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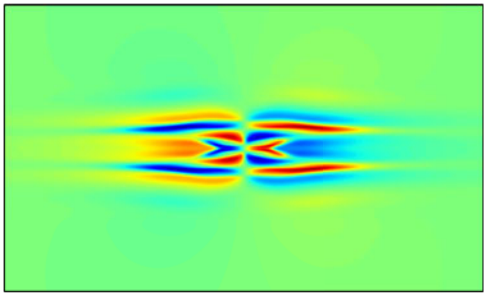
Doubly localized states in plane Couette flow



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Much of our understanding of the transition to turbulence in flows without a linear instability came with the discovery and characterization of fully three-dimensional solutions to the Navier–Stokes equation. The first examples in plane Couette flow were periodic in both spanwise and streamwise directions, and could explain the transitions in small domains only. The presence of localized turbulent spots in larger domains, the spatiotemporal decoherence on larger scales and the ability to trigger turbulence with pointwise perturbations require solutions that are localized in both directions, like the one presented by Brand & Gibson (*J. Fluid Mech.*, vol. 750, 2014, R3). They describe a steady solution of the Navier–Stokes equations and characterize in unprecedented detail, including an analytic computation of its localization properties. The study opens up new ways to describe localized turbulent patches.

Key words: instability, nonlinear dynamical systems, transition to turbulence

1. Introduction

Studies in many linearly stable shear flows have shown that exact three-dimensional (3D) solutions to the Navier–Stokes equation are key to understanding the transition mechanism. No turbulence is possible without them. However, their existence is not sufficient for the presence of turbulence, since they may be transient or confined to particular flow fields (Eckhardt *et al.* 2007). With increasing Reynolds number, more such states appear and the dynamical connections between them form an intricate web that has been dubbed the 'clockwork behind turbulence' (Cvitanović 2013).

Most studies in plane Couette flow, plane Poiseuille flow, pipe flow or boundary layers have considered small domains and spatially extended states. It is difficult to see how such extended states could be used to explain the localized puffs and slugs in pipe flow (Mullin 2011), the patterns in plane Couette flow and their spatiotemporal dynamics (Barkley & Tuckerman 2005; Duguet, Schlatter & Henningson 2010) or the decay of correlations in more turbulent situations at higher Reynolds numbers.

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The way out of this dilemma is provided by solutions that are localized. The numerical experiments by Lundbladh & Johansson (1991) were the first step in that direction: they monitored the temporal evolution of small localized perturbations added on top of plane Couette flow. When the Reynolds number was low, the perturbation became weaker and eventually disappeared. When the Reynolds number was high enough, the perturbation began spreading into the domain. Brand & Gibson (2014) thus identify and study a marginal state on the boundary between the decaying and the turbulent ones.

Spanwise localized states have been identified and related in an intriguing arrangement of symmetry-related states of increasing widths (Schneider, Gibson & Burke 2010*a*; Gibson & Brand 2014). Their origin in long-wavelength instabilities of the original spatially extended systems has been elucidated, in both the spanwise (Melnikov, Kreilos & Eckhardt 2014) and downstream directions (Chantry, Willis & Kerswell 2014). States that are localized in both directions have been described for plane Couette flow before (Schneider, Marinc & Eckhardt 2010*b*), but they were dynamically active and most likely chaotic. With these observations, the hunt was on for stationary states in plane Couette flow that are localized in both streamwise and spanwise directions. One of them has now been found and described by Brand & Gibson (2014).

2. Overview

Finding exact coherent states usually is a real challenge: the huge bag of tools from bifurcation theory that is so useful in Taylor–Couette and Rayleigh–Bénard flows is not available because the essential point of entry, a linear instability of the laminar profile, is missing. The way out are ingenious embeddings of the flow one wants to study into families of flows where an instability can be observed. Adding rotation (Nagata 1990) or heating from below (Clever & Busse 1997) paved the way to the identification of exact coherent structures in plane Couette flow. Artificial forces served the purpose in pipe flow (Faisst & Eckhardt 2003), and homotopies provided links between states in different flows (Waleffe 2003).

Brand & Gibson (2014) add to this toolbox a flexible method for finding localized states: they multiply a spatially extended solution with a localized windowing function (see also Teramura & Toh 2014). The position and the width and length parameters had to be adjusted with finesse, since the product naturally is not a solution to the Navier–Stokes equation, and much of the effort went into finding suitable parameters for the windows that can be refined to numerically exact solutions, and extended into ever-increasing domains until the influence of the boundaries was sufficiently small that the localization properties of the states could be studied.

The state that they find, as shown in figure 1, consists of two pairs of counter-rotating vortices, forming an X-shaped structure. Alternatively, one may describe the structures as consisting of a pair of Λ vortices, one connected with the top plate and one with the bottom plate, that try to move in opposite directions and end up being locked in a stationary state. Discrete symmetries along the axes assure that all interactions compensate, as is necessary for the stationary structure. Stacking several pairs of vortices should then give rise to other stationary structures, and breaking the symmetries should give travelling states. Clearly, the solution shown and described by Brand & Gibson (2014) is among the simplest stationary structures one can imagine.

In order to characterize the state further, Brand & Gibson (2014) analysed perturbations around the laminar Couette profile. Outside the spot, the deviation

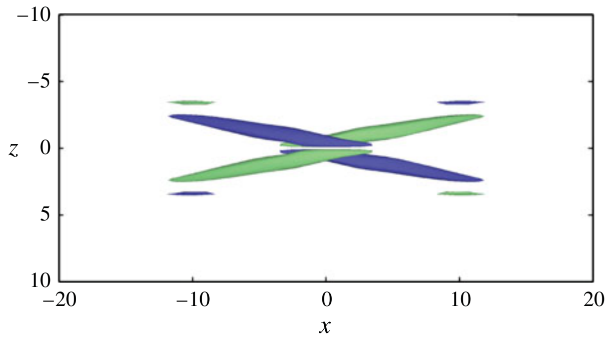

FIGURE 1. The vortex structure underlying the localized state, indicated by isosurfaces of signed swirling strength (adapted from Brand & Gibson 2014).

from the laminar state decays exponentially, only restricted by the condition that the decaying tails have to connect to the fringes of the turbulent region. The remarkable outcome of this analysis for the streamwise tails is a relation that gives the exponential decay constant in terms of a transverse wavenumber and the Reynolds number, and a functional form for the wall-normal velocity profile. The decay constant varies like $1/Re$, so that the structures become longer as the Reynolds number increases (Chantry *et al.* 2014). The agreement between the linearization and the full solution is excellent, and raises hope that other properties of localized coherent structures and turbulent patches will become accessible to analytical approaches as well.

For good measure, Brand & Gibson (2014) then also show what happens when the structure is perturbed: as befits a state that is intermediate between laminar and turbulent, some perturbations of the state increase and become turbulent (at least for a while), others return to the laminar state more directly. This is in many ways reminiscent of the crisis bifurcations that turn the stable attractor into an open saddle in small domains (Kreilos & Eckhardt 2012). Just how this bifurcation changes when there are other neutral directions remains to be seen.

3. Future

The identification of a localized state in plane Couette flow opens the way for a better understanding of the coherent structures in plane Couette and other flows.

The numerical experiments discussed here focus on states that are intermediate between the growing and decaying perturbations of the Lundbladh & Johansson (1991) experiment: in the flows described by Brand & Gibson (2014), the velocity fields are designed such that they do not decay, but at the same time they are not strong enough to grow and initiate turbulence. It should be promising to follow them through their bifurcations to also reach upper branch states around which longer-lived turbulent dynamics can be observed. It should then be interesting and rewarding to relate the properties of these localized states to the patterns seen by Barkley & Tuckerman (2005) and the correlation properties of the spatiotemporal patterns described in Duguet *et al.* (2010).

Given the observation of the families of localized states in the spanwise direction (Schneider *et al.* 2010*a*; Gibson & Brand 2014), it is natural to ask whether all spanwise solutions can be translated into localized ones, and whether the perturbation analysis that gave rise to the localization properties can be applied as well. Moreover, given the localization in the spanwise direction, one wonders whether it should not be

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possible to also obtain families of states that are localized in the downstream direction. The combination of both families would then give states that can be characterized by two 'quantum' numbers in the downstream and spanwise directions.

There is also a good chance that the analysis can be transferred to other parallel flows (e.g. plane Poiseuille flow, duct flow, pipe flow or the asymptotic suction boundary layer), and eventually one might be able to carry out a similar study for the edge state in the spatially developing Blasius boundary layer (Duguet *et al.* 2012). It remains to be seen whether dynamically active states, like the translating ones described in the asymptotic suction boundary layer (Khapko *et al.* 2010), can be tracked by similar methods.

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