AN ADDENDUM AND A SHORT COMMENT ON THE PAPER

Estimating the value of the Wincat coupons of the Winterthur Insurance convertible bond: a study of the model risk by U. Schmock

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1. INTRODUCTION

In January 1997, Winterthur Insurance, together with Credit Suisse First Boston (CSFB), issued the first listed CAT bond. The annual "WINCAT" coupons of this three-year convertible bond are knocked out if any single storm event damages more than 6,000 vehicles insured by Winterthur Insurance in Switzerland.

This was a completely new way of securing insurance risks. The main intention was to test the Swiss capital market for such products and to make investors acquainted with them. Thereby Winterthur, together with CSFB, set new standards in product transparency, fairness of pricing and investor education by making the historical data available via internet and by publishing a special brochure (CSFB (1997)), where the pricing and the mathematical modelling are described in detail. This is also a prerequisite to enable a scientific discussion on pricing aspects of such new financial products. The developers of the bond are therefore grateful to Mr. Schmock for this valuable scientific contribution which can be seen as a thorough and profound statistical analysis on the knock-out probability for the purpose of quantifying the model uncertainty.

In Section 2 we briefly summarize the whole pricing of the bond at the issue date and show that there were several risk premium elements in this pricing where the conservative estimation of the knock-out probability was just one of them. In Section 3 we consider the modelling of low frequency risks from a practitioner's standpoint and formulate some requirements from practice. In Section 4 we make some further comments on the modelling of the Wincat data. Section 5 is a short summary.

ASTIN BULLETIN, Vol. 29, No. 1, 1999, pp. 165-171

https://doi.org/10.2143/AST.29.1.504609 Published online by Cambridge University Press

2. The pricing of the Wincat bond

As pointed out in CSFB (1997), the value of the bond is steered by three value-driving components which can be evaluated separately:

(1) the present value of the principal that either becomes due at maturity or is applied towards conversion into shares,
(2) the value of the conversion right, and
(3) the present value of the expected coupon payments.

ad (1) Estimating the present value of the principal is straightforward. The discount factor consists of the risk-free interest rate and a spread. The spread depends on the creditworthiness of the issuer and is only arbitrary within relatively small limits.

ad (2) Next to the interest rate, the value of the conversion right depends on the knock-out probability of the last coupon, the expected dividend payments of the shares and the expected volatility of the underlying Winterthur stock. The latter three values have to be estimated. Since the conversion right was far out of the money, variations of the expected volatility result in large differences in the value of the conversion right.

ad (3) The present value of the expected coupon payments depends on the risk-free rate, the spread for Winterthur's creditworthiness and the knock-out probabilities of the annual coupons.

From all these value-driving factors, the determination of the knock-out probabilities (and their risk premiums) is the most interesting one. The paper by Schmock concentrates on this point. But it should be kept in mind that all the other factors also affect the value of the bond. Thereby Winterthur and CSFB had to respect the interests of different types of investors: the terms had to be interesting for investors looking for higher coupons or an attractive spread with respect to the Swiss Confederation Bonds; the knock-out probability P_{CAT} had to guarantee a risk premium for the CAT risk; and last but not least the implied volatility of the conversion right had to please investors mainly looking for a convertible bond. The pricing had to respect all of these interests and was therefore also a compromise in this respect.

The expected P_{CAT} amounts to 13.6% using a constant Poisson parameter model. In CSFB (1997), a fair value of 100.88% is calculated for the value of the convertible using a spread of 35 basis points over the zero-coupon yield on Confederation Bonds, an expected dividend of CHF 21, an implied volatility of 17%, and a P_{CAT} of 25%. The bond was issued at 100% and not at 100.88%.

The reader should note that there are several risk-premium components in the whole pricing, only one of them being a conservative estimation of P_{CAT}. For instance an obvious loading was the fact that the bond was issued at 100% and not at 100.88%. Furthermore the volatility of 17% of the underlying used for the valuation of the conversion right was below the actual market volatility of Winterthur shares and below the implied volatilities of comparable options or warrants, which was of course in favour of the investors. Indeed Winterthur was ready to pay a certain price for the development of this market and to grant a comfortable risk premium to the investors.

Those who have invested in the bond will be very satisfied with the performance of their investment so far. The coupons were paid out in the years 1997 and 1998 and the value of the bond on 12th April 1999 was 222% (100% = issue price).

3. Remarks on modelling of low frequency risks from a practioner's point of view

The subject of Schmock's paper is essentially the modelling of low frequency risks. The main problem dealing with such risks is that there are usually only few observations and that there are several models fitting to the scarce statistical data. Schmock investigates no less than 25 models for the Wincat data, and there are still more models which would be reasonable (cf. section 4). Of course different models will lead to different answers, and these different answers might be a guidance for the evaluation of the model risk. It is the merit of Schmock's paper to have drawn our attention to the substantial model risk inherent in pricing products like Wincat. On the other hand an actuary has to choose one model at the end of the day from the various thinkable models to base his calculation on. Moreover an actuary working in practice has often not enough time to examine too many different models. Therefore some "guidelines from practice" might we worthwhile.

A first point to be mentioned is that the scarce statistical data available for the specific problem in question is not the only source of information to the actuary. Actuaries who are regularly confronted with the evaluation of insurance risks have built up in the course of their professional career a considerable **a priori knowledge** which should be used in the model building process and which can reduce the model uncertainty to a certain extent. Indeed the quality of insurance risk models largely depends on the model builder's capabilities of incorporating such a priori knowledge into the model. For instance in the Wincat problem a simple "seasonal" model (assuming a long term cycle) would well fit to the observed data (= number of events with more than 1,000 damages vehicles in a given year), but it wouldn't make sense from an a priori point of view. Why should the number of heavy hail-storms follow such a cyclical pattern? There is no reason for this.

A first practical model requirement is **simplicity**. Models should be as simple as possible and as sophisticated as necessary. The actuary has to explain his calculations and findings to his "customers", and for this purpose simplicity can only be an advantage. Of course one should not "oversimplify" which is the meaning of "as sophisticated as necessary" in the above statement. In the case of Wincat, the customer was the financial market. It was important that the model was explainable to and understandable by this customer. A simple and natural way was to assume a constant Poisson parameter model. More sophisticated models are of course thinkable, but one should not abandon a simple model in favour of a more sophisticated one unless there are really strong arguments for the latter, be it from the data and/or be it from a priori considerations.

A further model requirement from practice is **robustness**. One should be reluctant to use a model where slight changes in the data have a great effect on the obtained results. In the case of Wincat it was desirable to have forecasts which are not too sensitive with respect to updating the model, because great variations in the forecasts could diminish the credibility of the pricing in the market. A look at the Table 12.1 in Section 12 of Schmock's paper reveals that the constant Poisson parameter model No. 2 is much more robust than for instance the more sophisticated modified linear trend model No. 13. This robustness aspect is an important point in most practical situations and a strong practical argument in favour of the simple model.

Finally **parsimony** is a third guideline for modelling. One should always aim to have a model with as few parameters as possible. This is a general statistical principle. More parameters usually give a better fit to the data, but this does not necessarily mean an improvement of the predictive power. On the contrary "overparametrisation" usually leads to poorer forecasts. In practice there is also another argument for parsimony. It is important to know what the parameters mean and what effect a change of a parameter value will have on the result. This is often not the case when using a model with many parameters.

As regards the Wincat problem we are in the comfortable situation of now knowing the outcomes of the years 1997 and 1998 of this "random experiment", namely one observed event with more than 1,000 damaged vehicles in each of these years. We can compare now these new observations with the forecasts of the different models. In the following we do this for the constant Poisson parameter model and for one of the more sophisticated models, namely the modified linear-trend model of Subsection 7.4 in the paper of Schmock.

The following Figure 1 shows the fitted curve and the forecasts of the two models as well as the observations used for the forecast (dotted points) and the two new observations (quadratic points). It reveals that the constant parameter model forecasted much better the two new observations than the trend model.

The following Figure 2 shows the forecasts of the modified linear trend model, but now evaluated at the end of 1996 (data available at the time of issuing the bond), a first update at the end of 1997 and a second update at the end of 1998. It illustrates the sensitivity of this model forecast. For 1999 the forecast is successively reduced from 5.27 to 2.82, i.e. by 46%! In contrast to this the forecast of the constant Poisson parameter model remains nearly unchanged during this period, which illustrates that the

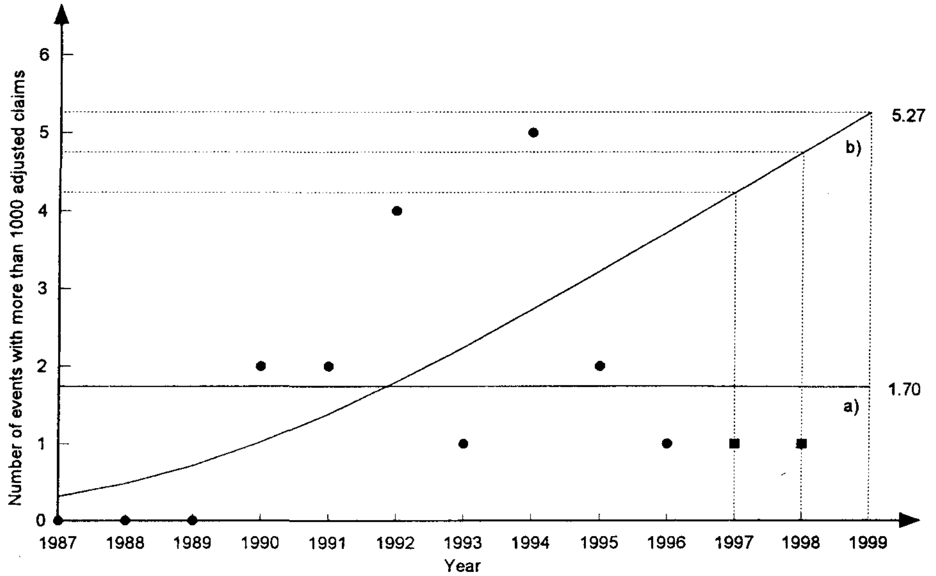

FIGURE 1
a) fitted line/forecast of the constant Poisson parameter model
b) fitted line/forecast of the modified linear trend model

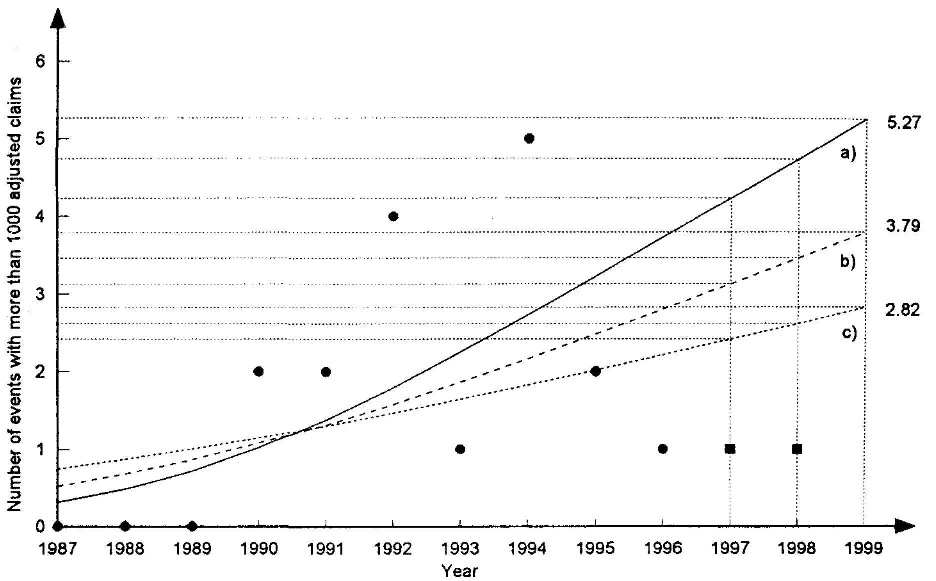

FIGURE 2
a) fitted line/forecast at the end of 1996
b) update fitted line/forecast at the end of 1997
c) update fitted line/forecast at the end of 1998

https://doi.org/10.2143/AST.29.1.504609 Published online by Cambridge University Press

simple model is much more robust. Note also that the first high forecasts of the trend model have successively moved in the direction of the value of the constant parameter model.

4. A FURTHER REMARK ON THE MODELLING OF THE WINCAT DATA

The careful reader of Schmock's paper might have noticed that the no-trend hypothesis is rejected on a 1.66% level in the test of the modified linear model carried out in section 7 of Schmock's paper. Of course this would no longer be the case if taking into account also the two new observations. But this is not an argument, as one has to consider the situation as it was at the issue date of the bond. Hence one might ask whether the simple model is not an oversimplification as there were strong reasons from the data against it. Indeed there might be a simplification in that model. Looking at the data one notices that the empirical variance is much bigger than the mean whereas for Poisson the two values should be about the same. Hence the Poisson assumption itself might be questionable. Indeed, a mixed Poisson assumption, which in the framework of generalised models means an overdispersion, would probably be more adequate, since it is also supported by the following a priori considerations. Heavy hail-storms emerge under special weather conditions, but given such weather conditions it is not unlikely that several hail-storms arise during a relatively short period. An adequate way to model such a situation would be to assume that the number of events in a given year is conditionally Poisson, given the general weather condition of that year, whereas the Poisson parameter is itself the outcome of a random variable reflecting the variation of the weather conditions in different years. But this means to assume a mixed Poisson distribution. However a mixed Poisson model will yield the same point estimates for the frequencies as the Poisson model. Hence there was no necessity of using the more complicated mixed Poisson model by the developers of Wincat. Looking at the different graphs of the different models in Schmock's paper one sees that the observed residuals are still big. Most of the residuals are outside the range of the fitted line plus/minus one standard deviation resulting form the Poisson assumption. Schmock is aware of that and carries out a test for overdispersion in subsection 5.1. Although the empirical variance (value 2.9) is much bigger than the observed mean (value 1.7), the Poisson assumption is not rejected on the 5% level. But of course this does not mean that a mixed Poisson assumption would not better describe reality. Given that the data **and** the a priori arguments go in the same direction, a mixed Poisson assumption would certainly be adequate. This shows that one could easily enlarge the number of reasonable models by just assuming for instance a Negative Binomial instead of a Poisson distribution. But again this would have no influence on the point estimates and hence on the forecasts and the pricing. However when it comes to testing a model it is crucial to take an eventual overdispersion into account. The present authors have therefore made the same test as mentioned above but by using the genmod-procedure in SAS (i.e. generalised linear model framework) assuming a "Poisson model" with overdispersion. They then got a *p*-value of 5.4% (to be compared with the 1.66% mentioned above), i.e. the no-trend model is rejected on the 5% level when allowing for overdispersion. Thus taking into account the overdispersion there was no evidence from the data against a constant parameter model.

5. Summary

The present authors thank Mr. Schmock for his valuable scientific contribution focusing on the substantial model risk inherent in pricing financial products like Wincat. However they believe that the constant parameter model used for pricing the Wincat is still a reasonable and adequate model also looked at from an a posteriori point of view given the analysis of Schmock and given the two new observations of the years 1997 and 1998.

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