STELLAR 5 MIN OSCILLATIONS*

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Abstract. Estimates are given for the amplitudes of stochastically excited oscillations in Main Sequence stars and cool giants; these were obtained using the equipartition between convective and pulsational energy which was originally proposed by Goldreich and Keeley. The amplitudes of both velocity and luminosity perturbation generally increase with increasing mass along the Main Sequence as long as convection transports a major fraction of the total flux, and the amplitudes also increase with the age of the model. The 1.5 $M_\odot$ ZAMS model, of spectral type F0, has velocity amplitudes ten times larger than those found in the Sun. For very luminous red supergiants luminosity amplitudes of up to about 0^m.1 are predicted, in rough agreement with observations presented by Maeder.

1. Introduction

There would be an obvious intrinsic interest in the discovery of stellar analogues to the solar 5 min oscillations of low degree. More important, however, is the fact that detection and detailed observation of such oscillations might enable the extension to other stars of the seismological investigations (e.g. Christensen-Dalsgaard and Gough, 1976) which are now beginning to yield information about the structure of the Sun (Scuflaire *et al.*, 1981; Christensen-Dalsgaard and Gough, 1980b, 1981; Shibahashi and Osaki, 1982). In addition, the variation of oscillation amplitude with stellar parameters would be of great interest in connection with the determination of the excitation mechanism for these oscillations.

An immediate problem facing any attempt to detect such oscillations, at least in solar-type stars, is their very small amplitude. Thus the velocity amplitudes of at most 15–40 cm s^{-1} for each mode of oscillation which is observed in the Sun (Grec *et al.*, 1980; Claverie *et al.*, 1981) are probably below the limit of present spectroscopic techniques; the relative luminosity amplitudes of $2–4 \times 10^{-6}$ (Deubner, 1981; Woodard and Hudson, 1983) would be observable in a star (Deubner found evidence

* Proceedings of the 66th IAU Colloquium: *Problems in Solar and Stellar Oscillations*, held at the Crimean Astrophysical Observatory, U.S.S.R., 1–5 September, 1981.
** NCAR is sponsored by the National Science Foundation.
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Solar Physics **82** (1983) 469–486. 0038–0938/83/0822–0469$02.70.

for the oscillations in sunlight reflected from Neptune), but only in observations dedicated to a single star for several nights.

Traub *et al.* (1977) attempted to detect stellar 5 min oscillations using the PEPSIOS spectrometer. The criterion used to select the stars observed was the presence of Ca emission, which was taken as evidence for waves heating a chromosphere. These observations clearly showed the solar 5 min oscillations, but failed to find oscillations in the other stars, giving an upper limit of about 5 m s^{-1} on the total velocity amplitude. In addition M. A. Smith (private communication) has found evidence for velocity fluctuations in two giants (Arcturus and Aldebaran) which he tentatively interprets as oscillations that may be related to the solar 5 min oscillations.

The obvious difficulty in observing these oscillations, and the long observing sequences needed for each star, make desirable some guidance for the choice of stars to observe. Such stars should clearly have as large amplitudes as possible. In addition the periods of oscillation should be sufficiently short to allow removal of drift in the observations (e.g. caused by variations in the electronics, or by atmospheric extinction). Finally, if frequency resolved observations are sought, the frequency separation between the dominant peaks in the spectrum of oscillation should ideally be large enough to be resolved with a single night's observations.

The purpose of the present paper is to give theoretical estimates of amplitudes, periods and frequency separations for a selection of stellar models. The periods, and hence the frequency separations, for a given model are easily calculated from linear theory. To obtain estimates of oscillation amplitudes a model for the excitation of the oscillations is needed, and this is at present far less certain. We shall adopt as premise that the solar 5 min oscillations are not self-excited (i.e. that they are stable according to linear theory), but that they are excited stochastically by convection. It is true that the linear non-adiabatic calculations of Ando and Osaki (1975, 1977) showed instability of the solar 5 min oscillations. However these calculations assumed that the equilibrium value of the mean intensity of radiation was equal to the Planck function everywhere (cf. Christensen-Dalsgaard and Frandsen, 1983), and, probably more importantly, they neglected the Lagrangian perturbation in the divergence of the convective flux. Later calculations by Berthomieu *et al.* (1980) and Baker and Gough (cf. Gough, 1980) that included the perturbation in the convective flux found the modes in the 5 min range to be stable. Under these circumstances the hypothesis of stochastic excitation appears to be the most likely alternative.

Goldreich and Keeley (1977b) made a simplified analysis of this excitation mechanism. They found that the resulting amplitude of oscillation is such that there is approximate equality between the energy, integrated over the star, in one mode of oscillation and the kinetic energy in one convective eddy whose time scale is the same as the period of the oscillation. Keeley (1977, 1980) and Gough (1980) showed that this equipartition of energy predicts roughly the observed amplitudes of the solar 5 min oscillations (cf. also Christensen-Dalsgaard and Gough, 1982). We shall use it, in a form made precise below, to estimate oscillation amplitudes for other stars.

2. The Calculation

Complete stellar models were calculated using the code described by Christensen-Dalsgaard (1982; in the following C-D82) with a small modification in the way the variation in the super-adiabatic gradient was taken into account in the determination of the mesh. The Cox and Tabor (1976) opacity tables were used, and the parameters were the same as for Model sequence 1 of C-D82. The models comprised a set of Zero-Age Main Sequence models with masses between 0.8 and 1.8 $M_\odot$ ($M_\odot$ being the mass of the Sun), as well as a continuation of sequence 1 of C-D82 to well into the hydrogen shell-burning phase.

In addition to the complete models envelope models were calculated, to explore the properties of stars further from the Main Sequence. These envelopes were assumed to be chemically homogeneous and with constant luminosity; the physics was the same as in the calculation of the complete models. To approximate stars in the hydrogen shell-burning phase sets of envelopes with varying effective temperature, but constant mass M and luminosity L were computed, the relation between M and L being approximately as found in Iben's (1964) evolution calculations.

The complete models were transferred to a mesh more suitable for pulsation calculations using four point Lagrangian interpolation. This mesh was based partly on the variation in pressure and temperature, partly on the asymptotic properties of high-order acoustic oscillations, in the manner of Christensen-Dalsgaard (1977); the same mesh was used directly in the calculation of the envelope models.

Observations of stellar oscillation, by necessity made in integrated light, are dominated by modes with values of the degree l less than about 4 (Dziembowski, 1977; Christensen-Dalsgaard and Gough, 1980a), and the relevant properties of such modes depend little on l (Christensen-Dalsgaard and Gough, 1982). Hence we have only calculated radial modes of oscillation, by solving the equations of linear non-adiabatic oscillation. Radiation was treated in the Eddington approximation (e.g. Unno and Spiegel, 1966), and we neglected $\delta(\mathrm{div}\,\mathbf{F}_c)$, where δ denotes Lagrangian perturbation and $\mathbf{F}_c$ is the convective flux. The mechanical surface boundary condition was derived from matching the solution onto the outward decaying solution of the adiabatic wave-equation in an isothermal atmosphere and was applied at optical depth $\tau = 0.01$. At the inner boundary the oscillations were assumed to be adiabatic; in the complete models a second inner boundary condition was obtained by expansion around the centre, whereas the displacement was assumed to vanish at the bottom of the envelope models. With these boundary conditions the frequencies of oscillation are determined as eigenvalues of the pulsation equations.

For every model considered we have calculated all modes of oscillation in a range sufficiently large to determine the maximum amplitudes in velocity and luminosity perturbation, and to study in some detail the variation in the amplitudes around the maximum. As an upper bound on the frequencies we have used Lamb's (1909) acoustical cut-off frequency

$$\nu_c = \frac{\Gamma_1 g_s}{4\pi c_s}, \tag{2.1}$$

where $\Gamma_1 = (\partial \ln p/\partial \ln \rho)_s$ is an adiabatic exponent, p, ρ, and s being pressure, density and specific entropy, respectively, g_s is the surface gravity and c_s is the sound speed in the (assumed) isothermal atmosphere bounding the model. The energy equipartition between convection and pulsation derives from the fact that turbulent viscosity appears to dominate the linear damping of the oscillations (Goldreich and Keeley, 1977a); thus the dynamics of convection controls both the excitation and the damping of the oscillations. However there is a tendency for modes with frequency ν close to or above ν_c to behave like running waves in the atmosphere; this leads to a relative increase in the atmospheric amplitude of such modes and so for these the radiative atmospheric damping may become dominant (see also Christensen-Dalsgaard and Frandsen, 1983). This would cause their amplitudes to be smaller than predicted by energy equipartition. (It might be noted that this argument suggests that claims of detection of stellar oscillations with frequencies significantly exceeding ν_s, which can of course easily be estimated, should be viewed with some suspicion. Such oscillations probably do not represent large-scale pulsation of the star).

The oscillation amplitude calculated on the basis of energy equipartition depends on the details of the description of convection and the precise definition of the convective time scale. We have adopted the mixing length formulation for a static model given by Gough (1977), with parameters chosen to make the convective flux agree with that of Böhm-Vitense (1958). The time scale was taken to be the mean lifetime τ_c of a convective eddy (Gough, Equations (4.23) and (4.26)); the mean kinetic energy in an eddy was calculated as

$$E_c = \tfrac{1}{2} p_t l_c^3 , \tag{2.2}$$

where p_t is the mean turbulent pressure (Gough, Equation (3.16)) and l_c is the mixing length, as usual taken to be a constant (in this case 1.6364) multiple of the pressure scale height. This choice of τ_c and E_c is clearly not unique; however as shown in Section 3 it does appear to give roughly correct amplitudes for the Sun, without the introduction of additional scaling factors. Furthermore the variation with stellar parameters in the predicted amplitude is probably not very sensitive to the precise formulation.

In a given model E_c and τ_c can thus be calculated as functions of the distance r from the centre of the model. At the edges of the convection zone the velocity tends to zero and τ_c tends to infinity; τ_c has a minimum $\tau_{c,\min}$ which is generally close to where the superadiabatic gradient is largest. For a mode with period Π greater than $\tau_{c,\min}$ there are at least two points, with $r = r_i$, say, in the convection zone where $\tau_c = \Pi$; we have calculated the oscillation amplitude by demanding that the amplitude of the kinetic energy of oscillation be equal to the sum of E_c over these points, that is

$$E_{\rm osc} \equiv \tfrac{1}{2} \int_V \rho |\mathbf{v}_{\rm osc}|^2 \, \mathrm{d}V = \sum_i E_c(r_i) . \tag{2.3}$$

Here $\mathbf{v}_{osc}$ is the amplitude of the oscillation velocity and the integral is over the volume V of the equilibrium model. The results would change little if the maximum over the energies of the resonant eddies, rather than their sum, had been used.

The present, grossly simplified, model of stochastic excitation predicts that modes with periods shorter than $\tau_{c,\min}$ are not excited. In reality there would be a contribution to the excitation of these modes from convective eddies with longer time scales, and from smaller eddies resulting from the turbulent breakdown of the dominant eddies. Thus one would expect a gradual decrease in amplitude at periods shorter than $\tau_{c,\min}$, rather than the sharp cut-off predicted here.

It is convenient to express the pulsational energy as

$$E_{osc} = \tfrac{1}{2} M v_s^2 \mathscr{E}_{osc} , \tag{2.4}$$

where v_s is the radial component of the surface velocity amplitude and

$$\mathscr{E}_{osc} = \int_V \rho |\boldsymbol{\xi}|^2 \, \mathrm{d}V / M \xi_r(r_s)^2 , \tag{2.5}$$

$\boldsymbol{\xi}$ being the eigenfunction of linear oscillation, ξ_r its radial component and r_s the surface radius of the star (notice that Equations (2.3)–(2.5) can be applied to non-radial as well as to radial oscillations). Thus $\mathscr{E}_{osc}$ can be found from linear theory, and then

$$v_s = \left[\frac{2 \sum_i E_c(r_i)}{M \mathscr{E}_{osc}} \right]^{1/2} . \tag{2.6}$$

Finally the relative surface luminosity perturbation can be calculated from v_s as

$$\delta L_s / L_s = \lambda_s v_s , \tag{2.7}$$

where $\lambda_s = (\delta L_s / L_s) / v_s$ may be found from a linear calculation.

3. Results

We first consider the calibration against observations of solar oscillations. It might be argued that the criterion for 'resonance', viz. $\Pi = \tau_c$, is arbitrary and should be replaced by $\Pi = \gamma \tau_c$, where γ is a factor to be calibrated against the solar data. On Figure 1 are shown the predicted velocity amplitudes, as functions of the frequency, for different values of γ. In each case the amplitude increases monotonically with ν until the cut-off frequency $\nu_{\max} = 1/(\gamma \tau_{c,\min})$, and the dominant effect of changing γ is clearly to shift $\nu_{\max}$. For $\gamma = 1$, $\nu_{\max}$ almost coincides with the observed position of maximum power, and the maximum velocity, about 15 cm s^{-1}, is consistent with the observed value. Of course the observed spectra show a gradual decrease in power towerds higher frequency, rather than the sharp cut-off found here; but as argued in the preceeding section this would be smoothed in a more detailed description of the excitation. Thus we have used $\gamma = 1$ in the following. The maximum amplitude of the relative luminosity perturbation

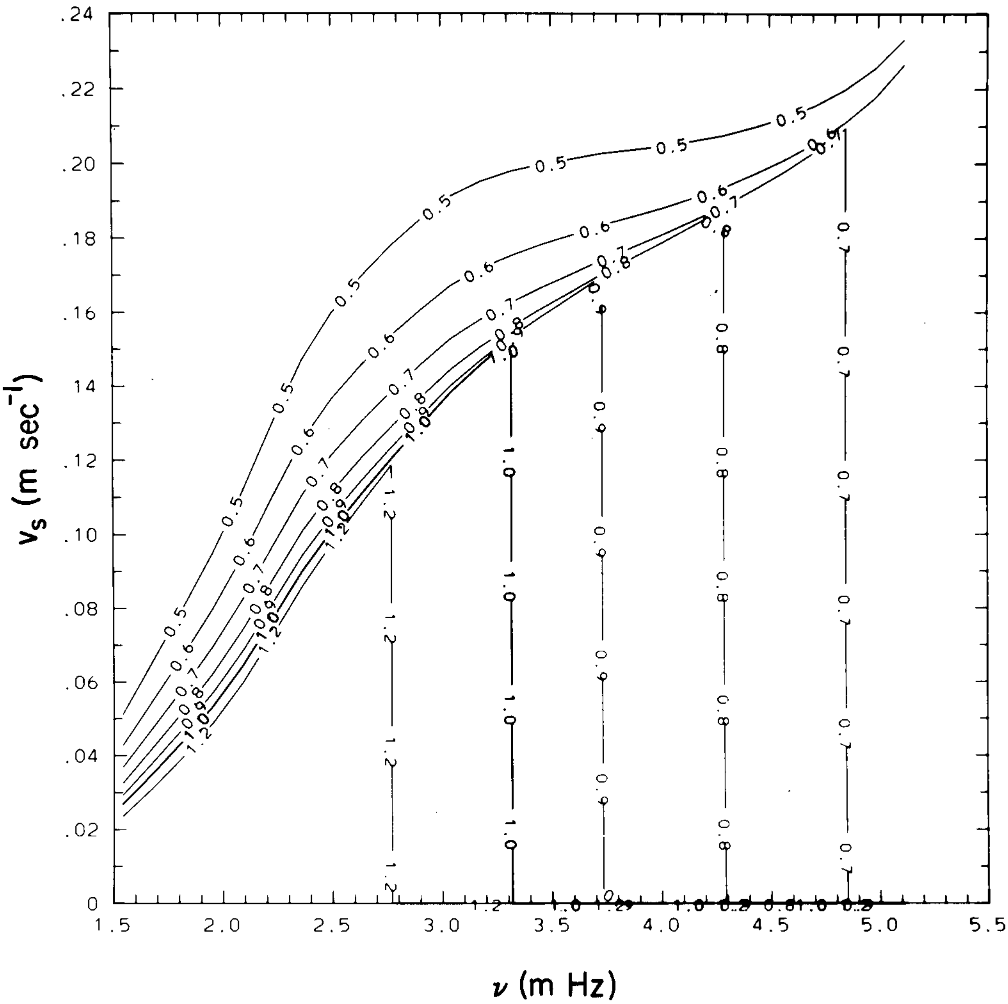

Fig. 1. Predicted surface velocity amplitudes v_s in a model of the present Sun, as functions of the cyclic frequency ν of oscillation, for various values of the ratio γ between the pulsation period and the time scale of the 'resonating' convective eddy. For clarity the values for the discrete modes of oscillation have been connected with continuous lines. The curves are labelled with the value of γ.

is then 3.5×10^{-6}, fairly close to the value observed by Woodard and Hudson (1983). It should also be noticed that the ratio found here between the luminosity perturbation and the velocity is in reasonable agreement with the value obtained by Gough (1980), who treated the radiation in the diffusion approximation but included the perturbation in the convective flux.

The main results concerning the complete stellar models are presented in Table I for the ZAMS models and in Table II for the $1\,M_\odot$ evolution sequence. The value given for $\Delta\nu$ is the frequency difference between two consecutive modes at the velocity maximum; as the frequency spacing is nearly uniform for high-order acoustic modes

TABLE I

Properties of oscillations of ZAMS models. M, T_{eff} and L are the mass, effective temperature and luminosity of the model, respectively, $M_{\odot}$ and $L_{\odot}$ being the solar values; $v_{s,\max}$ and $(\delta L_s/L_s)_{\max}$ are the maximum values of the surface velocity and relative luminosity perturbation, and $\Pi_{\max}$ is the period corresponding to, and $\Delta\nu$ the frequency difference between two adjacent modes at, the maximum velocity.

$M/M_{\odot}$	T_{eff}	$L/L_{\odot}$	$v_{s,\max}$ (cm s^{-1})	$(\delta L_s/L_s)_{\max}$	$\Pi_{\max}$ (min)	$\Delta\nu$ (μHz)
0.8	4880	0.25	3	6×10^{-7}	4	204
0.9	5285	0.44	6	1.5×10^{-6}	4	184
1.0	5646	0.71	10	2.5×10^{-6}	4	165
1.1	5916	1.13	15	3.4×10^{-6}	5	142
1.2	6178	1.70	20	4.2×10^{-6}	5	124
1.3	6457	2.49	25	5.1×10^{-6}	5	110
1.4	6778	3.49	37	6.2×10^{-6}	10	96
1.5	7184	4.74	144	1.3×10^{-5}	18	89
1.6	7640	6.26	72	1.0×10^{-5}	9	94
1.7	8072	8.06	71	2.7×10^{-6}	10	93
1.8	8475	10.2	72	2.7×10^{-5}	7	90

TABLE II

Properties of oscillations of the models in a 1 $M_{\odot}$ evolution sequence. The notation is as in Table I.

Age (10^9 yr)	T_{eff}	$L/L_{\odot}$	$v_{s,\max}$ (cm s^{-1})	$(\delta L_s/L_s)_{\max}$	$\Pi_{\max}$ (min)	$\Delta\nu$ (μHz)
0	5646	0.71	10	2.5×10^{-6}	4	165
2.65	5713	0.85	13	3.0×10^{-6}	5	150
4.75	5770	1.00	15	3.5×10^{-6}	5	137
9.68	5784	1.68	26	6.2×10^{-6}	8	96
11.71	5359	2.35	34	9.3×10^{-6}	15	59

(e.g. Vandakurov, 1967) this value is representative for the range of frequencies where the amplitudes are large.

The ZAMS models extend well into the region of the δ Scuti stars; as in other linear calculations (see e.g. the review by Dziembowski, 1980) a number of modes were found to be unstable in the models with $M \geq 1.4\ M_{\odot}$. For such modes the amplitude estimates based on energy equipartition are presumably invalid. However, except at 1.5 $M_{\odot}$, the predicted maximum amplitude occurs for modes that were found to be linearly stable.

There is clearly a marked increase in the predicted velocity amplitudes with increasing stellar mass until 1.5 $M_{\odot}$, and a similar, but more erratic, increase in the luminosity amplitude. Furthermore there is also a significant increase in the amplitudes as the 1 $M_{\odot}$ model evolves. To understand this behaviour we must study the dependence of the pulsational and convective energy on stellar parameters. The details are clearly quite complicated, but it is possible to get a qualitative understanding of the dominant features. The relation between the energy and the surface amplitude of the oscillations may be estimated from asymptotic theory. The modes are evanescent in a region close

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to the surface whose depth decreases with increasing oscillation frequency. Thus the energy density in the mode increases with increasing depth until the oscillatory region, where the amplitude of the energy density is roughly constant, is reached; the increase in the energy density decreases with increasing ν, and so does $\mathscr{E}_{\mathrm{osc}}$. From the asymptotic theory for high-order acoustic modes (e.g. Vandakurov, 1967), modified to take into account the nonvanishing surface temperature of a realistic model (Christensen-Dalsgaard and Gough, 1980b) one may show that

$$\mathscr{E}_{\mathrm{osc}} \approx \frac{3}{4\pi^2} \frac{\rho_s}{\bar{\rho}} \frac{c_s}{r_s} \mathscr{F}(\nu/\nu_c) \int_0^{r_s} \frac{\mathrm{d}r}{c} , \tag{3.1}$$

where ρ_s is the photosperic, and $\bar{\rho}$ the mean, density of the model, and c_s is the value in the atmosphere of the sound speed c; the dependence of $\mathscr{E}_{\mathrm{osc}}$ on ν is determined by $\mathscr{F}(\nu/\nu_c)$, where the acoustical cut-off frequency ν_c is given in Equation (2.1), and

$$\mathscr{F}(z) = \frac{1 + \Phi(z)^2}{\zeta[J_m(\zeta) - \Phi(z) Y_m(\zeta)]^2} , \tag{3.2}$$

$$\Phi(z) = \frac{J_{m+1}(\zeta) - \alpha(z) J_m(\zeta)}{Y_{m+1}(\zeta) - \alpha(z) Y_m(\zeta)} , \tag{3.3}$$

and

$$\alpha(z) = z^{-1} - (z^{-2} - 1)^{1/2} ; \qquad \zeta = (m+1)z . \tag{3.4}$$

Here m is the effective polytropic index of the region close to the surface and J_m and Y_m are Bessel functions. When $\nu/\nu_c \ll 1$,

$$\mathscr{F}(\nu/\nu_c) \approx \tfrac{1}{2}\Gamma(m+1)^2 [\tfrac{1}{2}(m+1)(\nu/\nu_c)]^{-(2m+1)} ,$$

where Γ is the Gamma function; this rapid increase of $\mathscr{E}_{\mathrm{osc}}$ with decreasing ν reflects the increasing depth of the outer evanescent region.

The behaviour of the convective energy is more difficult to analyze, but the general trend may be understood in simple terms. Clearly E_c may be expected to increase roughly in proportion to the convective flux and the volume of the dominant convective eddy. A more detailed analysis shows that when the heat loss from a convective eddy during its lifetime is small,

$$\begin{aligned} E_c &\sim T^{5/2}(\nabla - \nabla_{\mathrm{ad}})^{-1/2} g_s^{-3} F_c \\ &\sim \rho^{1/3} F_c^{2/3} (T/g_s)^3 , \end{aligned} \tag{3.5}$$

where $\nabla = \mathrm{d}\ln T/\mathrm{d}\ln p$ and $\nabla_{\mathrm{ad}} = (\partial \ln T/\partial \ln p)_s$; when evaluated at the depth corresponding to $\tau_{c,\mathrm{min}}$, $\rho^{1/3}T^3$ roughly scales as its photospheric value. The dependence of E_c on g_s reflects the variation in the volume of the convective eddy.

We can now qualitatively account for the variation in the oscillation velocity found in Tables I and II. Along the ZAMS g_s decreases with increasing mass for $M \lesssim 1.2\, M_\odot$

and is roughly constant for larger masses, whereas $T_{\rm eff}$ increases. Furthermore ρ_s decreases with increasing $T_{\rm eff}$; in fact we have approximately

$$\rho_s \approx \beta \frac{\mu_s g_s}{\mathscr{R} \varkappa_s T_{\rm eff}} \tag{3.6}$$

(e.g. Schwarzschild, 1958), where μ_s and $\varkappa_s$ are the mean molecular weight and the opacity in the photosphere, $\mathscr{R}$ is the gas constant and β is of order unity, and in the relevant temperature range $\varkappa$ is a rapidly increasing function of T. The net effect is an increase in E_c with increasing mass, until the point where convection ceases to transport

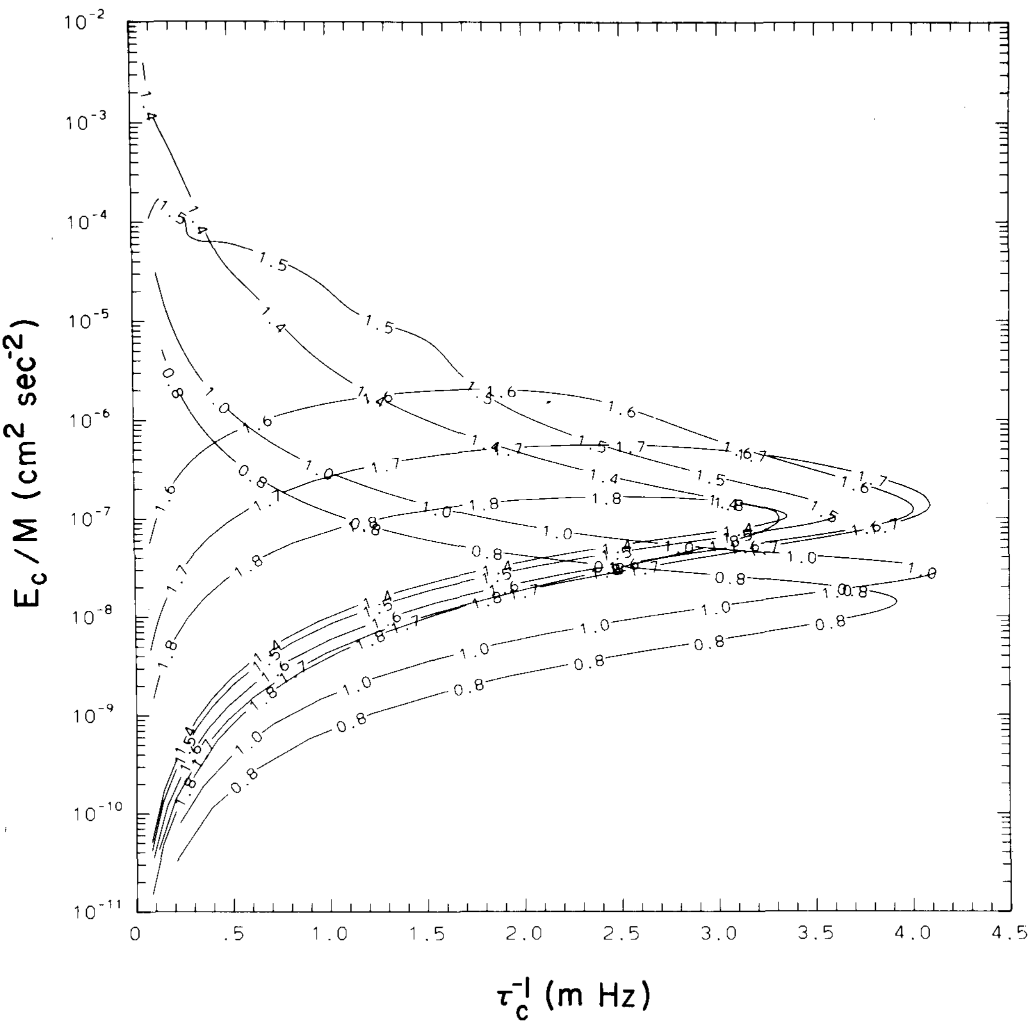

Fig. 2. The variation of E_c/M and $1/\tau_c$, E_c being the energy in a convective eddy and τ_c its time scale, through the upper convection zones in a selection of ZAMS models. The curves are labelled with $M/M_\odot$, where M is the mass of the model and $M_\odot$ the mass of the Sun.

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the major part of the total energy flux. The variation in $\mathscr{E}_{\rm osc}$ is dominated by the decrease in ρ_s with increasing mass (cf. Equation (3.1)). Thus the variations in E_c and $\mathscr{E}_{\rm osc}$ both contribute to the general increase in v_s with M shown in Table I. The beginning decrease in the most massive models is caused by a decrease in the efficiency of convection, which in the 1.8 $M_\odot$ model carries at most about 40% of the flux.

It is of some interest to study in more detail the variation of the computed quantities. Figure 2 shows the run of τ_c^{-1} and E_c/M in a number of ZAMS models. In each case the upper edge of the convection zone corresponds to large τ_c and small E_c. With increasing depth in the convection zone τ_c decreases and E_c increases, until τ_c reaches a minimum whose position is generally close to that of the maximum in the superadiabatic

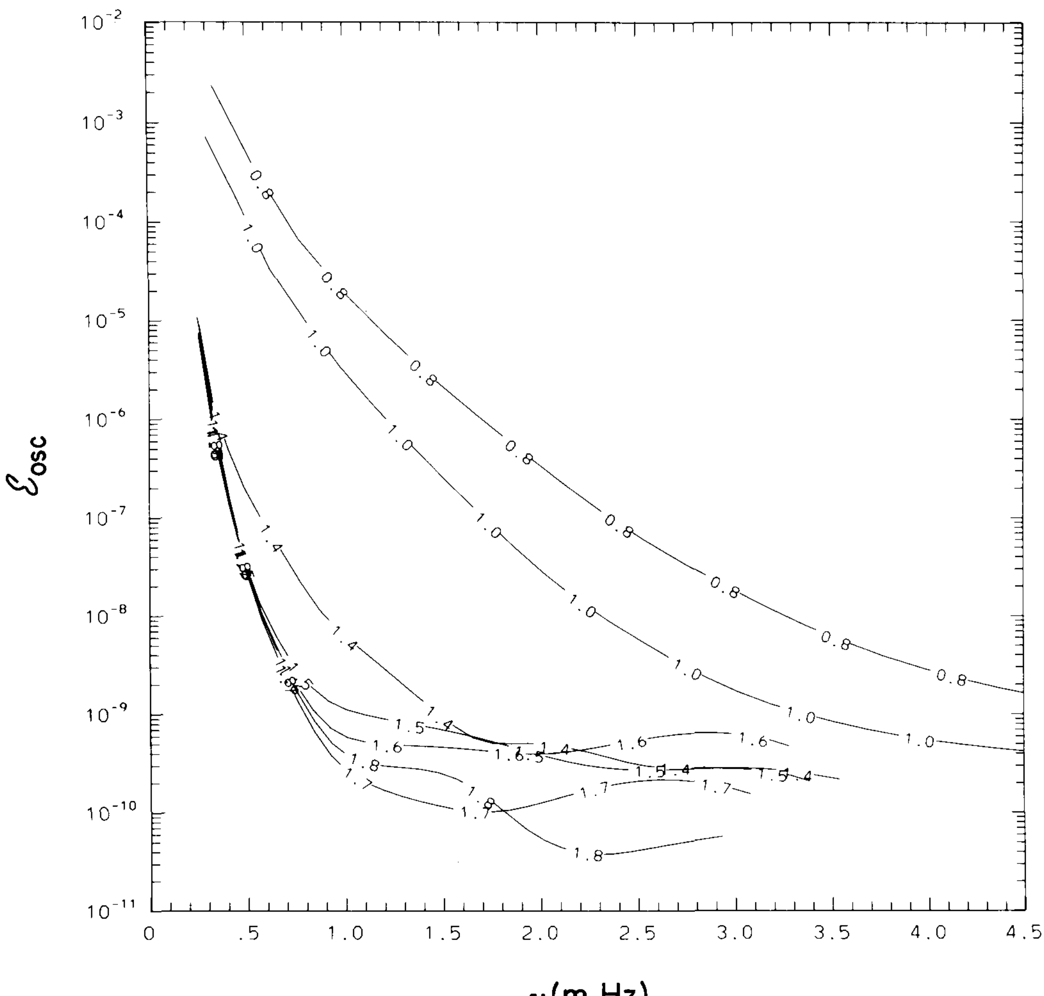

Fig. 3. The dimensionless pulsation energy $\mathscr{E}_{\rm osc}$ (defined in Equation (2.5)) for radial modes in the models on Figure 2, as functions of the cyclic oscillation frequency ν. For clarity the values for the discrete modes have been connected with continuous lines. The curves are labelled with $M/M_\odot$.

gradient. In the low-mass models, with deep convection zones, E_c continues to increase until close to the lower boundary of the convection zone where it tends to zero with the convective velocity. In high-mass models, on the other hand, convection only carries a significant fraction of the total flux near the middle of the convection zone, and E_c begins to decrease at a depth only slightly larger than the depth corresponding to the minimum in τ_c. In the models of highest mass in Table I mixing length theory predicts the existence of two separate convection zones, one corresponding to the ionization of H and the first ionization of He, and the other to the ionization of He^+. For these models only the H convection zone is included in Figure 1; the He^+ convection zone has time scales longer than the periods of the relevant oscillations.

The variation in $\mathscr{E}_{osc}$ with oscillation frequency, for the same models, is presented on Figure 3. All radial modes up to the acoustical cut-off frequency, or within the range of the convective time scales, have been included; for clarity the discrete values have been connected by continuous lines. The figure clearly shows the rapid increase in $\mathscr{E}_{osc}$ at small frequencies, predicted by Equation (3.1), as well as the decrease in $\mathscr{E}_{osc}$ with increasing stellar mass.

Finally Figure 4 shows the predicted velocity amplitudes as functions of the oscillation frequency. Both the total convective energy and the pulsational energy generally increases with increasing time scale, and hence the variation of the velocity with frequency depends on the balance between these two effects. For the lower-mass stars the variation in $\mathscr{E}_{osc}$ dominates, and the velocity is largest at the cut-off frequency $\tau_{c,\min}^{-1}$. For higher mass, however, v_c is reduced (partly because of the reduction in the mean molecular weight and in Γ_1 caused by ionization in the atmosphere) so much that $\mathscr{E}_{osc}$ varies relatively little for frequencies close to $\tau_{c,\min}^{-1}$. Hence here the increase in E_c with increasing τ_c dominates, leading to an amplitude maximum at intermediate frequencies; this is especially pronounced in the 1.5 $M_\odot$ model. For models of even higher mass, with two separate convection zones, the decrease in E_c at the lower boundary of the upper zone contributes to the decrease in the velocity at low frequencies and shifts the velocity maximum to somewhat higher frequencies.

In models with masses up to 1.6 $M_\odot$ the variation in λ_s is small, and the luminosity amplitudes follow the velocities fairly closely. At higher masses the behaviour of λ_s is less regular. Thus the 1.7 $M_\odot$ model has an exceptionally low value of λ_s, leading to the small $(\delta L/L)_s$ shown in Table I, whereas λ_s in the 1.8 $M_\odot$ model is large, causing a relatively high luminosity amplitude for this model. The reason for this behaviour of λ_s is not clear, but it may be related to differences between the two models in the structure of the convection zone. In particular the maximum efficiency of convection in the upper convection zone is reduced markedly from the 1.7 $M_\odot$ to the 1.8 $M_\odot$ model. Furthermore the upper edge of the convection zone is at a relatively small optical depth in these models, so that the surface luminosity perturbation is probably strongly affected by the behaviour of the luminosity perturbation in the convection zone. This also implies that the predicted luminosity perturbation might be quite sensitive to the neglect of the convective flux perturbation, and it should therefore be regarded as preliminary for the higher-mass models. In the lower-mass models, including the Sun, the upper boundary

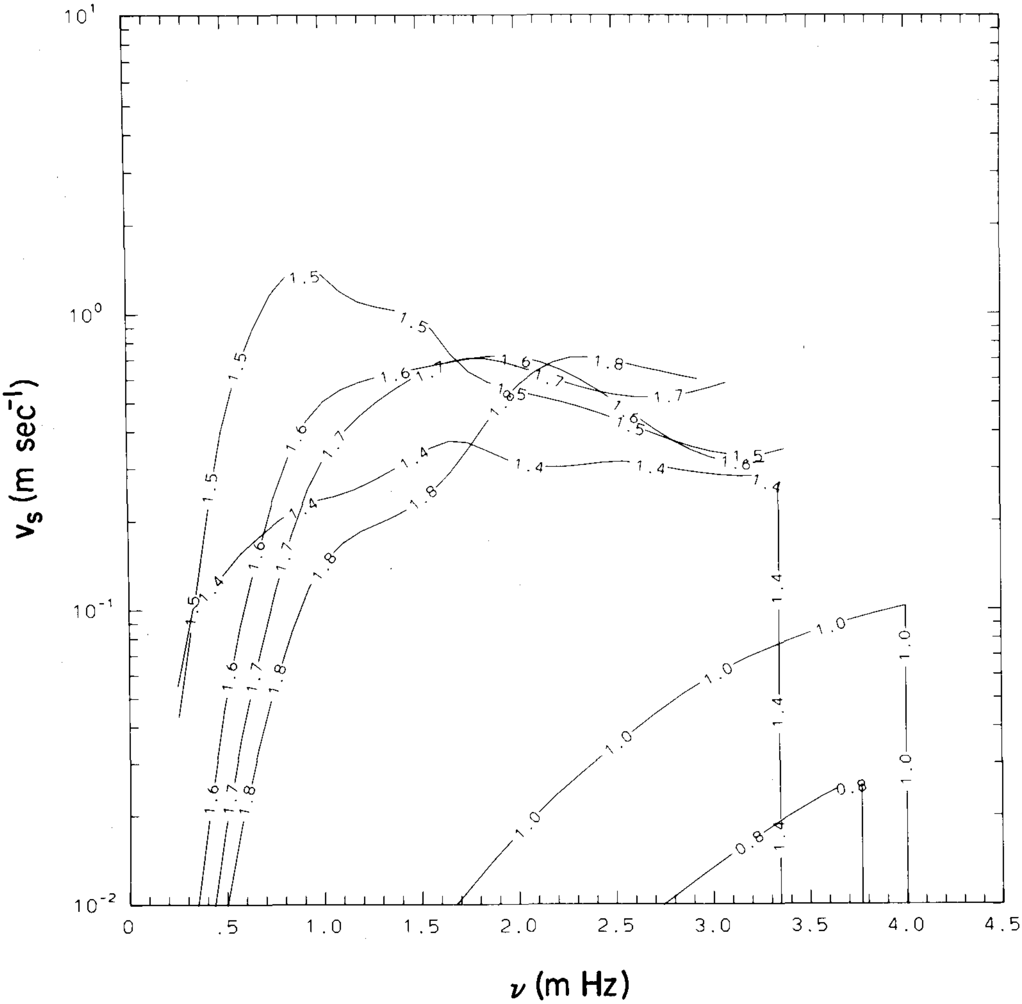

Fig. 4. Predicted surface velocity amplitudes v_s for the models on Figure 2, as functions of ν (cf. the caption to Figure 3).

of the convection zone is deeper and the effects of the neglect of the convective flux perturbation are probably less important. However a computation including the convective flux is clearly needed.

The mean frequency separation $\Delta\nu$ decreases with increasing mass of the model. This is largely an effect of the increasing radius. In fact $\Delta\nu$ can be estimated from asymptotic theory (e.g. Vandakurov, 1967) as

$$\Delta\nu \approx \left(2\int_0^{r_s} \mathrm{d}r/c\right)^{-1}, \tag{3.7}$$

and the variation in the sound speed with mass is relatively small.

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Along the 1 $M_\odot$ evolution sequence (cf. Table II) the increase in velocity amplitude comes predominantly from an increase in E_c, caused by the decrease in surface gravity (cf. Equation (3.5)); in all cases the maximum amplitude occurs at the cut-off frequency $\tau_{c,\mathrm{min}}^{-1}$. The change in the behaviour of λ_s is small, so that the variation in the luminosity perturbation follows the surface velocity closely. The decrease in $\Delta\nu$ with age largely reflects the increase in the radius.

From Equation (3.5) the amplitudes might be expected to generally increase as one moves away from the Main Sequence. In fact, as the luminosity varies much more rapidly than the mass, the amplitude is predicted to increase rapidly with increasing luminosity at fixed effective temperature. For variations in T_{eff} at fixed mass and luminosity the change in surface gravity dominates over the change in flux, and so the amplitudes are predicted to increase with decreasing T_{eff}. These predictions are largely confirmed by our results on envelope models of high luminosity, presented in Table III. The very long time scales and small frequency separations for these models are clearly caused by their low surface gravities and large radii. Furthermore the acoustical cut-off frequency decreases more rapidly with increasing radius than does the frequency ν_1 of the fundamental radial oscillation; hence the number of modes with $\nu < \nu_c$ is decreased, and in fact for several of the models in Table III the maximum amplitude is found for the fundamental radial mode.

TABLE III

Properties of oscillations of envelope models. The relation between mass and luminosity is derived from Iben's (1964) evolution calculations. The notation is as in Table I.

$M/M_\odot$	$L/L_\odot$	T_{eff}	$v_{s,\mathrm{max}}$ (km s^{-1})	$(\delta L_s/L_s)_{\mathrm{max}}$	Π_{max} (days)	$\Delta\nu$ (μHz)
5	10^3	6800	0.014	1.1×10^{-4}	0.32	2.65
–	–	5770	0.015	1.7×10^{-4}	0.56	1.82
–	–	5000	0.021	3.1×10^{-4}	3.5	1.13
–	–	4500	0.015	3.6×10^{-4}	1.2	0.75
9	10^4	6800	0.021	2.2×10^{-4}	2.0	0.64
–	–	5770	0.046	5.2×10^{-4}	2.9	0.43
–	–	5000	0.095	1.9×10^{-3}	5.2	0.30
–	–	4500	0.12	2.5×10^{-3}	37	0.21
–	–	4000	0.054	1.4×10^{-3}	60	0.15
12	3×10^4	5000	0.093	1.8×10^{-3}	12	0.16
–	–	4500	0.20	5.2×10^{-3}	18	0.12
–	–	4000	0.25	7.3×10^{-3}	130	0.085
–	–	3750	0.16	5.0×10^{-3}	170	0.064
15	10^5	6800	0.011	6.4×10^{-5}	45	0.11
–	–	5770	0.085	7.9×10^{-4}	20	0.086
–	–	5000	0.13	2.4×10^{-3}	31	0.074
–	–	4500	0.19	5.0×10^{-3}	230	0.064
–	–	4000	0.44	0.018	370	0.052
–	–	3750	1.0	0.046	480	0.040
–	–	3500	1.8	0.074	110	0.027

4. Observational Implications

Given the results of the preceeding section, we may consider the prospects for observing stellar analogues to the solar 5 min oscillations. On the basis of the amplitudes alone stars somewhat hotter or more evolved than the present Sun appear to offer the best chances. For main sequence stars with an effective temperature of 7000–7500 K velocity amplitudes of up to 10 times higher, and luminosity amplitudes up to 5 times higher, than for the present Sun were obtained. The amplitudes are still too small to be easily detectable; however the fact that evidence for the solar luminosity oscillations, reflected in the light of Neptune, was obtained by Deubner (1981) suggests that one should be able to observe luminosity oscillations with reasonable certainty in stars of spectral types somewhat earlier than the Sun. Velocity measurements are probably beyond present-day techniques, but may eventually become feasible.

For the detection of the oscillations high frequency resolution is not needed. However to be of use for seismology the observations must provide values of individual frequencies, and this puts constraints on the frequency separation between the individual modes. As shown by e.g. Loumos and Deeming (1978) the frequency resolution is $1.5/T$ for a single observation of length T; if spectra resulting from many such observations are averaged the resolution is improved to $1/T$ (Christensen-Dalsgaard and Gough, 1982). For observations with $T = 8$ hr, which is probably about the maximum duration of stellar observations from moderate latitudes, the corresponding resolutions are 52 and 35 μHz, respectively. When comparing these values with the theoretical estimates of $\Delta\nu$ given in Tables I and II should be kept in mind that the latter refer only to the radial modes. For measurements of luminosity or velocity oscillations in integrated starlight modes of degree 0, 1, and 2 are expected to dominate (Dziembowski, 1977). Unless very high resolution is achieved one would therefore (cf. Christensen-Dalsgaard and Gough, 1982) expect the observed spectrum to consist of almost uniformly spaced peaks corresponding alternately to modes of degree 0 and 2, and to a mode of degree 1, the spacing between adjacent peaks being half the value of $\Delta\nu$ given in Tables I or II. Thus to be resolved with a single night of observation, or an average of several such nights, the spectrum of oscillations of a star should have a $\Delta\nu$ of at least about 100 or 70 μHz, respectively. Evidently the higher-mass ZAMS stars or the more evolved $1\ M_\odot$ stars are close to these limits. For evolved stars of mass significantly greater than $1\ M_\odot$ frequency resolution is probably impossible if only single-night observations, analyzed separately, are used. Thus one clearly has to find a balance between the demands of high amplitude and large frequency separation, the optimum being probably slightly evolved stars somewhat, but not too much, hotter than the Sun.

With more extensive observations it may become possible to improve the resolution by combining several nights' observations coherently on the assumption that the oscillations have a sufficiently long lifetime, although the problems of aliasing would then have to be dealt with. For solar oscillations such analyses were carried out by Deubner (1981), Claverie *et al.* (1981), and Bos and Hill (1983). Alternatively a continuous record could be obtained by combining data from several different observatories spaced around the Earth; and it may ultimately become possible to observe stars for extended periods of time from the South Pole, as was done by Grec *et al.* (1980) in the case of the Sun.

Further from the Main Sequence the periods become so long, and $\Delta\nu$ so small, that frequency resolved spectra would require months or even years of observation, and $\Delta\nu$ may eventually get so small that the modes appear to merge due to their finite lifetime. However it might still be possible to observe the oscillations as apparently irregular fluctuations in brightness or velocity, and from such observations to determine the overall distribution of power with frequency. In the range of stellar parameters covered by Table III the predicted maximum amplitudes are so large that the fluctuations should be seen with reasonable ease in ordinary photometric observations. In fact there may be some evidence that such fluctuations have been observed. In an extensive set of luminosity measurements made by Rufener, Maeder (1980) found luminosity fluctuations of up to $0\overset{m}{.}1$ among giants and supergiants. The amplitudes of these fluctuations, as a function of effective temperature, appears to have a minimum in the neighbourhood of the Cepheid instability strip, and this suggests that different mechanisms are at work among high- and low-temperature stars. If so, it seems possible that stochastically excited oscillations might be responsible for the fluctuations in the low-temperature region. Indeed the observations have the same trend, of increasing amplitude with increasing luminosity or decreasing effective temperature, as the results in Table III, and the observed and predicted amplitudes agree in order of magnitude. Radial velocity fluctuations of the order of 1 km s^{-1}, observed by Gun and Griffin (1979) in stars belonging to M3, might also be caused by stochastically excited oscillations. It is tempting to speculate that irregular and semiregular variability among red giants represents a further extension of this phenomenon; the oscillations of the true red variables are probably caused by linear instability. – Clearly further observations and more theoretical work is needed to clarify these possibilities.

5. Discussion

The physical model used in predicting the oscillation amplitudes, viz. equipartition between pulsational energy in one mode and kinetic energy in one 'resonating' convective eddy, is undoubtedly very rough. It is perhaps best regarded as a scaling law which, after being calibrated against the Sun, may be used to estimate how the amplitudes vary with stellar type. Thus the general trends in the neighbourhood of the Sun, i.e. the increase in amplitude with mass or age, are probably correct if the oscillations are indeed caused by stochastic excitation, whereas the predicted properties for models further from the Sun are far less reliable.

A major uncertainty is the use of traditional mixing length theory to describe the dynamics of the convective motion. Although the inferred time scales for solar convection are not in obvious conflict with observations of solar granulation, we clearly have almost no observational evidence for the distribution of energy among the convective eddies, and for other stars we have information about neither time scales nor energies. In particular it appears from anelastic modal calculations (Latour *et al.* 1981) that mixing length theory overestimates the convective flux in relatively hot stars with thin convection zones. If this is the case the convective energies are almost certainly overestimated as well, and the predicted amplitudes for the hotter stars in Table I would have to be reduced. Estimates of convective time scales and energies from more detailed calculations would provide useful information on this issue.

Even if the spectrum of convection had been known, we would still need a consistent scheme for calculating from this the surface amplitudes of the oscillations. It is encouraging that Gough and Poyet (Poyet, 1983) have made progress towards formulating such a scheme. Thus there is hope that a more reliable basis for making the amplitude estimates may soon become available.

Given the uncertainties in the theoretical results any observational test would clearly be highly valuable. Detection of stochastically excited oscillations in other main sequency stars and measurement of their amplitude would provide such a test, but has not yet been made. However as mentioned in Section 4 Maeder (1980) has found luminosity fluctuations in relatively cool supergiants, which agree roughly in magnitude and in the dependence on stellar luminosity and effective temperature with the results obtained in Section 3. The association of these fluctuations with oscillations has not been demonstrated. They could perhaps be caused by direct fluctuations in the convective flux (Schwarzschild, 1975), or by inhomogeneities in a stellar wind. Further observations, in particular of the relation between the luminosity fluctuations and possible fluctuations in radial velocity (which should be relatively easy to measure, if the estimates in Table III are realistic), may permit a choice between these different possibilities. Confirmation that the fluctuations are caused by oscillations would not only provide an immediate test of the results obtained here, but would also imply that this type of oscillations could be studied over a fairly large region of the HR diagram, with amplitudes large enough to allow traditional photometric techniques to be used.

6. Conclusion

From the results obtained here it seems reasonable to hope that 5 min oscillations of main sequence stars will be detected in a not too distant future, and that it will become possible to measure individual frequencies of such oscillations. The observational effort required is undoubtedly large; but the results should give direct information about the structure of Main Sequence stars and would therefore be very valuable in testing the theory of stellar evolution. The first quantity to be determined would probably be the mean frequency separation $\Delta\nu$ which, as pointed out in Section 3, to a large extent is a measure of the radius of the star. However with observations of sufficiently high frequency resolution to resolve the individual modes, it should be possible to assign values of l to the individual frequencies from the observed amplitudes and the distribution of modes, as has been done for the solar oscillations, (see e.g. Christensen-Dalsgaard, 1980); the frequencies would then give more detailed information about the structure of the star. Even higher resolution might enable detection of rotational splitting of the frequencies of modes with $l > 0$; this has apparently been achieved by Claverie *et al.* (1981) for the Sun, and would give information about the rotation rate of the star, possibly eventually even about its variation with position in the star.

It is worth pointing out that observations of this type of oscillations might also be useful in the study of stellar convection. The calculation of the oscillation amplitudes resulting from a given convective velocity field is probably considerably simpler than a direct computation of the convective velocities. Furthermore, although identification of individual frequencies may be difficult for stars that are not close to the Main Sequence, it may still be possible in such stars to determine the broad variation of power with frequency. Thus, if the general idea that these oscillations are excited stochastically by convection is correct, one may hope to be able to perform at least a limited inversion on the observed amplitudes, to get information about the properties of the convection. Such information, which can then potentially be obtained over a wide range of stellar parameters, would clearly be very useful in testing theories of stellar convection.

Acknowledgements

We are grateful to J. Andersen for the suggestion that led to the present work, and to T. M. Brown, D. O. Gough, M. Jura, A. Reiz, and J. Toomre for useful discussions. JC-D would like to thank Professor G. K. Batchelor for hospitality at DAMTP, Cambridge, where much of the paper was written.

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