If in addition (iv) holds,

$$i.e., \text{ (iii) and } \left\| \begin{matrix} \phi_{xx}, & \phi_{xy} \\ \phi_{xy}, & \phi_{yy} \\ \phi_{tx}, & \phi_{ty} \end{matrix} \right\| = 0 \quad - \quad - \quad \text{(vi)},$$

the envelope has a cusp with the same cuspidal tangent.

(6) If in addition to (iii)

$$\phi_{tx} = 0, \ \phi_{ty} = 0, \quad - \quad - \quad - \quad \text{(vii)}$$

(*i.e.*, (i) and (iii)) the branches of the discriminant have 3-pointic contact with those of the curve.

(7) If (ii) and (iii) hold, the envelope has a singularity of the form $\eta^3 = \lambda\xi^4$, where $\eta = 0$ is the tangent to $\phi_t = 0$.

(8) But if this tangent should coincide with one of the two tangents to the curve at the double-point, *i.e.*, (iv), the form is $\eta = \lambda\xi^2$ thrice.

A Proof of the Theorem that the Arithmetic Mean of n positive quantities is not less than their Harmonic Mean.

By W. A. Lindsay, M.A., B.Sc.

Two Theorems on the factors of $2^p - 1$.

By George D. Valentine, M.A.

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