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Tensor-quark correlator

We shall be concerned with the two-point correlator:

$$\Psi_{2,\mu\nu\rho\sigma} \equiv i\int d^4x\, e^{iqx}\langle 0|\mathcal{T}\theta^q_{\mu\nu}(x)\theta^q_{\rho\sigma}(0)^\dagger|0\rangle$$
$$= \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\Psi_2(q^2)\,, \tag{34.1}$$

where

$$\theta^q_{\mu\nu}(x) = i\bar{q}(x)(\gamma_\mu\bar{D}_\nu+\gamma_\nu\bar{D}_\mu)q(x) \tag{34.2}$$

is the quark component of the energy-momentum tensor $\theta^q_{\mu\nu}(x)$. Here, $\bar{D}_\mu \equiv \vec{D}_\mu - \overleftarrow{D}_\mu$ is the covariant derivative. The previous current mixes under renormalization with the gluonic current:

$$\theta^g_{\mu\nu}(x) = -G_{\alpha\mu}G^\alpha_\nu + \frac{1}{4}g_{\mu\nu}G_{\alpha\beta}G^{\alpha\beta}\,, \tag{34.3}$$

as:

$$\theta^{q,R}_{\mu\nu} = Z_{11}\theta^{q,B}_{\mu\nu} + Z_{12}\theta^{g,B}_{\mu\nu}$$
$$\theta^{g,R}_{\mu\nu} = Z_{21}\theta^{q,B}_{\mu\nu} + Z_{22}\theta^{g,B}_{\mu\nu}\,. \tag{34.4}$$

The indices B and R refer respectively to bare and renormalized quantities. The renormalization constants have been evaluated in [450]. For the quark currents, they read in $4-\epsilon$ dimension space–time:

$$Z_{11} = 1+\left(\frac{\alpha_s}{\pi}\right)\frac{1}{\hat{\epsilon}}\frac{4}{3}C_F\,,\qquad Z_{12} = -\left(\frac{\alpha_s}{\pi}\right)\frac{1}{\hat{\epsilon}}\frac{4}{3} \tag{34.5}$$

with: $\hat{\epsilon}^{-1} = \epsilon^{-1} + (\ln 4\pi - \gamma_E)/2$. To the order, we are working, only Z_{11} is relevant. The corresponding anomalous dimension is:

$$\gamma_{11} = -\frac{\nu}{Z_{11}}\frac{dZ_{11}}{d\nu} = \left(\frac{16}{9}\equiv\gamma^1_{11}\right)\left(\frac{\alpha_s}{\pi}\right)\,. \tag{34.6}$$

https://doi.org/10.1017/9781009290296.046 Published online by Cambridge University Press

The renormalized perturbative contribution to the correlator to order α_s is [452]:

$$\Psi^R_{2,\text{pert}}(q^2 \equiv -Q^2) = -\frac{3}{10\pi^2} Q^4 \log \frac{Q^2}{\nu^2}\left[1 - \left(\frac{\alpha_s}{\pi}\right)\left(\frac{473}{135} - \frac{8}{9}\log\frac{Q^2}{\nu^2}\right)\right]. \quad (34.7)$$

The bare quark and mixed condensate contributions read:

$$\Psi^R_{2,q+m}(q^2) = \frac{1}{q^2}\left[-8m^3\langle\bar\psi\psi\rangle^B + \frac{16}{3}mg\left\langle\bar\psi\sigma_{\mu\nu}\frac{\lambda_a}{2}\psi G_a^{\mu\nu}\right\rangle^B\right]. \quad (34.8)$$

The evaluation of the gluon condensate is much more cumbersome. Evaluating the Feynman integrals for arbitrary mass and expanding the result in powers of m^2/q^2, one obtains:

$$\Psi^B_{2,G}(q^2) = \left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle^B\left[\frac{8}{9}\left(\frac{-2}{\hat\epsilon} + \ln\frac{m^2}{\nu^2}\right) + 3\frac{m^2}{q^2} + \frac{8}{9}\left(1 - 3\frac{m^2}{q^2}\right)\ln -\frac{q^2}{m^2}\right]. \quad (34.9)$$

In order to remove the IR logarithm appearing in the bare result, one has to write the heavy- to light-quark expansions of the condensates discussed in previous chapters:

$$\langle\bar\psi\psi\rangle = -\frac{1}{12\pi}\left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle + \cdots,$$
$$\left\langle\bar\psi\sigma_{\mu\nu}\frac{\lambda_a}{2}\psi G_a^{\mu\nu}\right\rangle = \frac{m}{2}\left(-\frac{2}{\hat\epsilon} + \ln\frac{m^2}{\nu^2}\right)\left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle + \cdots. \quad (34.10)$$

In this way, one obtains the bare gluon condensate contribution:

$$\Psi^B_{2,G}(q^2) = \left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle^B\frac{1}{3}\left[\frac{8}{3}\left(1 - 3\frac{m^2}{q^2}\right)\left(\frac{-2}{\hat\epsilon} + \log -\frac{q^2}{\nu^2}\right) + 7\frac{m^2}{q^2}\right]. \quad (34.11)$$

The remaining $m^2/\hat\epsilon q^2$ pole can be eliminated by the introduction of the renormalized mixed condensate [130] discussed in previous chapters:

$$\left\langle\bar\psi\sigma_{\mu\nu}\frac{\lambda_a}{2}\psi G_a^{\mu\nu}\right\rangle^B = \left\langle\bar\psi\sigma_{\mu\nu}\frac{\lambda_a}{2}\psi G_a^{\mu\nu}\right\rangle^R - \frac{m}{\hat\epsilon}\left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle^R. \quad (34.12)$$

Then, one obtains the renormalized result:

$$\Psi^R_{2,G}(q^2) = \left(\frac{\alpha_s}{\pi}\right)\langle G^2\rangle^R\frac{1}{3}\left[\frac{8}{3}\left(1 - 3\frac{m^2}{q^2}\right)\log -\frac{q^2}{\nu^2} + 7\frac{m^2}{q^2}\right]. \quad (34.13)$$

This explicit exercise has shown how delicate is the evaluation of the Wilson coefficients of the non-perturbative condensate contributions.

The four-quark condensate contribution is:

$$\Psi_{2,4q}(q^2) = \frac{64\pi}{9q^2}\rho\left(\frac{\alpha_s}{\pi}\right)\langle\bar\psi\psi\rangle^2, \quad (34.14)$$

where ρ is the deviation from the vacuum saturation estimate.

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