Topological Pumping in the Lower Overshoot Layer

Kristóf Petrovay

Eötvös University, Department of Astronomy[1]
Budapest, Kun Béla tér 2, H-1083 Hungary

Abstract: Problems associated with topological pumping in the lower overshoot layer suggest a strongly turbulent and strongly differentially rotating upper radiative zone as the seat of the dynamo and as flux reservoir.

1. Introduction

Nonlocal phenomenological models of the lower solar convective zone predict that the strongly turbulent regime is sharply bounded from below: in a very thin ($\leq 100\,\mathrm{km}$) transitional layer the turbulent velocity falls to zero and the subadiabaticity becomes quite large (e. g. Van Ballegooijen, 1982; Pidatella and Stix, 1986). In this "sharply bounded" case the radiative zone is obviously not an appropriate place for the storage of the toroidal flux tubes responsible for the formation of active regions. On the other hand, it is well known that buoyant instabilities in the bulk of the convective zone proper should lead to the loss of flux tubes with magnetic fluxes of 10^{22} Mx (the typical active region value) and field strengths of the equipartition value ($B_{eq}^2/8\pi = \rho v^2/2$) or higher on timescales short compared to the cycle time. The only region where these problem may be avoided is the lower half of the lower overshoot layer (Van Ballegooijen, 1982), so on the basis of this exclusional reasoning it is generally supposed that the flux is stored in the lower overshoot layer.

In Section 2, however, it will be shown that if the field strength in the flux tubes does not greatly exceed 10^5 G there exists a mechanism of flux transport that will lift the flux tubes very effectively out of the lower overshoot layer into the higher parts where buoyancy may take over. This mechanism is known as *topological pumping.* So, accepting the sharply bounded case, we are faced with the severe problem of being unable to find any layer in the Sun that would be suitable for dynamo operation and flux storage. Possible ways out of this impasse are discussed in Section 3.

[1] Present address: Dept. of Theor. Physics, University of Oxford, Oxford, UK

2. Topological pumping

In a horizontal cross section of a turbulent flow regimes of up- and downflow are in general topologically not equivalent: one of them is connected, while the other forms isolated "parcels" (or "columns", in 3D). E. g. in the lower overshoot layer it can be shown that the downflows are isolated and the upflows are connected (Petrovay, 1990). A one-dimensional object like a horizontal flux tube can evidently not be transported in its whole extent in the direction of the isolated flows, but it can be transported as a whole by the connected flow; in this way, a net transport of magnetic flux in the direction of the connected flow is expected. This topological pumping has been extensively studied for the case of laminar flows advecting passive magnetic fields (Drobyshevski and Yuferev, 1974; Arter, 1983; Moffatt, 1983).

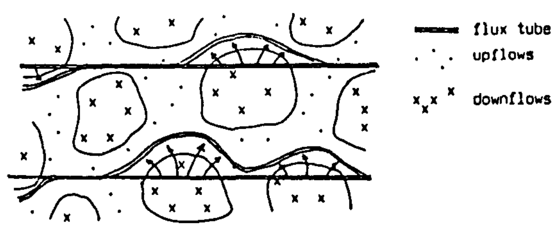

Fig. 1. Topological pumping: in a horizontal cross section of the lower overshoot layer the diverging isolated downflows sweep the flux tubes into the connected upflows. Then the upflows lift the tubes upward in their whole extent.

In order to judge if topological pumping is more effective than other transport mechanisms in the solar case, we take the typical values of different physical quantities in the lower overshoot layer and use them for reasonable estimates of the relevant forces and timescales. Our basic model is that of Van Ballegooijen (1982) fitted to the Unno *et al.* (1985) model, with crude anisotropy corrections on the basis of the results of Canuto (1989). Typical values of the physical parameters are as follows. Thickness of the layer available for flux storage: $h \sim 5 \cdot 10^8$ cm. Density: $\rho \sim 0.1\,\mathrm{g/cm^3}$. Gravity: $g \sim 5 \cdot 10^4\,\mathrm{cm/s^2}$. Scale height: $H \sim 5 \cdot 10^9$ cm. Horizontal correlation length: $5 \cdot 10^9\,\mathrm{cm} < l_h < 5 \cdot 10^{10}$ cm. R. m. s. vertical velocity: $\sigma_r \sim 3 \cdot 10^3$ cm/s. R. m. s. horizontal velocity: $\sigma_h \sim 2 \cdot 10^4$ cm/s. Typical horizontal velocities "felt" by the tube during this process are even higher than σ_h: $v_h \sim 4 \cdot 10^4$ cm/s. These high horizontal velocities arise as a consequence of the sharp boundary of the turbulent regime. The tubes are supposed to have a magnetic flux of $\Phi \sim 10^{22}$ G (the typical value for an active region).

It is found that the horizontal drag can overwhelm the curvature force acting on the tube unless the field strength is considerably stronger than 10^5 G:

$$F_{d1} \sim \rho v_h^2 B^{1/2} \Phi^{-1/2} \cong 0.5\,\mathrm{dyn/cm^3} > 0.2\,\mathrm{dyn/cm^3} \geq \frac{B^2}{4\pi l_h} \sim F_m .$$

The timescale of horizontal sweepout is

$$\tau_1 \sim \left(\frac{2\rho l_h}{F_{d1}}\right)^{1/2} \sim \text{a few days} \ll 1 \text{ month } \sim \tau_{turb},$$

so the typical flux tubes will lie in the connected flow for practically their whole extent. Now the vertical flow can sweep them upwards, out of the overshoot layer. Buoyancy cannot hinder this, except in the lowest ~ 1000 km thick layer:

$$F_{d2} \sim \rho v_r^2 B^{1/2} \Phi^{-1/2} \sim 10^{-3}\text{dyn/cm}^3 > 5{\cdot}10^{-4}\text{dyn/cm}^3 \sim \rho g \Delta \nabla h/H \simeq g \Delta\rho \simeq F_b$$

The timescale of vertical sweepout is

$$\tau_2 \sim \left(\frac{2\rho h}{F_{d2}}\right)^{1/2} \sim \text{ a few days} \ll 1 \text{ month } \sim \tau_{turb}.$$

In conclusion: all the conditions for the dominance of the topological pumping over other mechanisms of flux transport are fulfilled. *Strong horizontal motions in the sharply bounded case will sweep the flux tubes out of the downflows and then up from the overshoot layer on a timescale much shorter than that of turbulence.*

3. Location of the dynamo and flux reservoir

There are three possible solutions of the flux storage puzzle outlined above.

First, the flux tubes may actually be considerably stronger than 10^5 G. This however does not seem very plausible for observational and theoretical considerations summarized elsewhere (Petrovay, 1991).

Second, the buoyancy may not be so effective in transporting the tubes upward as generally thought, so flux confinement might still be possible above the lower overshoot regime. Parker's (1987) estimates however show that the thermal shadows can confine tubes with field strength of the equipartition value or stronger only if their flux is much higher than the observed value of 10^{22} Mx. Similarly, the turbulent pumping effect (Moffatt, 1983) seems to be unable to overwhelm the buoyancy for tubes with realistic properties. The simulations by Jennings *et al.* (1991) and Brandenburg *et al.* (1991) show the contrary: flux becomes concentrated in a relatively thin "cloud" a little above the lower turbulent boundary, between the regions dominated by topological and turbulent pumping, respectively; but the dependence of this result on flux tube properties is not clear. Besides, the boundary conditions used in these calculations amount to setting the buoyant flux removal timescale to infinity, and the competition of buoyancy and turbulent pumping is actually decided by their timescales – so these calculations can hardly be regarded as decisive. This problem deserves further investigation.

The most plausible solution to the flux storage problem is to accept Durney's (1989) proposal, that the layer where the solar rotational law changes so dramatically, from strong latitudinal differential rotation pervading the convective zone to nearly rigid rotation characterizing the deep interior, is situated at the top of the radiative zone, just below the overshoot layer. Kelvin-Helmholtz instabilities may be expected to generate strong turbulence here, so *the lower boundary of the turbulent regime is not sharp:* horizontal motions are less strong and topological pumping is ineffective.

We may conclude that the problems associated with topological pumping in the lower overshoot layer can serve as an argument in favour of a strongly turbulent and strongly differentially rotating upper radiative zone as the seat of the dynamo and as flux reservoir.

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