ing. (3) That propositions should not be repeated in a rigid form of words, but that the teacher insist on intelligent expression. (4) That geometry is a subject eminently fitted for oral exposition ; and that each proposition, before being prescribed to be learned, ought to be taught to the class. (5) That the text book should contain the propositions put as clearly as possible with easy exercises accompanying. (6) That symbols and contractions, as far as their use tends to simplicity, should be employed. (7) That the work be systematically reproduced in writing. (8) That revision might occasionally be made by retracing the chain of propositions. (9) That the quality of the geometrical work done, rather than its quantity, determines its educational value.

On Voting.

By A. Macfarlane, D.Sc., F.R.S.E.

Suppose that we have c candidates, e electors, s seats, v votes.

There are at least three different kinds of voting to consider: Simple, Combinational, and Cumulative.

I. *Simple voting.* By simple voting I mean any case in which an elector has only one vote. Denote the candidates by A, B, C, D. The possible ways in which elector No. 1 can vote are given by

$$A_1 + B_1 + C_1 + D_1 ;$$

similarly for elector No. 2,

$$A_2 + B_2 + C_2 + D_2.$$

The possible results of No. 1 and No. 2 voting are obtained by multiplying together the possible ways for each, hence they are :—

$$\begin{array}{l} A_2A_1 + A_2B_1 + A_2C_1 + A_2D_1 \\ B_2A_1 + B_2B_1 + B_2C_1 + B_2D_1 \\ C_2A_1 + C_2B_1 + C_2C_1 + C_2D_1 \\ D_2A_1 + D_2B_1 + D_2C_1 + D_2D_1. \end{array}$$

It will be observed that along one diagonal we have the cases in which the two electors vote for the same candidate. If it is considered inessential from whom the vote comes, then the ways to the left of the diagonal are duplicates of the ways to the right.

When there is a third elector, we have to multiply the result by

$$A_3 + B_3 + C_3 + D_3.$$

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From the mode of derivation, it is evident that the number of different ways in which the voting may result is c^e.

But if it is considered immaterial from whom the vote comes, the suffixes may be dispensed with, and the different ways are the homogeneous products of the symbols A, B, C, D. Hence the number of different ways is

$$\frac{c+e-1\,!}{e!\;\;c-1\,!}.$$

II. *Combination voting.* By combination voting I mean voting in which one elector has more than one vote, but must choose a combination. Suppose that the combination is of two, out of A,B,C,D. Then for the 1st elector

$$A_1B_1 + A_1C_1 + A_1D_1 + B_1C_1 + B_1D_1 + C_1D_1,$$

and for the 2nd elector

$$A_2B_2 + A_2C_2 + A_2D_2 + B_2C_2 + B_2D_2 + C_2D_2\,;$$

hence the different results of the two electors voting are

$$\begin{array}{l}
A_2A_1B_2B_1 + A_2A_1B_2C_1 + A_2A_1B_2D_1 + A_2B_1B_2C_1 + A_2B_1B_2D_1 + A_2B_2C_1D_1 \\
A_2A_1B_1C_2 + A_2A_1C_1C_2 + A_2A_1C_2D_1 + A_2B_1C_1C_2 + A_2B_1C_2D_1 + A_2C_1C_2D_1 \\
A_2A_1B_1D_2 + A_2A_1C_1D_2 + A_2A_1D_2D_1 + A_2B_1C_1D_2 + A_2B_1D_2D_1 + A_2C_1D_2D_1 \\
A_1B_2B_2C_2 + A_1B_2C_2C_1 + A_1B_2C_2D_1 + B_2B_1C_2C_1 + B_2B_1C_2D_1 + B_2C_2C_1D_1 \\
A_1B_2B_1D_2 + A_1B_2C_1D_2 + A_1B_2D_2D_1 + B_2B_1C_1D_2 + B_2B_1D_2D_1 + B_2C_1D_2D_2 \\
A_1B_1C_2D_2 + A_1C_1C_2D_2 + A_1C_2D_1D_2 + B_1C_1C_2D_2 + B_1C_2D_2D_1 + C_2C_1D_2D_1
\end{array}$$

The number of possible ways in which the combination may be chosen is

$$\frac{c-s+1\,!}{c-1\,!\;s!};$$

hence the number of different ways in which the voting may take place is

$$\left\{\frac{c-s+1\,!}{c-1\,!\;s!}\right\}^e$$

If it is considered immaterial from whom the vote comes, then all the terms on one side of the diagonal are cut off as before, but in addition some of the terms on the other side of the diagonal. For example, in the case above, there are two terms in the other diagonal which have to be cut out. There is evidently an expression for the number, but it is pretty complex.

III. *Cumulative voting.* In cumulative voting an elector has a plurality of votes, and he is not obliged to choose a combination; his votes are independent of one another. Hence the different ways in which he can vote are represented by the homogeneous products.

He may give any number of votes up to s, the number of seats, hence the number of different ways in which he can vote is

$$\frac{c+s\,!}{s\,!\;c\,!}.$$

When there are e electors voting in this way, the total number of ways (states of the poll) is the same as if one elector had es cumulative votes. Hence

$$\frac{c+es\,!}{es\,!\;c\,!}.$$

Mr J. S. MACKAY gave the following solution of Mr Edward's problem, (see p. 5):

Between two sides of a triangle to inflect a straight line which shall be equal to each of the segments of the sides between it and the base.

Let ABC (fig. 15), be a triangle, and let the side AB be less than AC. Draw any straight line DE parallel to BC, and cutting the sides AB, AC, or AB, AC produced either below the base or through the vertex, in D and E. Cut off CF′ equal to BD; with centre F′ and radius CF′ cut DE or DE produced at the points G′; and join F′G′. Let CG′ meet AB or AB produced at G, and draw GF parallel to G′F′. GF is the line required.

For through G′ draw A′B′ parallel to AB, and meeting the sides AC, BC, or AC, BC produced, in A′, B′.

Then B′G′ = BD = CF′ = F′G′.

Now, since the quadrilaterals CB′G′F′, CBGF are similar, and either similarly or oppositely situated, C being their centre of similitude; and since B′G′ = G′F′ = F′C;

therefore BG = GF = FC.

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