Basic real and abstract analysis, by John F. Randolph. Academic Press, New York and London, 1968. ix + 515 pages. U.S. $14.

Author's preface (first half):

"The keen precision toward which mathematics constantly strives is greatly enhanced by abstraction. Without sharp awareness of the sources of abstraction, however, there is danger of fostering an impression that abstraction is complication instead of simplification. Hence the minds visualized as maturing on this presentation are led from the concrete to the abstract rather than the other way around."

"Chapter 1 (Orientation) emerged last and was the hardest to write, since few, if any, instructors agree on an appropriate starting point for a course which could follow the two-year calculus sequence. Contact with past mathematical experience is made, the stage is set, abstractions to come are foretold by a taste of Boolean algebra, and careful reasoning by Dedekind cuts. Chapter 2 (Sets and Spaces) begins with the familiar, eases into generalizations, and includes just enough about transfinite cardinals to prove (in Chapter 5) that some Lebesgue measurable sets are not Borel sets. The classical material of Chapter 3 (Sequences and Series) shows how readily previous work may be tightened and extended. Limits inferior and superior are defined with care and their merit demonstrated by repeated use, not the least of which is l'Hospital's rule in Chapter 7. The first three chapters provide sufficient preparation for Chapter 4 (Measure and Integration)."

"One guiding light for this book was a strong conviction that time is overdue for Lebesgue theory to come into its own at an early stage without initial high abstraction. By starting with sets on a line, generalizing length, then sloughing off all except pertinent properties, Carathéodory's definition of measurability is revealed as a natural and intuitive requirement. Meaningful and significant theorems on integration are then proved with minimal effort and never a whisper of mystical visions. Those who continue to more advanced work will be equipped to fathom the thought processes that led to the Riesz or Daniell development of integration, either of which is excellent for a second exposure."

"Classical Stieltjes integrals are kept for their role in the procession of ideas from Riemann to Radón-Nikodym, even though they are subsumed (in Section 8-6) by Lebesgue integrals upon replacing the awkward integrator functions with measures. Riemann integration takes its place as an antique in the true sense of the word: not a piece of junk, but a prized relic from a former era honored for inspiring replacements of greater comfort and utility, and still in working order for many uses."

The book can be used as a text for advanced under-graduates; prerequisites would be two years of (serious) calculus, as indicated in the preface. The first three chapters are of a preparatory nature. (There is no detailed treatment of metric or metric linear spaces, but E^m is treated as a metric space, and later ℓ^2 and L^2 are discussed.) Chapter 6 deals with continuity (its last section is on continuous linear functionals). The central part of the book consists of Chapters 4 (Measure and Integration); 5 (Measure Theory); and 7 (Derivatives). The book concludes with a chapter on Stieltjes integrals; the very last section is on the Lebesgue decomposition and Radón-Nikodym theorems. At the end of every chapter there is a generous collection of problems, some other problems are scattered throughout the text. The bibliography lists fifty nine texts and monographs. At the

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end of Chapter 5 there is an additional bibliography of papers on measure theory.

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Mathematical methods, Vol. 1 (Linear algebra/ Normed spaces/ Distributions/ Integration), by Jacob Korevaar. Academic Press, New York, and London, 1968. xii + 505 pages. U.S. $14.

The volumes on mathematical methods, of which the first is being discussed here, grew out of an intensive beginning graduate course for students in the physical sciences and applied mathematics which the author has taught for many years. We quote the beginning of the preface:

"The volumes on mathematical methods are intended for students in the physical sciences, for mathematics students with an interest in applications, and for mathematically oriented engineering students. It has been the author's aim to provide

(1) many of the advanced mathematical tools used in applications;

(2) a certain theoretical mathematical background that will make most parts of modern mathematical analysis accessible to the student of physical science, and that will make it easier for him to keep up with future mathematical developments in his field."

In the present volume, the author has certainly achieved these aims extremely well. From a mathematical point of view the presentation is fairly rigorous, but certainly not abstract. The book as a whole is very well organised. Each chapter, and again each section, is preceded by some introductory and descriptive paragraph(s). Every section is broken up into titled subsections - a very useful arrangement in the present book. At the end of each section there is a generous supply of substantial and well-chosen exercises. Every chapter has its own, very detailed, bibliography. The book should be eminently suitable as a text, and it will make a very good reference book for any student who has worked through it. The exposition and writing are indeed lucid. The reviewer has not found a single "dull" or "dry" passage in the whole book.

The principal prerequisites for this text are: a year of advanced calculus; in addition, some knowledge of elementary linear algebra and elementary differential equations would be desirable. The "introductory and relatively general" (author's description) material of the first volume is to prepare the student for such subjects as orthogonal series, linear operators in Hilbert Space, integral equations. (All but the last topic are to be the subject of the second volume.) Let us have a detailed look at the contents of the present volume.

Chapter One deals with relevant topics in linear algebra. The emphasis is on an understanding of the basic concepts of vector space and linear transformation. The treatment is largely coordinate-free so that it applies to infinite dimensional as well as finite dimensional situations. Topics are: Basic material on vector spaces, linear transformations, and linear functionals; matrices and determinants; systems of linear equations; eigenvalues problems, eigenvectors and generalized eigenvectors (= root vectors); Cayley-Hamilton Theorem, Jordan canonical form, etc.; algebraic theory of tensors.

Chapter Two provides an introduction to functional analysis. It begins with a detailed discussion of the many different kinds of convergence for sequences (and