

---

# The Mathematical Gazette

---



Index to Volume 106  
Numbers 565 to 567  
2022



---

THE MATHEMATICAL ASSOCIATION



# *Index*

*to the*

# *Mathematical Gazette*

Vol. 106

2022

NO.	MONTH	PAGES
565	March	1 – 192
566	July	193 – 384
567	November	385 – 576

---

Editorials	Articles
Notes	Teaching Notes
Feedback	Correspondence
Problem Corner	Student Problems
Matters for Debate	Obituaries
Reviews	Editors

---

## Articles

AUTHOR(S)	TITLE	PAGE
Talaat Abdin, Hosam Mahmoud, Arian Modarres, Kai Wang	An index for betting with examples from games and sports	32
Michael Heinrich Baumann	An illustrative derivation of the sum of fifth powers	68
Paul Belcher	Approximating the volume of a solid of revolution – the frustum rule	408
A. F. Beardon	The real solutions of $x = a^x$	206
Mario Dalcín	New dualities in convex quadrilaterals	269
Subhranil De	The intriguing mechanics of a tractrix of cards	281
François Dubeau	Archimedes playing with a computer	427
Michael N. Fried	Locus problems concerning centroids of a cyclic quadrilateral and two classic cubic curves	247
Eric J. Friedman and Adam S. Landsberg	Winning strategies: the emergence of base 2 in the game of nim	212
Alexandru Gîrban, and Bogdan D. Suceavă	Power of a point: from Jakob Steiner to modern applications	41
Mowaffaq Hajja	The arbitrariness of the semi-angle-bisectors of a triangle	78
David Hopkins	Dropping plates	193
G. J. O. Jameson	Equal sums, sums of squares and sums of cubes	54

AUTHOR(S)	TITLE	PAGE
G. J. O. Jameson	The majorisation principle for convex functions	95
Martin Josefsson	New characterisations of bicentric quadrilaterals	414
Stephen K. Lucas and Amrik Singh Nimbran	Monotonic series for fractions near $\pi$ and their convergents	300
Lubomir Markov	Two short proofs of the formula $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$	28
Des MacHale	Dissecting attached squares	258
John D. Mahony	A mathematical approximation in the physical sciences	220
Abdel Missa. Chrif Youssfi	An alternative formula for the cubic equation	474
Abdel Missa. Chrif Youssfi	A novel method to solve the quartic equation	480
Toshio Nakata	Characterisation of equalisation problems via random walks	61
Saralees Nadarajah. Idika E. Okorie	On the tail integral formulae for real-valued random variables	487
Amrik Singh Nimbran	Evolution of the Euler-Maclaurin sum formula	443
Dario Pellegrinetti and Michael de Villiers	An extension of the six-point circle theorem for a generalised Van Aubel configuration	400
Chris Pritchard	Focus on the Visual (The 2022 Presidential Address)	386
Joscha Prochno and Michael Schmitz	A probabilistic way to discover the rainbow	103
Emrys Read	On the class of an integer triangle	291
Tim Rowland	Mathematics in 'the news': number theory and number sense	467
Aldo Scimone	Golden right triangles and the golden quadrilateral	9
John R. Silvester	The trisectrix and Langley's problem	21
Howard Sporn	Fibonacci fraction circles	1
Howard Sporn	Fibonacci-Lucas hyperbolas	242
Seán M. Stewart	A look back at a long-forgotten trigonometric function: the versine function and its inverse	84
Jerry Sullivan	A computer look at $N!$	233
Leonard M. Wapner	Probability: a questionable science of the uncertain	458

## Notes

March	106.01 – 106.16
July	106.17 – 106.37
November	106.38 – 106.49

AUTHOR	NO.	TITLE	PAGE
Mateus Alegri	106.41	Infinitely many series arising from $\cos^2 x + \sin^2 x = 1$	517
Elif E. Arikan, Hasan Unal	106.38	PWW: $\sin \alpha + \sin \beta$ and $\cos \beta - \cos \alpha$	514
Stephan Berendonk	106.16	Sums of Hex numbers are cubes - a planar Proof without Words	147
David Bevan	106.30	Threshold functions and the birthday paradox	343
A. F. Beardon	106.33	Fibonacci numbers and Cassini's identity	498
Ron Brown	106.31	What proportion of square-free numbers are divisible by 2? or by 30 but not by 7?	494
Günhan Caglayan	106.32	Pentagonal numbers and their relationships to other figurate numbers	497
Manishita Chakraborty	106.39	Visual proofs of sums of powers of positive integers	515
Himadri Lal Das	106.42	Another proof of Rolle's Theorem	521
Prithwijit De	106.06	An optimisation problem involving right circular cones	127
Prithwijit De and Sutanay Bhattacharya	106.17	An interesting spin-off	310
Sourangshu Ghosh	106.43	Another proof of $e^{x/y}$ being irrational	523
Sourangshu Ghosh	106.44	Another proof of the irrationality of $N^{p/q}$	525
Edney Freitas Gregorio	106.08	PWW: the Cauchy-Schwarz inequality using analytic geometry	132
Mehdi Hassani	106.09	Observations on a proof without words	133
Hailiang Hu	106.04	A geometric interpretation of Cramer's rule	124
Tran Quang Hung	106.12	A new proof of the $n$ -dimensional Pythagorean theorem	136
Jay Jahangiri	106.45	A generalisation of a classical open-top box problem	526
Wei-Dong Jiang	106.29	An improvement on the Garfunkel-Bankoff inequality	342
Petro Kolosov	106.37	An unusual identity for odd-powers	509
Francesco Laudano	106.40	The law of tangents and the formulae of Mollweide and Newton	516
Paul Levrie and Amrik Singh Nimbran	106.21	A class of interesting integrals	323
Martin Lukarevski	106.14	Exarc radii and the Finsler-Hadwiger inequality	138
Martin Lukarevski and J. A. Scott	106.25	Three discs for the incentre	332
Melvyn B. Nathanson	106.03	Real-rooted polynomials and a generalised Hermite-Sylvester theorem	120
Nick Lord	106.19	Some observations on inequalities related to Huygens' inequality	316
Nick Lord	106.26	The nested polygons problem revisited	335
Hiroshi Ohyama	106.18	Impossibility of solving the quintic using Cardano's solution	312
Victor Oxman and Moshe Stupel	106.10	PWW: Trigonometric inequality	134
R. Padmanabhan and Alok Shukla	106.20	When do we have $1 + 1 = 11$ and $2 + 2 = 5$ ?	319
Sammedkumar M. Patil	106.36	A method to calculate the harmonic number and other related sums	508
Dario Pellegrinetti	106.11	On a synthetic proof of Bottema's theorem	135
Ángel Plaza	106.07	A function-based proof of the harmonic mean– geometric mean – arithmetic mean inequalities	130
Ángel Plaza	106.24	Proof without words: a Riemann sum	331
J. A. Scott	106.46	On the Gerretsen inequalities in trigonometrical form	532

AUTHOR	NO.	TITLE	PAGE
J. A. Scott	106.48	On the first Fermat point for the triangle	539
Sabu Sebastian	106.34	A theorem on divisibility by congruence	501
Juhaina A. Shahbari and Moshe Stupel	106.13	A triangle inequality	138
Angad Singh	106.01	The number of divisors of the LCM of the first $n$ natural numbers	116
K. B. Subramaniam	106.23	Proof without words: $\sin 3x = 3 \sin x - 4 \sin^3 x$	330
Nguyen Xuan Tho	106.02	The rational distance problem revisited	117
Nguyen Xuan Tho	106.05	On the irrationality of sums of square roots	125
Nguyen Xuan Tho	106.15	A proof of Lukarevski's conjecture	143
Nguyen Xuan Tho	106.27	An interesting application of Ptolemy's inequality	338
Nguyen Xuan Tho	106.28	Inequalities involving the inradius and altitudes of a triangle	341
Vladimir Volenec	106.49	An area problem using barycentric coordinates	541
Thanh Tung Vu	106.35	On the representation of rational numbers in the form $(\phi(m^r))^a / (\phi(n^s))^b$	504
Robin Whitty	106.47	Halving a triangle in a given direction	534
Norio Yoshida	106.22	The golden ratio represented by a tangent	325

## Teaching Notes

Nick Lord	A quick simultaneous evaluation of $\sum k^3$ and $\sum k^2$	149
Nick Lord	How to impress a chemist (again!)	149
Nick Lord	Two unusual optimisation problems	151
Nick Lord	The full story of invariant lines	547
Félix Martínez de la Rosa	Rewriting polynomials: a tool for teaching secondary mathematics	544

## Feedback

March	John D Mahony	Diagonal Problem Conjecture	p. 155
	Hugh Porteous	Problem, Number 425	p. 155
	Peter Giblin	On 105.28	p. 156
	Owen Toller	On 105.49	p. 158
July	Martin Lukarevski	On 'What makes a good Proof without Words	p. 349
	Graham Jameson	On 'A pretty series revisited'	p. 350
	Lars Lund-Hansen	On 'Correct answer – dodgy method'	p. 350
	Alan Beardon	On 105.28	p. 351
November	Robert M. Young, Jack Calcut	On 106.06	p. 549
	Alan Beardon	On 106.17	p. 550
	Paul Stephenson	On 'How to impress a chemist – again!'	p. 551
	Bob Burn	On Feedback July 2022	p. 552

**Reviews**

AUTHOR	TITLE	REVIEWER	PAGE
Mark J. Ablowitz, Athanasios S. Fokas	Introduction to complex variables and applications	<i>Nick Lord</i>	570
David Acheson	The wonder book of geometry	<i>Gerry Leversha</i>	377
Hamza Alsamraee	Advanced calculus explored	<i>Seán M. Stewart</i>	181
Ariel Amir	Thinking probabilistically	<i>Owen Toller</i>	378
Allan Bickle	Fundamentals of graph theory	<i>Mark Hunacek</i>	379
Ethan Bueno de Mesquita, Anthony Fowler	Thinking clearly with data	<i>Owen Toller</i>	572
Samia Challal	Introduction to the theory of optimization in Euclidean space	<i>Owen Toller</i>	185
Francois Digne, Jean Michel	Representations of finite groups of Lie type (2nd edn.)	<i>Mark Hunacek</i>	372
Radek Erban, S. Jonathan Chapman	Stochastic modelling of reaction-diffusion processes	<i>David Hopkins</i>	186
Raymond Flood, Tony Mann, Mary Croarken (eds)	Mathematics at the meridian: the history of mathematics at Greenwich	<i>Tony Crilly</i>	170
Sophie Goldie, et al	OCR A level Mathematics (A) for Year 2	<i>Gerry Leversha</i>	564
Frederick P. Greenleaf, Sophie Marques	Linear algebra I	<i>Mark Hunacek</i>	178
Frederick P. Greenleaf, Sophie Marques	Linear algebra II	<i>Mark Hunacek</i>	180
Geoffrey R. Grimmett, David R. Stirzaker	Probability and random processes (4th edition)	<i>Owen Toller</i>	568
Geoffrey R. Grimmett, David R. Stirzaker	One thousand exercises in probability (3rd edition)	<i>Owen Toller</i>	568
Adam Hart-Davis	Fibonacci's rabbits	<i>Peter Hall</i>	191
Julian Havil	Curves for the mathematically curious	<i>Samuel Hewitt</i>	184
Anton Howes	Arts & minds: how the Royal Society of Arts changed a nation	<i>Tony Crilly</i>	362
Dirk Huylebrouck	Africa and mathematics: from colonial findings back to the Ishango Rods	<i>Firdous Ahmad Mala</i>	370
Nets Hawk Katz	Calculus for cranks	<i>Nick Lord</i>	569
T.W. Körner	Where do numbers come from?	<i>Nick Lord</i>	188
Snezana Lawrence	A new year's present from a mathematician	<i>Peter Shiu</i>	563
Panos Louridas	Algorithms	<i>Hugo Aaronson</i>	380
Des MacHale	Number and letter puzzles	<i>Owen Toller</i>	191
Elizabeth S. Meckes, Mark W. Meckes	Linear algebra	<i>John Baylis</i>	174

AUTHOR	TITLE	REVIEWER	PAGE
Cheryl Misak	Frank Ramsey: a sheer excess of powers	<i>Tony Crilly</i>	363
Mircea Pitici (ed.)	The best writing on mathematics 2019	<i>Gerry Leversha</i>	381
Chris Pritchard	Pack up a penguin, journeys into the mathematics of area	<i>Peter Hall</i>	361
Henry J. Ricardo	A modern introduction to differential equations (3rd edn.)	<i>Owen Toller</i>	376
David Lindsay Roberts	Republic of numbers	<i>Mark Hunacek</i>	172
James C. Robinson	An introduction to functional analysis	<i>Mark Hunacek</i>	375
Tony Royle	The flying mathematicians of World War I	<i>Tony Crilly</i>	367
Edward Scheinerman	The mathematics lover's companion	<i>Owen Toller</i>	383
Ben Sparks, Claire Baldwin	OCR A level Further Mathematics for Core Year 2 (A)	<i>Gerry Leversha</i>	566
Gilbert Strang	Linear algebra for everyone	<i>Peter Giblin</i>	574
Matthew C. H. Tointon	Introduction to approximate groups	<i>Mark Hunacek</i>	175
Fabio Toscano	The secret formula	<i>Owen Toller</i>	366
Godfried T. Toussaint	The geometry of musical rhythm: what makes a “good” rhythm good?	<i>David Hopkins</i>	187
Daniel J. Velleman	How to prove it (third edition)	<i>Peter Giblin</i>	567
Cédric Villani	Mathematics is the poetry of science	<i>Gerry Leversha</i>	371
Steven Weintraub	Linear algebra for the young mathematician	<i>Mark Hunacek</i>	176
Kit Yates	The maths of life and death	<i>Peter Shiu</i>	189
Eric Zaslow	Quantitative reasoning	<i>Peter Hall</i>	374

## Student Problems

Numbers	2021 – 5 & 6	2022 – 1 & 2	2022 – 3 & 4	2022 – 5 & 6
Statement	<b>105</b> p. 559	<b>106</b> p. 167	<b>106</b> p. 358	<b>106</b> p. 560
Solutions	<b>106</b> pp. 168-160	<b>106</b> pp. 359-360	<b>106</b> pp. 561-562	<b>107</b> p.##

## Problem Corner

Numbers		<b>105</b> E, F, G, H	<b>105</b> I, J, K, L
Statements		<b>105</b> p. 358	<b>105</b> pp. 552-553
Solutions		<b>106</b> pp. 161-166	<b>106</b> pp. 353-358
Numbers	<b>106</b> A, B, C, D	<b>106</b> E, F, G, H	<b>106</b> I, J, K, L
Statements	<b>106</b> p. 159-160	<b>106</b> p. 352	<b>106</b> p. 553
Solutions	<b>106</b> pp. 554-559	<b>107</b>	<b>107</b>



## Cover Pictures

	<i>Image</i>	<i>Author</i>	<i>Pages</i>
March	The trisectrix and Langley's problem	<i>John R. Silvester</i>	17-21
July	The intriguing mechanics of a tractrix of cards	<i>Subhranil De</i>	281-290
November	Nightingale's polar area graph	<i>Chris Pritchard</i>	386-399

## The 2022 *Gazette* editorial team

<i>Editor</i>	Dr Gerry Leversha
<i>Production Editor</i>	Mr Bill Richardson
<i>Reviews Editor</i>	Mr Owen Toller
<i>Problem Corner Editor</i>	Mr Nick Lord
<i>Student Problem Corner</i>	Ms Beth Woollacott, Ms Tuya Sa
<i>Assistant Editor</i>	Mr Tom Roper
<i>Proofreaders</i>	Mr Colin Davis, Mr Michael Fox, Mr Graham Howlett, Professor Adam McBride

Thanks also to Graham Howlett for his help in checking this Index.