations of symmetry in the world around them, men took pleasure in making use of them in their artistic reproductions and in their techniques. During the course of the centuries scholars have studied the attributes of symmetry in order to establish a perfect axiom, self-sufficient, and able to provide the solution for numerous problems. Just as trigonometric relationships can be used to measure the distance that separates the place one occupies from another place, often inaccessible, so it is possible, by a study of symmetry, to know in particular the spatial constitution and the attributes of certain molecules without having direct access to them.

*Symmetry* signifies “with measure” (*sun*: with, *metron*: measure) or, in its generally accepted meaning, exact proportion, measurability, parity, and harmony.

*Asymmetry* (from *a*, the privative prefix) denotes a “lack of symmetry.” An asymmetrical body is one which possesses no symmetrical element whatsoever.

*Dissymmetry* (from the prefix *dis*, bad or difficult) is a term used by Pasteur as early as his first studies on crystals. In the mind of that scientist, a molecule is dissymmetrical if it possesses neither a design nor a center of symmetry, yet it may have axes of symmetry.

Symmetry is an equilibrium (the diverse levers, the balance), which

Translated by Elaine P. Halperin.

controls the stability of objects. It is indispensable to our means of transportation—an airplane that suddenly loses a wing is a dramatic example of this!

Without going into developments of too mathematical a character, let us simply state here that symmetry can be applied to finite or to infinite figures. A solid cube, all of whose surfaces are equal to each other, is a simple example of the first category, while a roll of wallpaper upon which the same pattern is repeated at regular intervals represents an infinite figure, since the design on it can be reproduced by *transfer* as many times as one wishes. The face of a cube occurs similar to itself only four times if the cube is turned on an axis passing through the center of the two faces perpendicular to the axis so chosen. We can say that in finite figures symmetry consists in the *repetition* of one *part* of the object itself. In concluding these brief technical observations, we must point out that symmetrical *operations* are divided into two entirely distinct categories. The first comprises *transfer* and *rotation* (and the combination of both, the *helical* movement) which yields figures called *congruent*, the kind that can be overlaid. An illustration of this is the movement of a hand in space. Operations of the second kind are *mirage* and *inversion*. They produce figures called *enantiomorphic* (of contrary form), figures that cannot be overlaid, for example, the right and left hands. Operations of the first kind, it should be noted, permit you to change from a first to a second position in a continuous fashion, while operations of the second kind are absolutely discontinuous. You cannot pass by means of a series of displacements of your person to its reflection in a mirror.

With these very brief definitions in mind, we shall now see how men made contact with the idea of symmetry in nature.

Because of the existence of weight, the appearance of a design of vertical symmetry is an absolutely natural thing. If one spreads over the ground sand made up of grains of different categories (the heaviest falling first), horizontal stratifications would occur and thus one would witness, if the heap is even, the appearance of a *vertical design of symmetry*. Men, most of the animals that live around us, and numerous plants represent a design of this kind. It is quite certain that the existence<sup>1</sup> of this design among men predisposes them to react more naturally to such a symmetry and obliges them to utilize this attribute in their construction of buildings

1. "Symmetry is what one sees from a point of view based on that which there is no reason to make otherwise and based also on man's shape, which explains why one desires symmetry in width only, not in height or in depth." *Pensées*, Pascal, p. 25.

and furniture.<sup>2</sup> However, although we do not generally notice the presence of this attribute, we are surprised when it is missing; for example, we are struck by the lack of symmetry wherever we customarily encounter it, as in an automobile that lacks a wheel or a man who has lost a leg or a setting different from the others on an impeccably arranged table. To see again a face that one recognizes makes an impression that can be either pleasant or unpleasant but which, in either case, does not leave us completely indifferent.

The existence of identical twins is an example of symmetry that is well known to everyone. In monozygotic twins (issued from the same egg and therefore of the same sex), one encounters an opposite tendency (symmetry in relation to a design) of certain elements. If one of the two has a right eye that is larger than the left, the inverse might be true of the other twin. The same could hold for the way the hair grows or for fingerprints. If historians are to be believed, Johann Sebastian Bach's father had a twin brother who resembled him so closely that their respective wives found it difficult to tell them apart. They both had the same talent for music, were often ill at the same time, and died almost simultaneously.

A more detailed analysis provides us with numerous cases of symmetry in the three kingdoms of nature. Among the animals, we encounter radiating symmetry in the Heliozoa, which are practically spherical. The Foraminifera have spiral shapes which, according to the species, are either right or left (the enantimorphic objects of our introduction). The Coelenterata possess axes of the 6 and 8 kind and the Echinodermata axes of the 5 kind (a symmetry not found in crystals). Man and the vertebrates, however, must content themselves with a simple vertical design!

It is also important to observe the labor of animals: spiders' webs are symmetrical in relation to a design if one disregards the two spirals, but it is mainly the bees that teach us geometry.

The following is a quotation from the work of Professor Léon Bertin, *La vie des animaux*: "Apparently no one observed the rhomboidal form of the bottom (of the bee-hive) before the eighteenth century. In 1712 a nephew of Cassini, Maraldi, an astronomer at the Observatory of Paris, determined the angles of the lozenges precisely and experimentally. He

2. The symmetry of the human body is merely an approximate one. If one reproduces the two right or the two left sides of a face on a photograph (which can be done most easily by means of a mirror placed perpendicularly to the portrait) the effect obtained is often monstrous. The right side of the face is olive-shaped and the left almost spherical. These very special reproductions apparently enable one to exteriorize certain characteristics and are employed in some countries for the purpose of criminological research.

found them equal to  $109^{\circ}28'$  and  $70^{\circ}32'$ . Réaumur suspected that the bees must be guided for reasons of economy, in the construction of the bottom. He suggested to the German geometrician, Koenig, without first acquainting him with Maraldi's findings, the resolution of the following problem: 'Among all these hexagonal cells of the bottom composed of three equal rhombuses, determine which cell can be constructed with the least matter.' Koenig dealt with the problem by differential calculus and found that the angles of the lozenge must be  $109^{\circ}26'$  and  $70^{\circ}34'$ . This much agreement with Maraldi's measurements is already quite surprising; but there is still more. In 1743, MacLaurin proved that Koenig made a mistake in his calculations and that, in resolving the problem one discovers that the correct measurements of the angles are precisely those indicated by Maraldi and achieved by the bees—in other words,  $109^{\circ}28'$  and  $70^{\circ}32'$ . After this, should one not write on the top of the bee-hive this phrase so dear to Pythagoras: 'Let no man enter here unless he is a geometrician?'

In the vegetable kingdom we observe innumerable cases of symmetry which everyone can easily recognize, particularly among flowers and fruits and also in the posture of trees. The growth of vegetables can assume various forms. The most simple cases are those concerning elongation in cylindrical or conical shapes, but there are also spherical growths and those that present spirals which, according to the case and to the species, can develop into right or left helices. We must quote in passing some studies on phyllotaxy which deal with the more or less regular arrangement of leaves around a stem. Curiously enough, this problem has concerned a very large number of thinkers, artists or scholars, such as Leonardo da Vinci (1515), Robert Brown (1658), Malpighi (1675), J. W. Goethe (1749–1832), Bravais (1837), d'Arcy-Thompson in 1942 and Lucien Plantefol in 1945. All the scholars who preceded Plantefol in this investigation claimed that the growth of leaves around a stem formed a kind of spiral (this term is employed here in the popular and not in the mathematical sense of the word). In going over the curve in question, and in measuring certain sizes concerning the number of leaves they found between determined intervals, they attempted to identify their results with those of a known series in mathematics, called the Fibonacci series. The recent studies of Professor Plantefol have enabled us to establish that in reality the leaves are scattered over several helices surrounding the stem and not over one unique curve. This is an illustration of the danger that exists in the field of science when one attempts to verify preconceived ideas at any price!

But it is above all in the mineral kingdom that the best and most char-

acteristic examples of symmetry are evident. When men acquired knowledge about the bowels of the earth they were struck by the peculiar arrangements of the polyhedrons whose admirable geometric patterns they encountered. It was in the study of crystals and their properties that the doctrine of symmetry achieved great progress. This it did by effecting a harmonious synthesis between the data of pure mathematics compiled by numerous researchers and the observations of mineralogists, crystallographers and geologists.

Impressed by the relationships and the attributes of symmetry, man naturally was led to utilize multiple combinations of them in the arts and in the sciences in order to embellish and improve his life.

In the arts man concentrated on reproducing the various manifestations that he had observed in the kingdoms of nature, conceiving, as well, many different combinations. It is impossible to develop here the theories of symmetry in regard to art, one interesting example being the evolution of symmetry in theatrical scenery throughout the centuries.

In ancient times the architecture of the theatre embodied the usual pattern of vertical symmetry, this being true as well for scenery designed for the stage. In the Middle Ages the "miracle" and "mystery" plays in the various places where they were produced employed a device that was called the "simultaneous scene" as distinguished from the customary "successive scene." During this period no characteristic plan of symmetry existed, but one notes a symmetry that stems from symbolism (the three divisions: heaven, earth and hell) or from repetition (the stations of the Cross).

The symmetrical plan returned with the Italian humanists of the Renaissance. The great problem was that of perspective. The following quotation from Pierre Sonrel's excellent book contains some curious details about the work of the great scenic designer, Sabbattini:

"First of all Sabbattini placed upon the stage canvasses upon which façades of houses were drawn and painted. His design was limited only by the dimensions of the theatre, which he made as large as possible in order to leave room for the actor's entrances. The height and the details of the houses were as yet undetermined. Into the center of the rear wall of the stage and at the level of the Prince's eye, he hammered a nail. From the Prince's seat and at the level of his vision he drove another nail into a post. Finally, he connected these two nails with a string that served as a line of vision. Then, a light in his hand, he moved to the left of the string to fix the

line of the canvasses on the right. He next did the same on the opposite side. His assistants then drew the shadow that the string cast upon the canvas frames at various levels, determining thus the height of the roofs, the cornices, windows, balconies, doors. With the aid of a second horizontal string suspended perpendicularly to the first, Sabbattini thus obtained all the other lines of his construction directly on his canvasses. An empirical method, undoubtedly, but quick and ingenious, useful to his contemporaries who resorted to perspective without always possessing its secrets."

During the classical period, no matter what play was being given, the backdrop, which represented a palace, a triumphal arch, a garden, or some other such place, was handled in an absolutely symmetrical manner. But, as Hélène Leclerc observed in her very fine work, *Les origines italiennes de l'architecture théâtrale moderne*, Servandoni replaced the "symmetrical view" of the seventeenth century with the "oblique view"—an example of the influence of the Baroque trend of the eighteenth century.

In keeping with the wish to break "deliberately" with the symmetry of the latter part of the last century (melodrama), we have now returned to a great variety of concepts. I believe that a number of decorators in our day are seeking a certain symmetry (balance and harmony in forms and in colors) and are blending it quite rightly with the room that they are supposed to "dress."

We must add that symmetry is also to be found in the very composition of our plays. The classical theme, for example, provides us with the unfolding of two parallel actions of two pairs: masters and servants mixed up in analogous intrigues.

As René Bray has indicated, "the rhythm is of major importance in Molière's comedies, just as it was in the *Commedia dell'Arte*. The comic fragments are handled with skill, as the function of play and not of intrigue, and in accordance with a cadence that stems almost from the choreography."

Apropos of this, we owe it to ourselves to point out that symmetry is employed currently in the art of the dance. A dancing couple can represent a "transferred" symmetry when the two partners carry out similar movements (the two right legs moving together in the same step), or else a "mirrored" symmetry (symmetry in relation to a design) when they execute opposite movements, one turning to the left and the other to the right. If we think of a group of chorus girls who in unison reproduce the same pattern at a given moment, we can see that we have here too a

characteristic effect of symmetry by transfer, but achieved in a dynamic manner reminiscent of the frieze of a decorative design that has been executed in a static manner.

In his *Dictionnaire raisonné*, Viollet-le-Duc expresses himself in the following terms: "Symmetry today means in the language of architects not a balance, not a harmonious relation of the parts to the whole, but a similarity of opposite parts, the precise reproduction to the left of an axis of that which is to its right. One must render this justice to the Greeks, authors of the word 'symmetry': they never gave it such an empty meaning." I believe that this is mainly a quarrel about words. It is quite certain that in all times architects and decorators never limited themselves to considering the kind of banal and oversimplified symmetry that consists in flanking the clock on the mantelpiece with two identical candlesticks. Rather, they attempted in a much more subtle fashion to enlarge the notion of right and left in the form of an equilibrium, of a mass relationship. Measurable relations exist between the sizes of objects that we use for our pleasure. In particular, the candlesticks made by the famous Germain were all fashioned in the proportion of four to seven from the base to the top. Furthermore, the size of the base was never less than the fraction six over eleven. We should remember that in decorative patterns the symmetry of colors (their balance) plays as important a role as the symmetry of geometrical forms. In my opinion Oriental rugs owe their beauty to the fact that they possess what I would call a pseudo-symmetry. They are never fashioned perfectly from the standpoint of pure geometry; the colors change progressively from the beginning to the end of the rug, apparently in order to exorcise the evil eye!

The question of right and left has often been brought up in regard to painting as well as to scenery. The historian of art, Heinrich Wölfflin, demonstrated, using famous examples, that something of a peculiar nature is often to be found on the right-hand side of painting that does not exist at the left. I have often myself experimented with projecting reproductions of paintings on a screen both on the right and on the left side. Most of the audience, made up of people who had no special training in art, realized which was the correct side. And so one can say that very often the average observer (if he is not left-handed) is attracted by what is on the right side rather than by what is on the left. This theory in regard to painting, which fully accords with the opinion of René Huyghe, was rather amply developed by the German physiologist, Wilhelm Ludwig, in his book entitled, *Das rechts-links Problem im Tierreich und bei Menschen*.

In this connection it is interesting to remember that Goethe, in his rules for the theatre, indicated that to make a dramatic impression, the scene must be laid in the courtyard and not in the garden. An inquiry that I am conducting at the moment in competent artistic circles confirms this opinion. I must also call attention to the numerous observations demonstrating that people who are racing on foot, on horseback, in a car, on a bicycle or on ice skates, always turn counter-clockwise. Such examples only confirm this distinction between the right and the left, a distinction which often appears when one remarks upon scientific research that takes into account the attributes of symmetry.

Louis Pasteur showed that bodies (that were called racemic) are capable of dividing themselves by certain methods (which this scientist indicates) into two components, a right one and a left one, and each the image of the other in a mirror. Pasteur calls attention to the very important fact that the material objects that we encounter can be divided into two categories: "In the one case, objects placed in front of a mirror reflect an image that can be superposed on them; the image of the others cannot cover them, although all their details are faithfully reproduced. A straight staircase, a stem with distichous leaves, a cube, the human body—these are in the first category. A curved staircase, a stem with leaves that grow in spirals, a screw, a hand, an irregular tetrahedron—these fall into the second category. The latter have no symmetrical design."

On the other hand we know that composite bodies are aggregates of identical molecules, themselves composed of elementary atoms distributed according to laws that regulate their nature, their proportion, and their arrangement. The individual component of every composite body is its chemical molecule, which is a homogeneous group of atoms with a very definite arrangement. All physicians envisage the constitution of bodies in this way.

Once this is understood, it would surely be surprising if nature, so varied in its effects, and whose laws allow for the existence of so many species of bodies, did not offer us, in the atomic groups of composite molecules, both of these two categories which include all material objects. In other words, it would be truly extraordinary if there were not some individuals with superposable images and others with non-superposable images among all chemical substances, natural or artificial. And this is actually the case, all chemical combinations without exception being distributed equally into the two categories stated above.



In nature, this manifests itself in a particular way, since one currently encounters only one of the component elements—for example, glucose and most of the sugars in the form of the *right* constituent, asparagine and the bodies called amino-acids in the *left* form. Why this is so is impossible to explain here. To do so it would be necessary to go back to the beginning of the world and to examine the reasons why a particular orientation happened to be given to the body in question. However, we must point out, as Louis Pasteur has done, that we find ourselves in a dissymmetrical universe. Actually, our globe turns in a definite direction on its own axis and around the sun; the terrestrial magnetic field is oriented. It is therefore natural, from our standpoint (since a phenomenon can spring only from a dissymmetry) that we do not find the molecules of left and right bodies equally distributed, as purely statistical considerations would dictate.

We are indebted to the great French scientist, Pierre Curie, for very important works on the relation of symmetry to physical phenomena. Just as Pasteur established a theory of molecular dissymmetry, so Curie demonstrated that for a phenomenon to occur, certain elements of symmetry must be nonexistent. If we place two equal and parallel iron blades in acidulous water and if we connect these two elements by a conducting rod, we do not obtain an electric current. But if we magnetize one of the blades a current may be produced, and, actually, experiments show that it is produced. It would be the same if the two blades were made of different metals.

Let us heat a crystal of tourmaline under a gas flame for a few seconds, and then place it on a glass slide. Let us blow a mixture of sulphur and red lead powder onto the crystal. The sulphur which is electrified negatively by being breathed upon fastens itself to one of the pyramidal extremities of the tourmaline. The red lead, positively electrified, will become fixed at the other end. Thus one obtains a crystal presenting a yellow “pole” and a red “pole.” Electricity has differentiated between colors! This stems from the fact that by means of heat or pressure (in the case of piezoelectricity) one is able to develop electrical currents of a different voltage.

The three examples show, as Pierre Curie has often said, that it is dissymmetry that creates the phenomenon. He added the following remarks: “It would be much more logical to call a design of dissymmetry any design that is not symmetrical, to call an axis of dissymmetry any axis that is not a symmetrical one, etc., and in general to give the list of operations which are not recovery operations of the system.” [He is referring to finite groups.] “But in the groups under consideration, there are an infinite number of

operations of recovery: and so it is much simpler to supply the list of these operations.”

Curie has demonstrated by experiments and by discussion of a theoretical nature that when different and naturally diverse phenomena are superposed, the final system that results embraces solely those elements of symmetry which are common to each of the systems under consideration. He also showed that the symmetrical elements of causes are to be found again in the effects produced. “The dissymmetry of effects,” he says, “must be recovered in the causes.” But we must call attention to one remark of his that is of major importance: the reverse is not true, and effects can be more symmetrical than causes because certain causes of dissymmetry apparently are not effective.

A fine experiment made by the German physicist Wiedemann consists in superposing an electric and a magnetic field and in demonstrating that the resultant phenomenon no longer possesses more than those elements of symmetry that are common to the two component phenomena. All that would remain in space would be an axis (called isotropy).

The following is the experiment in detail:

1. If a metal wire is magnetized lengthwise and one shoots an electric current through it, the wire bends.
2. Let us take the same magnetized wire and twist it. At this moment, an electro-moving force springs up in the wire that has been shot through with a current, provided it is placed in a closed circuit.
3. A wire shot through with a current is entirely magnetized lengthwise when it is twisted.

Let us now go on to chemical biology. We note with some surprise that microbes can differentiate between a *right* and a *left* body.

In 1857 Pasteur observed that certain mushrooms utilized the right composite of tartric acid and that the left remained intact. Since this memorable discovery, numerous studies have been made on the subject which show that microbes accept the right bodies of the series of sugars as a source of carbon and the left bodies of the series of amino-acids as a source of nitrogen. We have even demonstrated during recent years that the right bodies (called right antipodes) of certain amino-acids that are not adaptable to the micro-organism in question possess a certain inhibiting power in regard to the cultures of certain microbes. It is interesting to point out in this connection that “nonnatural” bodies are to be found in various antibiotics.

We must add that sight is not the only means we have of distinguishing the diverse forms of symmetry; taste and smell perform this function as well! It is through these two senses that we can recognize the orientation of molecules. The word molecule is particularly apt here since it is thanks to the molecular scale that we can smell the slightest trace of a subtle perfume in a large theatre. In the physiological domain one has always observed obvious differences in the taste of the *right* and *left* components of the same body. Without going into a detailed discussion, we would merely like to point out that as early as 1886 Piutti demonstrated that one of the antipodes of the asparagine has a sugary taste, while the other is tasteless.

These principles of symmetry can very often be applied in scientific research. When a chemist attempts to establish the representative formula of a body, he frequently finds himself in the presence of numerous combinations; he can make a selection only by relying upon the different characteristics which diverse methods provide. However, a good knowledge of the principles of symmetry immediately enables him to eliminate a very large number of schemata, thus facilitating his task. A great many physical constants, which nowadays are obtained with great accuracy, are directly connected with the symmetry of the bodies studied. Rotative power, the existence of the piezo and of pyro-electricity and even the temperature of fusion provide us with useful information concerning the structure of bodies. Microbes themselves are capable of distinguishing the degree of symmetry of the products that they need. We have shown by a great many experiments that these organisms can utilize the right or left antipode of a body, but not both. But even more remarkable is the fact that the "bad" antipode is in many cases an inhibitor of microbic growth, its effect being like that of the antibiotics.

Hence, everywhere, and in all domains, we encounter this notion of symmetry, among living beings as well as in the mineral kingdom. The relationships it imposes determine a great many phenomena.

We must add that mathematics is not a pure creation of the mind. The elements of this science happen to be within our reach, and we discover them when we analyze the world that surrounds us and to which we belong. The symbol  $\pi$  (the ratio of the circumference to the diameter) is one of the constituents of our universe; it is not something that mathematics invented. All the attributes of symmetry, all the relationships which this discipline presents, exist independently of us. As we have shown, perhaps

too rapidly, symmetry is a powerful and subtle means of investigation; it aids scientific progress and enables us to resolve problems, the solution of which would not otherwise be directly accessible to us. It shows us the possible structure of those constituents of matter which our senses do not reveal—at least not at the present moment.

Science traversed successive stages before assuming the appearance it has today. In the beginning men observed the world that surrounded them. Afterward they began their experiments. The means of producing fire was the first great discovery. Then, in the course of the centuries, came the three other fundamental discoveries: steam, electricity, atomic energy.

Observation at first played the principal role. Experimentation and then prediction followed. We have seen, in the course of this brief exposition, that man was called upon to encounter in various domains very numerous manifestations of symmetry (twins, flowers, crystals). Next we saw that he was able to utilize the attributes of symmetry in their artistic representations in order to fashion the utensils and objects he needed. Afterward he began to experiment. Thanks to quartz crystals (and to those properties of crystal that are directly connected with symmetry), it became possible to produce the ultra-sounds that enabled man to take soundings of the bottom of the sea and thus to detect submarines. Pasteur's accomplishments in the most diverse domains stemmed directly from his first studies on molecular dissymmetry. Now we have arrived at the stage of prediction. Pierre Curie, as we pointed out, established propositions which demonstrate that by making use of the attributes of symmetry, one can ascertain whether the realization of a phenomenon is possible or not. I believe that in time we will be able to determine, in many cases, the biological attributes of certain molecules when we acquire knowledge of the precise role of their groupings in relation to questions of positions in space.

Problems of right and left in art as well as in science force us to reflect upon the privileged orientations that we encounter in the world's system—the rotation of the earth and the stars in a definite direction, the appearance of the first molecules of certain bodies in either the right or the left form, the predominance of the right hand over the left. These are fascinating problems, which demonstrate that what exists is not “sciences” but only Science and its applications. Let us repeat, on the subject of this very special discipline with which we have here dealt, the remark of the great physicist Louis de Broglie: “Symmetry constitutes one of the most important orientations of contemporary scientific thought.”

## BIBLIOGRAPHY

- L. Bertin, *La vie des animaux*. Paris, Larousse, 1949.
- R. Bray, *Molière, homme de théâtre*. Paris, Mercure de France, 1954.
- P. Curie, *Œuvres*. Paris, Gauthier-Villars, 1908.
- M. Lamy, *Les Jumeaux*. Corrèa, 1949.
- H. Leclerc, *Les origines italiennes de l'architecture théâtrale moderne*. Paris, Droz, 1946.
- H. Leclerc, Article in the *Revue d'histoire du théâtre*, Tome IV, p. 259, 1953.
- W. Ludwig, *Das rechts-links Problem im Tierreich und bei Menschen*. Berlin, J. Springer, 1932.
- J. Nicolle, *La symétrie et ses applications*. Paris, Albin Michel, 1950.
- J. Nicolle, *Un maître de l'enquête scientifique Louis Pasteur*. Paris, La Colombe, 1953.
- J. Nicolle, *La symétrie dans la nature et les travaux des hommes*. Paris. La Colombe, 1955.
- L. Pasteur, *Œuvres complètes*. Paris, Masson et Cie, 1922.
- L. Plantefol, *La théorie des hélices foliaires multiples*. Paris, Masson et Cie, 1948.
- P. Sonrel, *Traité de scénographie*. Paris, O. Lieutier, 1943.
- Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française du XIème au XVIème siècle*. Paris, A. Morel, 1869.