NEW RESULTS ON THE MOTIONS OF ASTEROIDS IN RESONANCES

R.DVORAK
Institute of Astronomy
University of Vienna
Türkenschanzstrasse 17
A-1180 Vienna
e-mail: DVORAK@AVIA.UNA.AC.AT

Abstract. In this article we present a numerical study of the motion of asteroids in the 2:1 and 3:1 resonance with Jupiter. We integrated the equations of motion of the elliptic restricted 3-body problem for a great number of initial conditions within this 2 resonances for a time interval of 10^4 periods and for special cases even longer (which corresponds in the the Sun-Jupiter system to time intervals up to 10^6 years). We present our results in the form of 3-dimensional diagrams (initial a versus initial e, and in the z-axes the highest value of the eccentricity during the whole integration time). In the 3:1 resonance an eccentricity higher than 0.3 can lead to a close approach to Mars and hence to an escape from the resonance. Asteroids in the 2:1 resonance with Jupiter with eccentricities higher than 0.5 suffer from possible close approaches to Jupiter itself and then again this leads in general to an escape from the resonance. In both resonances we found possible regions of escape (chaotic regions), but only for initial eccentricities $e \geq 0.15$. The comparison with recent results show quite a good agreement for the structure of the 3:1 resonance. For motions in the 2:1 resonance our numeric results are in contradiction to others: high eccentric orbits are also found which may lead to escapes and consequently to a depletion of this resonant regions.

Key words: Kirkwood Gaps - Chaotic Motion - Numerical Experiments

1. Introduction

One of the most interesting progresses in Celestial Mechanics was the discovery of chaotic motions in the 3:1 resonance with Jupiter in the main belt of asteroids (Wisdom, 1982 and 1983). Wisdom derived his surprising results using a mapping method in the framework of the averaged elliptic restricted problem. After a long time period of a quasiperiodic moderate change of the eccentricity ($0 \leq e \leq 0.1$) a sudden increase up to 0.4 was discovered for some specific orbits. Fig. 1a shows this effect, which could even occur several times within the lifetime of an asteroid. The phenomenon of sudden qualitative changes is commonly observed in non linear dynamics, and is a major topic of research in theoretical mechanics of recent years. Wisdoms discovery was the first proof of the existence of such "chaotic" orbits in the Solar System. Since those first appearance of chaos in planetary dynamics much progress took place concerning this question (e.g. Laskar,1986, for the planetary system and Chirikov and Vecheslavov (1986), Froeschlé and Gonczi (1988) for comets).

The chaotic behaviour of the motion of asteroids in the gap could explain the lack of bodies in this resonance, because orbital eccentricities higher than 0.3 make the asteroid a Mars crosser and consequently the body will be kicked out sooner or later by this planet. The described scenario as reason for the 3/1 gap in the asteroids' main belt is commonly accepted by the astronomical community.

For the gap at the 2/1 resonance with Jupiter the latest results seem to indicate that they *cannot* be explained by a simple gravitational model as the elliptic

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 145–152.
© 1992 *IAU. Printed in the Netherlands.*

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

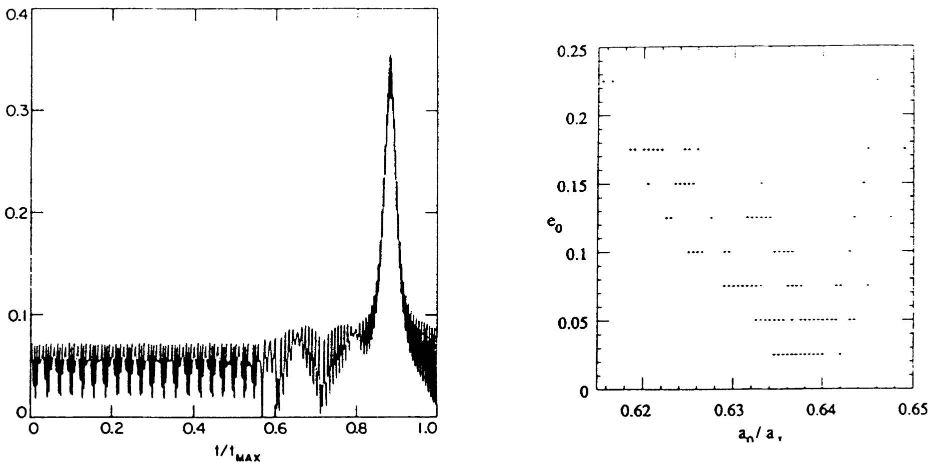

Fig. 1. a (left side): Mapping in the 3/1 Resonance, abscissa: t_{max} =250000 years, ordinate: eccentricity of the fictitious asteroid (after J. Wisdom, 1983). Fig.1.b (right side) Initial condition diagramm for the 2/1 Resonance, initial semimajor axes versus initial eccentricity, Points mark "chaotic orbits" (after J.Wisdom, 1987).

restricted three body problem.

The numerical results shown in this paper agree quite well with the analytical and numerical ones for the 3/1 resonance. They disagree for the the 2/1 resonance with the ones derived by other authors (see below) because many of the fictitious asteroids in this gap suffered from a close approach to Jupiter itself and then escaped.

2. The 2:1 Resonance

Historically the first one since Kirkwood in the last century in trying to explain the nonexistence of asteroids in some resonances was Poincaré (1902). He treated the problem in the framework of the circular restricted problem and reduced it by averaging to a 1 degree of freedom problem (2:1 resonance). For the same resonance Giffen (1973) used Schubart's averaging in the elliptic restricted problem and found a small zone of "very complex motion" which "might be related to the formation of gaps". Numerical experiments in the averaged elliptic problem by Froeschlé and Scholl (1981, 1982) did not put in evidence, that a former low eccentric orbit in the gap could become a high eccentric one. A big progress in a better understanding the physics of the resonance was made by Sessin and Ferraz-Mello (1984) who included in their analytical model of the motion in the 2:1 resonance also high order terms. Following Wisdom's mapping technique Murray (1986) deduced results which are valid only for very low eccentric orbits. Wisdom (1987) published numerical results in which he claims to have found broad chaotic zones in this resonance. Recently

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

Lemaitre and Henrard (1990) developed a semianalytical method for the 2:1 resonance and concluded that their results do "not support the theory of formation of the 2/1 Kirkwood gap by removal of the asteroids through close encounters with Mars". This statement agrees with Yokishawa (1991), who doubted that the pure gravitational forces are responsible for the formation of this gap. A new analytical treatment based on the stability of periodic orbits by Morbidelli and Giorgilli (1991) could find for the 2/1 Kirkwood gap "no mechanical explanation in the framework of the complete restricted problem of three bodies".

Fig.1b shows the numerical results by Wisdom (1987) derived with the Digital Orrery for orbits in the 2:1 resonance where he integrated for a net of initial conditions $0.62 \leq a \leq 0.65$ and $0.0 \leq e \leq 0.25$ (a and e are the semi major axes respectively eccentricities). In the diagramm the chaotic orbits are characterized by points.

The goal of our work was to explore the structure of asteroidal motion in the gaps concerning the time evolution of their eccentricities. Therefore we integrated the equations of motion (using the Lie-series integration technique e.g. Hanslmeier and Dvorak, 1983) for Wisdom's set of initial conditions for a very large number of fictitious asteroids. Fig.2 shows the detailed results of our own integrations in the three dimensional elliptic restricted problem.

In the three dimensional graph the highest eccentricity value during the whole integration is plotted in the z - direction, the initial conditions a_0 and e_0 in the x respectively the y-direction. In general we integrated for 10^4 periods of Jupiter which corresponds approximately to the 10^5 years in Wisdoms' integration. Note that only the highest peaks in fig.2 lead finally to a close approach with Jupiter itself, and then consequently to an escape from the asteroid main belt. Both graphs (fig.1b and fig.2) show the following characteristics:

– on the left side there is a broad area of higher eccentricity variations visible; it is chaotic in fig.1b but in fig.2 the eccentricities are not high enough to lead to a close approach - and an escape.
– a sharp small region to the right with very high peaks of the eccentricity (=escape) on fig.2 and only few chaotic orbits in fig.1b

The triangular structure seen in the initial conditions diagram was explained by Lemaitre and Henrard (1990): they found that it stems from the appearance of second order resonances at the borders to regular motion.

Very similar graphs were published recently by Yoshikawa (1989 and 1991), where results from analytical calculations and numerical integrations are shown. The main different point is, that in the calculations undertaken by us we found that even with moderate initial eccentricities ($e \sim 0.15$) escapes from the gap are possible.

A typical orbit is shown in fig.3a, where it seems to be in a quasiperiodic state, but after a sudden close approach to the second primary body (Jupiter), the orbit starts to be quite irregular. Note that in the calculations in the circular model the orbit stays quasiperiodic with a much lower maximum value of the eccentricity e (the existence of the Jacobi-Integral does not allow high increases of the eccentricity). From this it is evident, that the eccentricity of Jupiter's orbit (e=0.048 during all the integrations) leads to a drastic change of the orbit. Fig.3b and Fig. 3c show

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

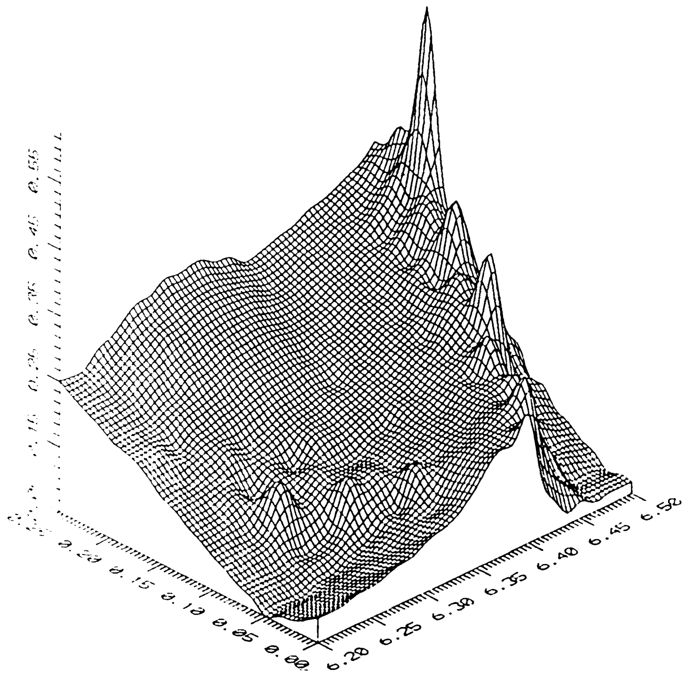

Fig. 2. 3-D Initial condition diagramm for the 2/1 Resonance, initial semimajor axes (semi major axe of Jupiter's orbit is 10 AU) versus initial eccentricity. In the z-direction we plotted the maximum value of the eccentricity during the whole integration time (10^4 periods of the primaries)

another quasiperiodic orbit, where one recognizes a circulating $\tilde{\omega}$. In some cases we discovered also librating orbits.

It is an interesting point that the corresponding diagramm (to fig.2) is very similar when the motion of the massless asteroid is not restricted to the plane of motion of Jupiter. The similiarity may be due to the initially very low inclined orbit of the fictitious asteroid (0.5 degrees) in the 3-dimensional integrations. It should be pointed out, that in other numerical experiments undertaken we found a significant difference in the resulting orbits in the 2 models, but the role of the 3^{rd} dimension will be discussed in a later paper.

3. 3:1 Resonance

For the 3/1 resonance the results we derived in our numerical experiments do very well agree with the existing analytical and numerical ones. There exists a complete study of this gap using Wisdom's mapping method by Murray and Fox (1984), where they determined the structure of the gap concerning the chaoticity of the orbits with the Liapunov exponents. The bifurcation of the chaotic domain from the initial e=0 on to higher values of e (0.4) is mainly a consequence of the sec-

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

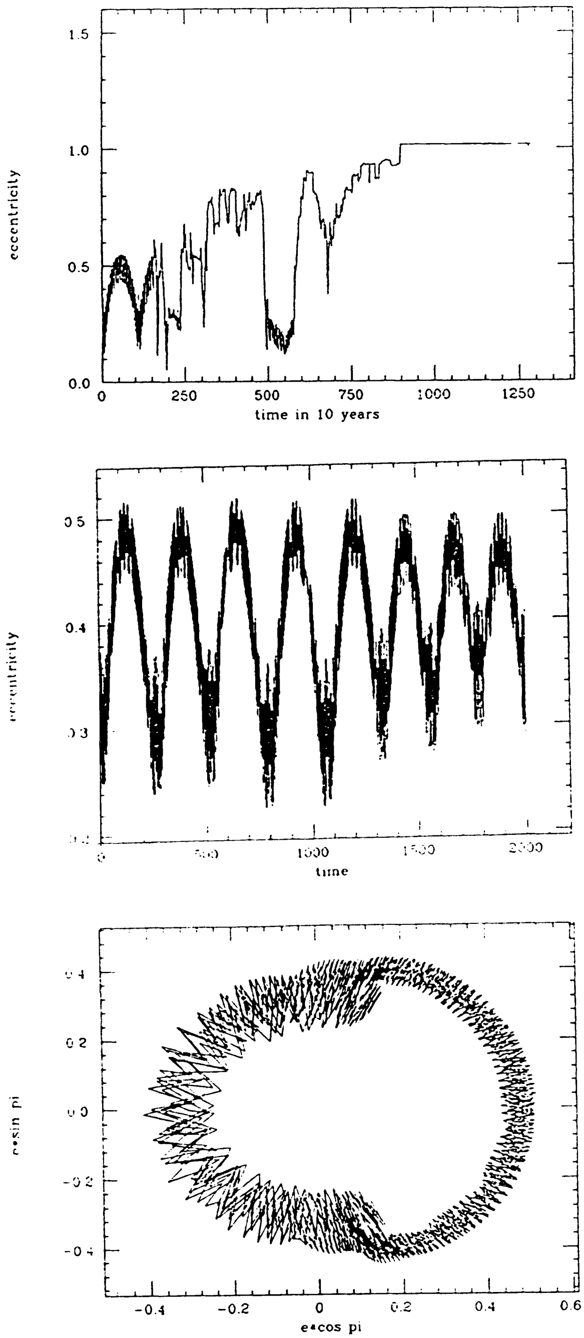

Fig. 3. a (upper graph), Typical chaotic orbit in the 2/1 resonance, time in 10 periods of the primaries, Fig.3b (central graph), Typical quasiperiodic orbit in the 2/1 resonance, coordinates as above, Fig.3c (lower graph), Kozai diagramm for the same orbit, abscissa: $e.\cos\tilde{\omega}$, ordinate: $e.\sin\tilde{\omega}$.

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

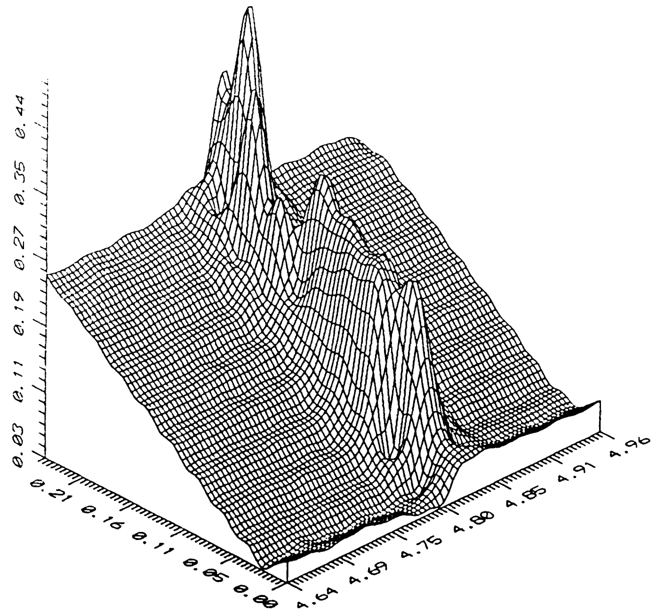

Fig. 4. 3-D Initial condition diagramm for the 3/1 Resonance, initial semimajor axis (semi major axis of Jupiter's orbit is 10 AU) versus initial eccentricity. In the z-direction we plotted the maximum value of the eccentricity during the whole integration time (10^4periods of the primaries)

ondary resonances (Henrard and Caranicolas, 1990). The structure of the gap was found also by Yoshikawa (1990) in his analytical and numerical approach. In our calculations we did not find such a bifurcation, but a very sharp line of chaotic motion (fig. 4) around the distance 0.48 (in dimensionless coordinates). It should be noted that for an asteroid in the 3/1 gap an eccentricity of $e \geq 0.4$ leads (after a long enough time span) to a close approach with Mars.

Two example of a chaotic orbit are shown in fig. 5 where we can see on the upper graph a moderate change of the eccentricity at the beginning, and then a sudden increase up to 0.4. The other example is an orbit with peaks of the eccentricity up to 0.6; the fictitious asteroid is then a possible Earth crosser. In principle these types of orbits show the same qualitative chaotic behaviour as the orbit shown in fig.1.

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

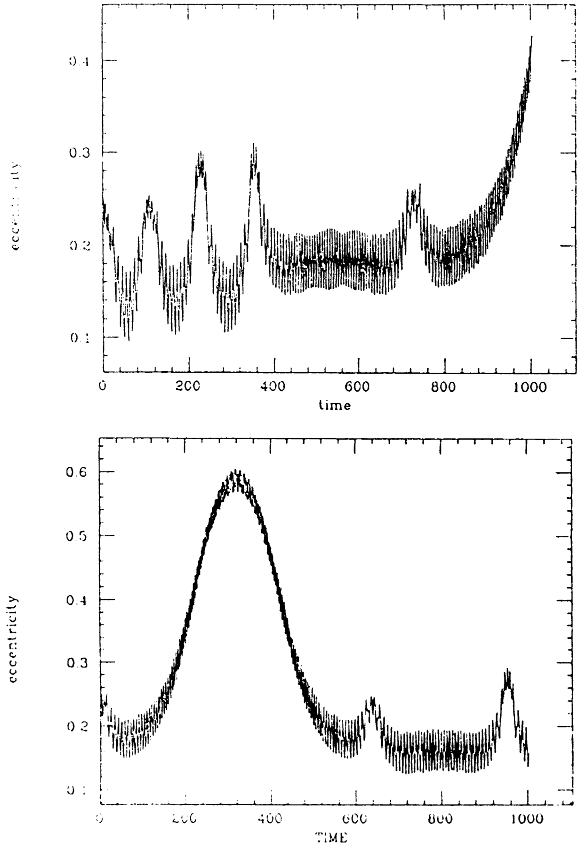

Fig. 5. Typical chaotic orbits in the 3/1 resonance, time in 10 periods of the primaries

4. Conclusions

In our diagramms we have shown the structure of the 2/1 and 3/1 Kirkwood gaps concerning the possible increase of eccentricities of the orbits of fictitious asteroids. As pointed out the value of eccentricity is the crucial parameter for the appearance of chaotic motion in the gap: it allows, from a certain value on, that the orbit of the asteroid may suffer from a close approach with a planet (Mars or Jupiter or even the Earth) and consequently an escape from the mean motion resonance may occur.

The presented results indicate, that the pure gravitational cause for the Kirkwood gaps is highly probable. The scenario described above is already well established for the lack of asteroids in the 3/1 resonance with Jupiter. In contrary to recent results we found also escape orbits in 2/1 resonance but it is not yet clear, whether the chaotic zone, which leads to an escape, is large enough. The role of

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press

the initial conditions (especially ω) in this resonance and in the 3/2 resonance is also very important. We do not have complete results for the 3/2 resonance but first calculations show, that for many initial conditions the fictitious Hildas have in fact stable orbits. We could not solve the problem of the role of the free motion out of Jupiter's orbital plane; in all our results initially moderate inclined orbits ($i \leq 3^o$) show qualitatively the same orbital behaviour as asteroids treated in the plane elliptic model. Nevertheless we expect a different behaviour for higher inclined orbits.

References

Chirikov, B.V., and Vecheslavov,V.V.:1986, "Chaotic Dynamics of Comet Halley", Novosibirsk, preprint

Froeschlé, C., and Scholl, H.: 1981, "The Stochasticity of Peculiar Orbits in the 2/1 Kirkwood Gap", Astron.Astrophys., 93, p.62

Froeschlé, C., and Scholl, H.: 1982, "A Systematic Exploration of Three-dimensional Asteroidal Motion at the 2/1 Resonance", Astron.Astrophys.111 p.346

Froeschlé, C., and Gonczi, R.: 1988, "On the Stochasticity of Halley like comets", Cel.Mech.43, 325

Giffen, R.: 1973, "A Study of Commensurable Motion in the Asteroid Belt", Astron.Astrophys. 23, p.387

Hanslmeier, A., and Dvorak, R.:1984, "Numerical Integration with Lie-Series", Astron.Astrophy. 132, 203

Lemaitre, A., and Henrard, J.: "On the Origin of Chaotic Behaviour in the 2/1 Kirkwood Gap", Icarus, 83, p.391

Henrard, J., and Caranicolas, N.D.: 1990, "Motion near the 3/1 Resonance of the planar elliptic restricted three body problem", Cel.Mech. 47, p.99

Laskar, J.: 1989, "A numerical experiment on the chaotic behaviour of the solar system, Nature, 338, No.6212, p.237

Morbidelli, A. and Giorgilli, A.:1990, "On the dynamics in the asteroids belt. Part II: Detailed study of the main resonances. Cel.Mech., 47, p.173

Murray, C.C., and Fox, K.: 1984, "Structure of the 3 : 1 Jovian Resonance: A Comparison on Numerical Methods", Icarus, 59, p.221

Murray, C.D.: 1986, "Structure of the 2:1 and 3:2 Jovian resonances", Icarus, 65, p.70

Poincaré,H. :1902,"Sur les planètes du type d'Hécube", Bull. Astron. 19, p.289

Scholl, H., and Froeschlé, C.: 1975, "Asteroidal motion at the 5/2, 7/3 and 2/1 resonances"' Astron.Astrophys. 42, p. 457

Sessin, W., and Ferraz-Mello,S. : 1984; "Motion of two Planets with periods commensurable in the ratio 2/1", Cel.Mech.32, 307

Wisdom, J. : 1982, "The origin of the Kirkwood gaps: A mapping for Asteroidal motion near the 3/1 commensurability", Astron. J. 87, p.577

Wisdom, J. : 1983, " Chaotic behaviour and the origin of the 3/1 Kirkwood gap", Icarus, 56, p.51

Wisdom, J. : 1987, "Urey Prize Lecture: Chaotic Dynamics in the Solar System", Icarus, 72, p.241

Yoshikawa, M.: 1989, "A Survey of the Motions of Asteroids in the Commensurabilities with Jupiter", Astron.Astrophys. 213, p.436

Yoshikawa, M.: 1990, "Motions of Asteroids at the Kirkwood Gaps I. On the 3:1 Resonance with Jupiter, Icarus, 87, p.78

Yoshikawa, M.: 1991, "Motions of Asteroids at the Kirkwood Gaps I. On the 5:2, 7:3, and 2:1 Resonances with Jupiter", Icarus92, p.94

https://doi.org/10.1017/S0074180900091051 Published online by Cambridge University Press