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ARTICLE

Optimal retirement with disability pensions

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Abstract

This paper develops a general equilibrium life-cycle model with endogenous retirement that focuses on the interplay between old-age pensions (OAP) and disability pensions (DP) in Germany. Germany has introduced a phased-in increase of the normal retirement age from age 65 to 67 (Reform 2007) and closed off other routes to early OAP retirement. This reform was followed by a phased-in expansion of future DP benefits (Reform 2018). Our simulation results indicate that the first reform will induce a shift toward DP retirement, while the Reform 2018 will even neutralize the financial and economic gains of the Reform 2007 if current DP eligibility and benefit rules remain unchanged. We therefore highlight the increased relevance of DP when reforming the retirement system and retirement incentives in an aging society. Securing the financial stability of public pensions requires activation and rehabilitation of sick elderly in the workforce and tight access to disability benefits.

Keywords: disability pensions; endogenous retirement; overlapping generations; stochastic general equilibrium

JEL codes: C68; D91; H55; J24

1. Introduction

Almost all developed economies in the Western world are confronted with a rapidly aging population which places considerable pressure on the future sustainability of pay-as-you-go-financed pension systems. As a consequence, countries such as the United States, France, Italy, or Germany try to delay retirement and encourage labor force participation and employment of older workers by increasing the normal retirement age (NRA) and reducing early retirement incentives. Although these policies may yield a double dividend in the form of financial and distributional benefits (Cremer and Pestieau, 2003), late retirement is typically very unpopular. Therefore, households seek alternative routes of retirement, which may seriously dampen the effectiveness of the original reforms.

The difficulty of raising effective retirement ages is at least partially due to the simultaneous provision of disability pensions (DP) and old-age pensions (OAP). DP allows individuals to retire earlier when they are sick with severe mental or physical work limitations. However, in practice, it is often difficult to observe the mental or physical condition of an agent who applies for DP. Consequently, fairly healthy agents who want to retire early may try to use DP as an alternative pathway into retirement.[1]

The present paper analyzes this interplay between OAP and DP retirement in Germany. Like many other countries, Germany has successively reduced OAP and DP benefits as well as incentives for early

[1]For theoretical contributions to deal with this problem, see Diamond and Mirrlees (1978), Diamond and Sheshinski (1995), or Simonovits (2006). DP retirement could be also seen as a pathway out of unemployment. A recent study by Maestas *et al.* (2021) found that the Great Recession during 2008–12 increased the number of disability insurance beneficiaries in the United States by almost 9 percent.

retirement and enacted in 2007 a phased-in increase of the NRA from age 65 to 67 during the period 2012 until 2031. Börsch-Supan *et al.* (2020) document that these reforms steadily increased employment rates of the elderly as well as pension-claiming ages in Germany between 2000 and 2015. However, the increase of the NRA also turned out to be very unpopular among the German population due to the widespread fear that those with physically demanding and often low-paid jobs would be the main losers.[2] To mitigate this opposition and to reduce old-age poverty risk, various reforms between 2014 and 2018 sequentially will raise the generosity of future DP by increasing the maximum assessment age (MAA) for the benefit calculation. Both reforms may induce a shift toward DP applications (and enrolments) in the future that lowers effective retirement ages and threatens future sustainability.

This paper aims to quantify such possible effects. Therefore, we capture the linkage between DP and OAP within the framework of a dynamic general equilibrium (GE) life-cycle model that features the institutional structure of the German tax and pension system. Individuals consume, save, and decide whether to file a DP application and when to stop working, facing idiosyncratic health, earnings, disability claiming, and mortality risks. We distinguish between severe and less severe health shocks, which reduce work capacity. While the government can only partially identify severe and less severe work limitations, the DP application is associated with social stigma costs. Therefore, depending on the prospects of recovery, the severity of the screening process, the benefit generosity, and the social stigma cost associated with disability, households with work limitations will either apply for DP or continue to work on the labor market.

Our initial equilibrium reflects the situation, where the NRA for OAP was still at age 65 and the generosity of DP benefits was quite low. Then we successively alter the benefit and eligibility rules for OAP and DP retirement and compute the resulting behavioral reactions and financial consequences. Our simulations indicate that although DP only play a minor role in the pension budget and are often neglected in the pension debate, they may have significant economic effects. First, similar to previous studies, we find that the higher NRA will induce a shift toward DP retirement which dampens the increase in effective retirement ages. Nevertheless, the Reform 2007 will increase long-run employment and savings and reduce the financial pressure on the pension system. Second, raising DP generosity as in the considered DP reforms induces even stronger shifts toward DP retirement, so that the benefits from higher NRAs may be almost neutralized or even reversed. We show that this result is compatible with empirically plausible retirement elasticities and robust to various parameter configurations. As a final result, we also show that just increasing eligibility requirements for OAP benefits makes little sense since it encourages further substitution toward DP. To reduce DP applications upfront, one has to increase efforts for activation and rehabilitation and/or introduce stricter rules for the medical test which increase rejection rates in the future.

Our paper is related to the macroeconomic literature on social security reforms in overlapping-generations models pioneered by Auerbach and Kotlikoff (1987). While the early studies typically neglected the retirement decision, various recent contributions focus on retirement behavior and the interaction between health and retirement. In the United States, İmrohoroğlu and Kitao (2012) apply a model with endogenous labor supply at the intensive and extensive margin to analyze the increase in the NRA from 66 to 68. Fehr *et al.* (2012) provide an analysis of the 2007 pension reform and alternative options to increase the effective retirement age in Germany. Börsch-Supan *et al.* (2018) model labor market participation costs which rise with age to quantify the impact of deductions combined with an earnings test for early retirement. They propose actuarially neutral deductions in combination with an elimination of the earnings test. All these studies abstract from disability risk and DP pensions. These features are considered by Díaz-Giménez and Díaz-Saavedra (2009), Erosa *et al.* (2012), Fehr *et al.* (2013), Kitao (2014), or Jones and Li (2023), who study retirement in models with earnings uncertainty and disability risk. However, DP retirement is treated there as a pure exogenous process without an individual application decision and application process. Most closely

[2]For example, Scheubel *et al.* (2013) report on opinion polls which show that about 80–90 percent of the population opposed this reform.

related to our approach are therefore Laun and Wallenius (2015, 2016), Laun *et al.* (2019), Li (2018), and Galaasen (2021), who explore how the interaction between OAP and DP affects the labor supply of older workers in Sweden, the United States, and Norway, respectively. These studies highlight that including DP is quantitatively important when analyzing pension reforms. For example, Li (2018) finds that in the United States an increase in DP receipt may offset about 40 percent of the fiscal gain from the higher NRA. In Norway, where DP recipients are much more important than in the United States, the fiscal gain from OAP reforms could be completely offset by increases in DP receipt, see Galaasen (2021).

This is exactly where our paper sets in and extends the literature in various directions. First, while previous studies focus on OAP reforms, we mainly study the impact of DP reforms on retirement behavior and aggregate welfare. The latter are intended to reduce future old-age poverty which is not only in Germany an important policy issue. Second, we model the productivity process and the interaction between health and productivity in greater detail. Following Kindermann and Püschel (2021) by distinguishing households according to their educational background and career path, we account for workers that are trapped in a low-earning state, so that they accumulate very low pension claims during the working phase. As in French (2005) or Capatina (2015), education-specific health shocks have a direct impact on productivity. Chronic illness may lead to an interrupted employment biography that eliminates eligibility for early OAP claiming. In this way we reproduce the fact that in Germany DP pensioners often enter retirement after health-related periods of unemployment, see Mika (2017). Previous simulation models have not been able to capture the retirement pattern in such detail. Third, the DP claiming process is modeled as a recurring event where the claiming decision is made at the beginning, and acceptance/rejection is decided at the end of the period. This allows us to capture income losses due to significant processing times and possible health recoveries that were only indirectly taken into account in previous studies. Finally, similar to Seibold (2021) or Dolls and Krolage (2023), we introduce reference points (RPs) (that may change due to policy reforms), which determine the stigma cost of DP claiming. This allows us to map the pikes in the retirement distribution and to include non-economic behavioral characteristics.

The next section introduces the structure of the German pension system, the DP application process, and a discussion of recent OAP and DP benefit reforms. Then the GE model which is applied for the quantitative analysis is presented. The fourth section discusses the calibration of the baseline equilibrium, section five reports the results from the simulation exercises and section six concludes.

2. OAP and DP in Germany

The statutory German pension system is a compulsory social insurance program that offers benefits to elderly and disabled citizens as well as survivors. It is mainly financed by social security contributions by the working population and by public funding.[3] Table 1 provides some key indicators for 2019 concerning OAP and DP. The total budget of the system amounted to roughly 9.3 percent of GDP, mostly financed by payroll taxes of employers and employees which amount to 18.6 percent of the gross wage up to the contribution ceiling which is roughly the double of average income.

When employees retire from the workforce, they may either apply for OAP or – in case of partial or full disability – DP. Both systems require a minimum contribution period of five working years to receive benefits. Eligibility for DP also requires a contribution record of at least three years during the last five years before filing the DP application. Workers with a contribution record of at least 35 years are eligible for OAP benefits starting at the early retirement age (ERA), all others have to wait until they pass the NRA. OAP claiming is fairly standard, the processing time for eligibility check and benefit calculation takes roughly three months.[4] As shown below, households typically

[3]Public funding mostly finances non-contribution-related benefits. Note also that civil servants are not covered by the statutory system, while self-employed can voluntarily join.

[4]The resulting delay in retirement mostly explains the difference between OAP applications and inflow in Table 1.

Table 1. Key pension indicators in Germany 2019

Total statutory pension insurance (GRV) budget 320 bn. € (9.3% of GDP)				
Contribution ceiling 80,400€ (207% of average income)				
Contribution rate 18.6%				
	OAP	DP	Total	DP (in %)
Applications	868,373	369,499	1,237,872	29.8
Inflow	816,129	161,534	977,663	16.5
With mental health problems (in %)		41.7		
With musculoskeletal problems (in %)		12.5		
Retirement age 2019	64.3	52.7	62.3	
Retirement age 2011	63.5	50.5	60.8	
Avg. annual benefit (men, in €)	14,244	9,924		69.7
DP (in % of pension budget)				6.7

Source: Deutsche Rentenversicherung (2021); own calculations.

transit into OAP retirement when they reach an eligibility age, so pension claiming in most cases happens during full employment. Labor income during early retirement is subject to an earnings test, which reduces incentives to work. Delayed retirement beyond the NRA is very uncommon, and working after retirement at older ages is still an exception.[5]

Compared to OAP retirement, the transition into DP retirement is more complicated, time-consuming, and highly uncertain, see Aurich-Beerheide and Brussig (2017). Figure 1 provides a simplified timeline of the process of dealing with work inability. Typically the health shock inducing the reduction in earnings capacity occurs in an active employment relationship. If the employee cannot recover within six weeks (with continued salary by the employer), a wage replacement benefit through the health insurance sets in. This sickness benefit is paid for up to 18 months. During that time there will be recurring examinations and checks of work capacity and the patient may be transferred to the employment agency. This period ends either with a return to the labor market (in case of full or partial recovery) or with a DP application in case a permanent incapacity is likely or when the sickness benefit has expired. In the latter case, the pension insurance first initiates a check of eligibility and then a detailed assessment of the earnings capacity and the prospects of recovery based on the previous records and medical examinations. DP requires a likely persistent inability to work for more than three (full disability) or more than six hours (partial disability) per day in a regular job (i.e., not necessarily the previous one). Based on this assessment, the pension insurance can either approve a temporary or an unlimited DP, grant a rehabilitation benefit, or reject the application. In case of a temporary DP or rehabilitation measure the assessment process is renewed.[6] In case the application is rejected, the applicant can file an objection against the decision made, which starts a new examination process at the pension insurance. If in the end the previous decision is confirmed, the applicant may in principle still take legal action in the social court and subsequent instances, but in practice, this is very unlikely.

Therefore, any DP application requires to be at least partially out of the labor force, extensive documentation of the disability, and a final positive medical examination. In 2019 the average processing time until final rejection and acceptance was ten months and six months, respectively. The acceptance rate of applications was only about 50 percent which explains the difference between the number of applications and the inflow of new pensioners in Table 1. About 35 percent of those who were rejected did not fulfill either the minimum waiting time or the required contributions before the DP

[5]In 2019 only 3.7 percent of the retirement inflow was older than the NRA, see Deutsche Rentenversicherung (2021). Concerning retirees, Romeu Gordo *et al.* (2022) report that about 15 percent of the age group 65–69 are employed and more than two-thirds of them are working in so-called mini-jobs.

[6]In practice, DP is often approved temporarily with regular medical examinations required after retirement to check the disability status. However, it is extremely rare that people re-enter the labor market from a temporary DP, see Drahs *et al.* (2022).

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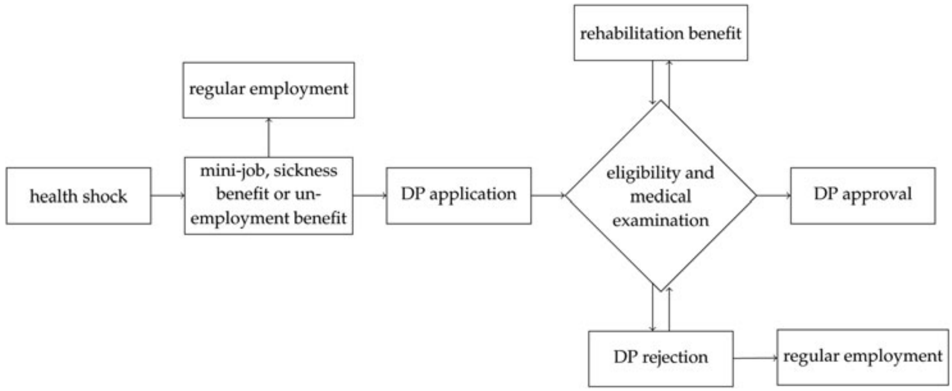

Figure 1. Timeline for DP application process.

application and more than 55 percent were rejected because they did not pass the medical examination. Those who are finally rejected have to go back to work and may apply again in a future period. As shown in Table 1, among those who were accepted, by far the most important diagnoses were mental health and musculoskeletal problems (back injuries, etc.), which are both difficult to diagnose. The average retirement age for OAP was 64.3 years in 2019, while the respective retirement age for DP was 52.7 years. Working in regular jobs after retirement is also very unlikely. Six years after retirement <20 percent of disability pensioners have some form of employment, but most of them in mini-jobs, see Zink and Brussig (2022).

The calculation of benefits is based on a point system, where an annual earning point reflects the relative income position of a worker in that year. A year's contribution at the average earnings of contributors earns one earning point, contributions based on lower or higher income (up to the contribution ceiling) earn proportionally less or more points. When the employee retires, the sum of the accumulated earning points is multiplied by the annual 'point value', which defines the benefit amount for each earning point. The latter is adjusted every year based on changes in wages and demographics. Since disability pensioners are usually forced to retire at an early age, accumulated earning points are upgraded to the MAA, which in effect assumes that employees have continued to contribute up to the MAA with average contributions. Finally, employees who retire with OAP before the NRA and with DP before the disability retirement age (DRA) face deductions that reduce the benefit permanently by 0.3 percent for each month (or 3.6% for each year) of early retirement. These deductions, however, are capped at a maximum of 10.8 percent for disability pensioners. OAP benefits claimed after the NRA are increased permanently by 0.5 percent for each month (or 6% for each year) of late retirement. In 2019 the annual pension value of an earning point was 396€, so the 'standard pensioner benefit' of a 65-year new retiree, who had contributed for 45 years at the average income amounted to ($45 \times 396 =$) 17,820€ or roughly 46 percent of average labor income. However, as shown in Table 1, the average annual OAP benefit of retired men was much lower than the standard benefit, reflecting interrupted employment, earlier retirement, and deductions. The average DP benefit in the same year amounted to roughly 70 percent of the OAP benefit. Overall, DP amounted to 6.7 percent of total pension benefits. This number was fairly stable during the last 20 years. Note that it understates the importance of DP retirement since DP recipients are automatically relabeled as OAP recipients (with unchanged benefits) after passing the NRA.

During the last decade, Germany implemented several pension reforms which affected retirement incentives quite dramatically. Until 2012, the NRA was set at 65 and employees who had contributed for at least 35 years could retire earlier starting at age 63 with OAP benefits (i.e., NRA = 65, ERA = 63). Of course, individuals who retired at age 63 had to accept a reduction of their benefits by 7.2 percent. Disability pensioners, on the other hand, could already retire without deductions at age 63 (i.e.,

Table 2. Recent changes of key retirement parameters in Germany

	Initial equilibrium	Reform 2007	Reform 2014	Reform 2017	Reform 2018
Reflecting year	2011	2031			
NRA	65	67			
NRA (contribution years $\geqslant$45)		65	(63)		
ERA (contribution years $\geqslant$35)	63				
DRA	63	65			
MAA	60		62	65	67

DRA = 63). For those who retired before age 60, earning points were calculated as if they had worked until age 60 (i.e., MAA = 60). Then a maximum deduction of 10.8 percent was applied. This situation reflects the calibrated initial equilibrium of our model. Table 1 shows that due to these (and other) provisions, early retirement was quite common in 2011. To induce individuals to retire later and to stabilize the financial perspective of the system, the pension reform of 2007 introduced an announced and gradual increase of the NRA from 65 to 67 and the DRA from 63 to 65 (Reform 2007 in Table 2).

Starting in 2012 and the birth cohort of 1947, the NRA increased by one month per year for each successive birth cohort. After 2023 for birth cohorts born in and after 1959 the NRA increases by two months per year so that cohorts born in 1964 and later face an NRA of 67. Early retirement is still possible at age 63 with a contribution record of 35 years and a permanent benefit reduction of 14.4 percent. In addition, those with a contribution record of at least 45 years could retire already at age 65 without deductions.

Concerning disability retirement, the reform of 2007 introduced a maximum of 10.8 percent deductions to all DP pensioners, who retire at age 62 or earlier. Surprisingly, the MAA for DP benefits was not altered initially. As a result, DP benefits remained fairly low compared to OAP benefits and were identified as a major source for the observed rise in old-age poverty. To dampen this development, various recent reforms successively increased the MAA significantly. First, the pension reform of 2014 increased the MAA for new disability pensioners by two years to 62 and introduced an advantageous check for the calculation of average earnings.[7] Then the reform of 2017 implemented a phased-in increase of the MAA up to age 65 until 2024 for new DP entrants. Finally, the so-called 'Pension Pact' of 2018 introduced an MAA of age 65 for new retirees of 2019 and phased in a further increase of the MAA up to age 67 until 2031. Gasche and Härtl (2013) show that the reform of 2014 increased DP benefits on average by 5.5 percent. The attractiveness of DP relative to OAP will further increase in the future. Simple extending these calculations into the future indicates that the considered reforms increase average DP benefits by about 15–18 percent. In Table 1 the reported relative DP/OAP benefit level in 2019 reflects the situation *before* 2014, since it is based on the *actual balance* of retirees in that year. However, for those who retired in 2019 or 2022, the relative DP/OAP benefit level increased to 73.4 percent and 79.6 percent, respectively.[8]

It should be quite clear that agents will increasingly try to retire with DP instead of OAP benefits, which counteracts the intended postponement of retirement and all budget consolidation efforts. Duggan *et al.* (2007) forecast the rise in DP enrollment in the United States due to the NRA increase from 65 to 67. They project that after 2024 when the reform is fully phased in, the implied benefit reduction will raise the disability enrollment by 1 percentage point for male and 1.56 percentage points for female employees between the ages of 45 and 64. Given a DP enrollment rate of 6.7 percentage

[7]This reform also reduced the NRA for employees with a very long contribution record to 63 years, see Dolls and Krolage (2023). Although heavily disputed in the public debate, these provisions are not relevant for our simulations since they only apply to those born before 1953.

[8]See Deutsche Rentenversicherung (2021) and the release in 2022 at https://statistik-rente.de/drv/extern/rente/rentenzugang. The dramatic improvement for specific groups is also confirmed by Jess *et al.* (2019, p. 109) who find that retirees with a long contribution record could be up to 27.4 percent better off with a DP instead of an OAP in and after 2031.

points in this age group in 2005 this is a significant increase by about 15 percent. Mullen and Staubli (2016) analyzed the elasticity of disability claiming concerning disability generosity in Austria for the period 1987–2010. Their estimated elasticity of DP applications concerning DP generosity is 1.4. This figure applied to German birth cohorts after 1964 implies a rise in DP applications of about 25 percent.

Of course, all these figures are very vague and only provide a first intuition of the reform effects. A more comprehensive analysis requires detailed modeling of the individual decision process concerning retirement, which reflects health and productivity risk over the life cycle and takes into account the incentives and associated uncertainties provided by the German pension system. The aggregation of the derived behavioral reactions also allows for quantification of the fiscal consequences of the reforms. The following section presents a simulation model which is based on these criteria.

3. The model economy

To quantify the likely effects of the above-described reform packages, we apply a GE simulation model with overlapping generations which is developed in this section. The theoretical structure is in the spirit of Li (2018), where households face survival risks, and face shocks to their labor productivity and health which reduce their work capacity at least temporarily. During their life cycle, they have to decide whether to be employed, whether to claim DP or OAP, and how much to consume and save. Besides the individual health and productivity situation, these decisions depend on the incentives provided by the tax and pension system and some psychological factors that differ across households and may change over time. The government taxes consumption and income from capital and labor to finance public goods and operates a pay-as-you-go-financed pension system that reflects the institutional structure described above. The model considers a closed economy where wages and the interest rate are determined to balance the respective factor markets. Due to our steady state analysis, we omit the time index in the following whenever possible.

3.1 Demographics

The economy is populated by J overlapping generations with a model period that equals a calendar year. At the beginning of each period, a new generation is born. The population grows at a constant rate n and the cohort size of newborns is normalized to unity. Individuals enter the economy at (model) age $j = 1$ (i.e., age 20) with perfect health and may live up to a maximum of J years after which they pass away with certainty. After working in the labor market they may be hit by health shocks which may induce them to apply for DP. Alternatively, if eligible, they may claim (and immediately receive) an OAP pension starting at the ERA j_B. Throughout their entire life, agents face idiosyncratic survival risk, which is determined by age and individual health status h. The conditional survival probability of an agent to survive from age $j-1$ to age j is denoted by $\psi_j(h)$ with $\psi_{J+1}(h) = 0$.

3.2 Preferences and endowments

Households have preferences over stochastic streams of consumption $c_j \geqslant 0$, work and non-work $l_j \in \{0, 0.2, 0.5\}$ and the application for disability benefits $d_j \in \{0, 1\}$. They maximize discounted expected utility

$$U = \mathbb{E}\left[\sum_{J}^{j=1} \beta^{j-1} u(c_j, l_j, d_j)\right] \quad \text{with } u(c_j, l_j, d_j) = \frac{c_j^{1-1/\gamma}}{1-1/\gamma} - \chi_{j,s} I_{l_j>0} - \xi_{j,s} d_j.$$

Expectations are formed concerning survival, productivity, health, and employment risk and future utility is discounted with the constant time discount factor β. The intertemporal elasticity of substitution (IES) in consumption is denoted by γ. Agents face participation cost $\chi_{j,s}$ when employed depending on age and education. The indicator variable $I_{l_j>0}$ is zero when retired and one when working.

Finally, following Laun *et al.* (2019) and Galaasen (2021), we assume that applying for DP induces stigma cost $\xi_{j,s}$, meant to capture psychological factors of accepting work limitations but also time cost and hassle to write a DP application. Stigma cost also depends on age and education, they are iid across households with a log-normal density function, so that[9]

$$\xi_{j,s} \sim LN(\mu_{j,s}, \sigma_{\xi}^2),$$

where $\Psi_{\xi}(\cdot)$ denotes the cumulative distribution function (at a specific age).

Labor productivity: The modeling of productivity risk is taken from Kindermann and Püschel (2021) who have studied administrative data from the German pension insurance system to investigate the properties of individual labor earnings dynamics over the life cycle. They found that workers are exposed to significant earnings risk which could not be captured by the standard AR(1) process for log-earnings. On the one hand, individual contribution records show periods where employees do not pay contributions because they are out of official employment due to informal work, illness, etc. On the other, some employees typically spend a significant fraction of working years in low-income episodes where they only make about 10 percent of average annual labor earnings. Due to these 'mini-jobs', the distribution of labor earnings becomes bimodal with quite distinct dynamics across educational groups.

Kindermann and Püschel (2021) therefore distinguish between households with high-school education ($s = 0$) and college education ($s = 1$), where the initial fraction of college-educated households is denoted by ω_s. All workers of education level s share a common deterministic age-specific labor productivity profile $e_{j,s}$. Knowing their educational level, workers are again divided into two permanent subgroups m, which indicate whether they face a stable ($m = 0$) or an unstable career path ($m = 1$). The probability to draw the state $m = 1$ is independent of education and denoted by ω_m. Throughout their working life, individuals' labor productivity is due to idiosyncratic shocks η. Productivity of individuals with a stable career ($m = 0$) follows a standard AR(1) process

$$\eta^+ = \rho\eta + \epsilon^+ \quad \text{with } \epsilon^+ \sim N(0, \sigma_{\epsilon,s}^2), \tag{1}$$

where innovations ϵ^+ are iid across households with education level s. In the following, we omit all state indices and only indicate the next period variables with '+' to simplify notation. To capture low-income episodes for individuals with unstable careers ($m = 1$), this standard shock process (1) is augmented by a persistent (but not permanent) low productivity shock η_0. Productivity in this low-productivity state is independent of age and education. For households with unstable careers, the transition into and out of low earnings is modeled as a first-order discrete Markov process. Those who are currently normal earners (i.e., where $\eta \neq \eta_0$) face the education-specific probability $\pi_{low,0}^s$ to transit into the low-earning state in the next period (i.e., where $\eta^+ = \eta_0$), while currently low earner households face a probability $\pi_{low,1}^s$ to remain in the low-earning state. In the initial year, a fraction ω_{low}^s of individuals with education s in the unstable career subgroup starts as a low-earning individual.

Health and employment risk: In our model households are also exposed to health risks that affect their work capacity. We distinguish a good health state ($h = 0$), where households can work without impairment, a medium state ($h = 1$), where they face some minor injuries and suffer from mental problems that reduce work capacity slightly and a bad health state ($h = 2$), where work capacity is reduced severely.[10] These shocks again follow a first-order discrete Markov process where households face probabilities of the future health state $\pi_h(h^+|j, s, h)$ that depend on age, education, and the current health state:

$$h^+ = f(j, s, h). \tag{2}$$

[9]In practice such psychological factors may also depend on health, but this is disregarded for technical reasons.

[10]Note that we do not model any additional cost (hospital, etc.) associated with these health shocks, since in Germany full coverage of health insurance is mandatory.

To model the impact of these health shocks on productivity, we further augment the productivity process described above by assuming that a fraction π_h^{nw} of employees in the health state h stay at home (i.e., do not receive income and pay no pension contributions), while the productivity of the remaining fraction is reduced by $\theta_{h,s}$ percent. Bad health therefore not only reduces labor income and pensions, it may also reduce the eligibility for early OAP receipt if the number of contributory years is not reached.[11]

Consequently, labor productivity $z(j, s, m, \eta, h)$ depends on age, education, career stability, the idiosyncratic productivity shock, and the health state. Individual labor income y is then given by

$$y = w \times z(j, s, m, \eta, h) \times l,$$

where the wage per efficiency unit w is multiplied by labor productivity and the working hours.

Individuals enter the labor market without assets ($a_1 = 0$), during their working years they accumulate assets to finance retirement and to self-insure against uncertainty. Since our model abstracts from annuity markets, individuals who die before the maximum age of J may leave accidental bequest b that will be distributed equally in a lump-sum fashion among all individuals below age 60.

3.3 Accumulation of pension wealth and benefit calculation

As already explained above, the statutory German pension system is based on so-called 'earning points', which reflect the relative income position (and therefore contribution level) during employment years. Agents who receive an average income $\bar{y}$ in a specific year accumulate one earning point in their retirement account. In the case of higher income, earning points are increased proportionally (up to the contribution ceiling which is roughly $2\bar{y}$) and vice versa in the case of lower income. Consequently, earning points ep in the model are accumulated as

$$ep^+ = ep + \min\,[y/\bar{y};\, 2.0] \tag{3}$$

until agents decide to stop working at age j_R and receive retirement benefits. We abstract from temporary retirement and from separating the decisions of benefit claiming and labor market exit as in İmrohoroğlu and Kitao (2012) or Jones and Li (2023). As already discussed above, OAP claiming typically happens during employment, and although there are only a few restrictions there is hardly any full-time work after retirement. Consequently, retirement is an absorbing state and no earning points are accumulated in the model after retirement.

Figure 2 shows the retirement windows for OAP and DP in the model. Early retirement with an OAP is possible either with or without deductions in and after the ERA j_B if the contribution record is either more than 35 years or more than 45 years, respectively. All other employees retire with an OAP at the NRA j_N or delayed up to the maximum retirement age j_E in which all remaining employees are forced to retire. Since those with health problems might end up without labor income in some periods, not all employees qualify for early retirement. To check eligibility for early retirement, we keep track of the number of non-contributory years before retirement[12]:

$$nc^+ = \begin{cases} nc & \text{if } y > 0 \\ nc + 1 & \text{otherwise.} \end{cases} \tag{4}$$

In contrast to OAP, retirement with DP depends on the individual health status. Agents enter the labor market in full health ($h = 0$), but after one year their health may deteriorate which reduces or even temporarily eliminates their work capacity. Employees may then either stay in the labor market and hope for future health improvements or restrict labor market participation from $l = 0.5$ to $l = 0.2$

[11]Note, however, that (mainly for technical reasons) we do not include non-working as a specific dimension of the state space, i.e., current and future non-working are only indirectly linked by the development of the health state.

[12]The maximum retirement age j_E, as well as the number of non-contributory years (instead of contributory years) are specified for purely technical reasons to reduce the state space.

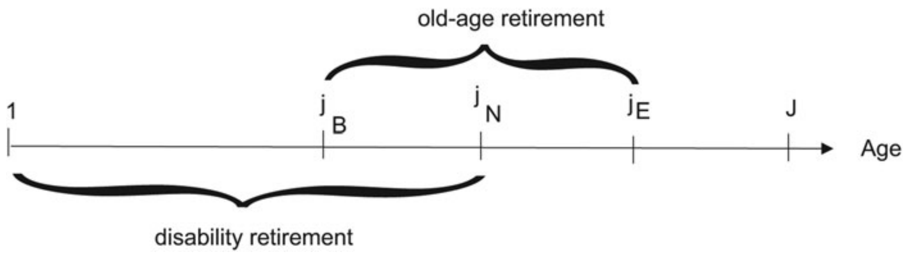

Figure 2. Overlapping retirement windows for OAP and DP.

and submit an application for DP.[13] After one year, the pension insurance accepts an application with an (exogenous) probability of $1 - q$ and the agent starts receiving DP benefits.[14] If the application is rejected with probability q, the agent either works again full time or files another application depending on the actual health condition or decides to retire with OAP. The acceptance/rejection of the application depends on the health status or other characteristics such as age, etc. As shown in Figure 2, DP retirement is possible before reaching j_N, so that the two retirement windows may overlap in some periods for specific individuals.

If agents are eligible for OAP benefits they may retire without any delay. At the retirement age j_R, the accumulated earning points ep_{j_R} are multiplied with an adjustment factor $\nu(j_R)$ which takes into account early or late retirement, and the annual point value (APV), which reflects the worth of one earning point in pension income. For simplicity, we derive the APV from the replacement rate of a so-called 'standard pensioner', who has worked for 45 years and always received an average income:

$$p = \nu(j_R) \times ep_{j_R} \times \underbrace{\kappa \times \bar{y}/45}_{\text{APV}}. \tag{5}$$

Since all agents in the model enter the labor market at age 20 ($j = 1$) and the NRA for OAP in the initial equilibrium is 65 years ($j_N = 46$), the standard pensioner has worked for 45 years and accumulated $ep_{46} = 45$ earning points at retirement. The adjustment factor at j_N is unity, so that OAP benefits of the standard pensioner are simply computed by $p = \kappa \times \bar{y}$ where κ denotes the replacement rate. For each year of early retirement before j_N, OAP benefits are reduced by 3.6 percent, and for each year of delayed retirement they are increased by 6 percentage points, i.e.:

$$\nu(j_R) = \begin{cases} 1 - (j_N - j_R) \times 0.036, & j_R < j_N \\ 1 + (j_R - j_N) \times 0.060, & j_R \geqslant j_N \end{cases}.$$

In principle, the calculation of DP benefits is based on the same formula (5). However, earning points and the adjustment factor are computed slightly differently. Since DP retirement is typically much earlier than OAP retirement (see Table 1), accumulated earning points are upgraded to the MAA j_{MA}:

$$ep_{j_R} = ep_{j_R} \times \max\left[1.0; \frac{j_{MA} - 1}{j_R - 1}\right].$$

[13]The reduction in labor market participation aims to capture two distinct aspects: on the one side, in 2019 about 65 percent of DP retirees had a job subject to social security contributions one year before receiving benefits, see Deutsche Rentenversicherung (2021). On the other side – and as already explained above – full DP benefits are only provided to applicants with a work capacity of <40 percent of full working time.

[14]As mentioned above, the average DP processing time is much less than one year. However, the processing time in the model may also include part of sick time shown in Figure 1 before the DP application.

In the initial equilibrium, the MAA is set at 60 ($j_{MA} = 41$). Someone who receives DP benefits starting at age 50 ($j_R = 31$) therefore receives a 33 percent increase in accumulated earning points. In addition, the DP adjustment factor $\nu(j_R)$ is defined relative to the DRA j_D, reductions of DP benefits are limited to 10.8 percent, and no increase in DP benefits is granted for delayed retirement, i.e.:

$$\nu(j_R) \quad = \quad \begin{cases} 1.0 - \min\,[0.108;\, (j_D - j_R) \times 0.036], & j_R < j_D \\ 1.0 & j_R \geqslant j_D. \end{cases}$$

3.4 The dynamic optimization problem

To describe the individual decision process we need to distinguish three retirement states rs for each household, which specify the labor market situation in the previous year. If the agent has worked before ($rs = 0$) he/she needs to decide about retirement (with DP or OAP), consumption, and savings. If the agent has either filed a successful DP application or has already received a DP benefit before ($rs = 1$), he/she only needs to decide about consumption and savings. Of course, the same applies when the agent was already retired with OAP benefits before ($rs = 2$), but the benefit calculation is different. Therefore, the current state of a household is described by a vector

$$x = (j, s, m, rs, a, ep, nc, \eta, h)$$

where $a \in [0, \infty]$ defines the financial assets at the beginning of the period and the remaining variables summarize the household's current age, education, career stability, retirement state, earning points, non-contributory periods, labor productivity, and health. In each period the age-j cohort is fragmented into subgroups, according to the initial distribution at age $j = 1$ as well as idiosyncratic shocks to productivity and health and optimal household decisions. Let $\Phi(x)$ be the corresponding cumulated measure so that[15]

$$\int \mathrm{d}\Phi(x) = 1 \quad \text{with } x = (1, s, m, 0, 0, 0, 0, \eta, 0)$$

must hold since we have normalized the initial cohort size to be unity and endowed initial agents with productivity η. In the following, we will omit the state index x for every variable whenever possible.

Optimal household decisions regarding consumption, savings, and retirement with DP or OAP will be formulated recursively starting in the last phase of life (see Figure 2). Since we abstract from bequest motives, households who have survived until the final age of J simply consume their resources:

$$V(x) = u(c, 0, 0) \quad \text{with } c = [(1 + r)a + p - T(p, ra)]/(1 + \tau^c),$$

where p either denotes DP ($rs = 1$) or OAP ($rs = 2$) benefits, $T(p, ra)$ defines the (progressive) income tax levied on pension income and asset income, and τ^c is the consumption tax rate (see below).

Younger households who are beyond the maximum retirement age j_E must be also retired (i.e., $rs \in \{1, 2\}$), but they may still face health shocks that affect their life expectancy. Optimal consumption $c(x)$ and savings $a^+(x)$ are then derived according to

$$V(x) = V_2(x) = \max_{c, a^+} u(c, 0, 0) + \beta \psi_{j+1}(h) \mathbb{E}[V(x^+)|h] \tag{6}$$

subject to (2) and the budget constraint

$$a^+ = (1 + r)a + p - T(p, ra) - (1 + \tau^c)c.$$

[15]To account for their longer education, we set the initial non-contributory years for high-skilled to $nc = 3$.

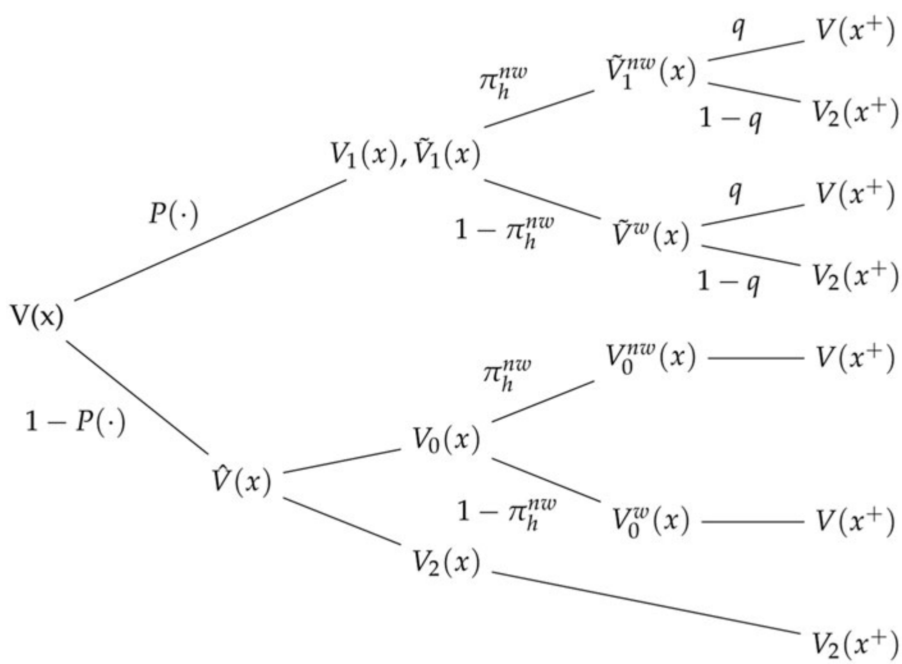

Figure 3. Decision problem of household.

Of course, the same also applies to all retired households who are below the maximum retirement age j_E and in one of the absorbing states $rs \in \{1, 2\}$. Figure 3 illustrates the decision problem of the household. The lower part describes someone who does not apply for DP so that the value function $V_2(x)$ applies to OAP recipients. The upper part of Figure 3 shows someone who applies for DP, so that the value functions $V_2(x^+)$ refer to DP recipients.

Those who are in or beyond j_N and were still working full time in the previous year (i.e., $rs = 0$) have to decide whether to continue working or to exit the labor market and receive OAP benefits. In the case of working, they need to account for employment risk which is captured by the (expected) value function

$$V_0(x) = \pi_h^{nw} V_0^{nw}(x) + (1 - \pi_h^{nw}) V_0^w(x),$$

where the exogenous employment probability $1 - \pi_h^{nw}$ is independent of age and increasing with better health. If the household is employed, optimal consumption and saving is derived from

$$V_0^w(x) = \max_{c,a^+} u(c, 0.5, 0) + \beta\psi_{j+1}(h)\mathbb{E}[V(x^+)|\eta, h] \tag{7}$$

subject to (1)–(4) and the budget constraint

$$a^+ = (1 + r)a + y - T_p(y) - T(y - T_p(y), ra) - (1 + \tau^c)c, \tag{8}$$

where $T_p(y)$ defines the contribution to the pension system.[16] If the household is not working the same optimization problem (7) is solved to derive $V_0^{nw}(x)$, but in this case $y = l = 0$. Since at this

[16]For simplicity we assume that taxation of pensions is completely back-loaded, although this will be fully implemented in Germany only after 2040.

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age DP application is not possible anymore, the expected value function $V_0(x)$ is shown in the lower part of Figure 3.

In the case of retirement, the optimization problem (6) applies. The participation decision $l(x)$ is then derived from maximizing

$$V(x) = \hat{V}(x) = \max_{l} [V_0(x), V_2(x)], \tag{9}$$

shown again in the lower part of Figure 3. If the household decides to retire, the future retirement status changes to the absorbing state $rs = 2$. Otherwise, he/she remains in $rs = 0$. The same decision applies to working agents in good health who are in the early retirement window for OAP.

Agents younger than j_N, who are in the medium or bad health state (i.e., $h \in \{1, 2\}$) and who have worked in the previous year may also consider applying for DP. In the latter case (shown in the upper part of Figure 3) they also face employment risk as in the previous situation. The expected value function is therefore defined by

$$\tilde{V}_1(x) = \pi_h^{nw} \tilde{V}_1^{nw}(x) + (1 - \pi_h^{nw}) \tilde{V}_1^{w}(x).$$

When computing the value functions in the two situations they need to take into account that on the one hand they can work only part-time and face stigma cost during the application period and on the other their application may be rejected with probability q. In the case of employment, optimal consumption and savings are derived from

$$\tilde{V}_1^w(x) = \max_{c,a^+} \frac{c^{1-1/\gamma}}{1-1/\gamma} - \chi + \beta\psi_{j+1}(h)\{q\mathbb{E}[V(x^+)|\eta, h] + (1-q)\mathbb{E}[V_2(x^+)|h]\} \tag{10}$$

subject to (1)–(4) and the budget constraint (8), which now may also include unintended bequest b received at ages younger than 60 (i.e., $j < 41$). As before, the situation without employment is solved quite similarly but then $y = l = 0$. Note that $\tilde{V}_1(x)$ denotes the expected value function in case of an application *net of stigma cost*. If the application is accepted with probability $1 - q$, agents receive DP benefits in the next period, they are assigned to the retirement state $rs = 1$ and face no further productivity uncertainty.[17] In case of denial, expected future utility depends on future productivity and health conditions. In case of good health, they may still decide (if they are eligible) to retire early with OAP benefits, or they have to return to the labor market. If they end up again in the medium or bad health state, they may apply again for DP benefits.[18]

The application decision $d(x)$ depends on individual stigma cost $\xi(x)$. Since the distribution of this cost is given by the age- and education-dependent cumulative function Ψ_ξ, the probability that an individual in state x decides to apply is

$$P(d = 1|x) = \Psi_\xi(\tilde{V}_1(x) - \hat{V}(x)).$$

Ignoring stigma cost, the difference $\tilde{V}_1(x) - \hat{V}(x)$ reflects the expected utility gain from a DP application. The term $\Psi_\xi(\cdot)$ on the right side then denotes the fraction of households with state x with stigma cost below this utility gain, who will apply for DP. The utility in case of not applying for DP

$$\hat{V}(x) = \begin{cases} \max [V_0(x); V_2(x)] & j_B \leqslant j < j_N \\ V_0(x) & j < j_B. \end{cases}$$

[17]Note that the value function $V_2(x)$ applies to both, old-age pensioners and disability pensioners.

[18]As described above, households may end up in legal court if their DP application is rejected. But there is no information on that since the pension insurance does not trace those who were rejected.

reflects the options for those in and above the ERA who may either continue to work or receive OAP and the younger ones who can only continue to work. The value function at this age (shown on the left side of Figure 3) before DP application is therefore a weighted average of the value functions with and without disability application:

$$V(x) = P(d = 1|x)V_1(x) + (1 - P(d = 1|x))\hat{V}(x),$$

where now $V_1(x)$ (in contrast to $\tilde{V}_1(x)$) also includes the expected stigma cost of applicants.

Finally, the value function of healthy young employees at age $j < j_B$ is given by (7) except that the budget constraints (8) now include unintended bequest b. Table 3 summarizes the final decision problem of an agent in retirement state $rs = 0$ depending on age and health status.

When entering the labor market everybody is healthy and working, so there is only the decision problem of how much to consume and save. Those who are younger than j_B and not healthy have to decide whether to file a DP application or work. Of course, healthy agents will always work or may stay at home. The period between the early and the NRA is most interesting. Healthy agents (i.e., where $h = 0$) with a long contribution record have to choose whether to work or to retire early with OAP. Those with work limitations (i.e., $h \in \{1, 2\}$) may in principle file a DP application, work or retire (if qualified) with OAP. These are exactly the groups where the government may induce even more instead of less retirement. Finally, at ages between the NRA j_N and the end of the retirement window j_E, all households still working have to decide whether to continue working or receive an OAP.

3.5 The government sector

The government in our model splits into a general budget and a pension system. Both budgets are closed separately. We abstract from public debt and corporate taxes so that the general government expenditure G, which is fixed in absolute terms, is financed by income and consumption taxes, i.e.:

$$T_y + \tau^c C = G, \tag{11}$$

where C defines aggregate consumption and T_y the revenues of income taxation. The latter is computed from

$$T_y = \int T(y, p, ra)\mathrm{d}\Phi(x) \quad \text{with } T(y, p, ra) = 2T_{16}(\tilde{y}/2) + \tau_r ra.$$

Due to deferred taxation of pensions, taxable income $\tilde{y}$ is either computed as the difference between gross labor income net of pension contributions $T_p(y) = \tau^p \min[y, 2\bar{y}]$ or – after retirement – as public pensions:

$$\tilde{y} = y - T_p(y) + p.$$

Given taxable income, we apply the German progressive tax code of 2016 $T_{16}(\cdot)$ to labor income and assume that all households are married couples (i.e., full income splitting). Concerning taxable interest income we apply a constant rate τ^r which reflects the flat capital income tax in Germany.

Table 3. Decision problem for working individuals

	$h = 0$	$h = 1$	$h = 2$
$1 \leqslant j < j_B$	V_0	$\max(V_0, V_1)$	
$j_B \leqslant j < j_N$	$\max(V_0, V_2)$	$\max(V_0, V_1, V_2)$	
$j_N \leqslant j < j_E$		$\max(V_0, V_2)$	

The pension system pays in every period disability and old-age benefits $p(x)$ to retired households and collects payroll contributions from labor income below the contribution ceiling. The budget of the pension system must be balanced in every period by adjusting the contribution rate τ^p:

$$\int p(x)\mathrm{d}\Phi(x) = \tau^p \int \min[y(x); 2\bar{y}]\mathrm{d}\Phi(x). \tag{12}$$

3.6 The production sector

The production sector is populated by firms that employ capital K and effective labor L from perfectly competitive factor markets to produce a single good according to the Cobb–Douglas production technology

$$Y = AK^{\alpha}L^{1-\alpha},$$

with α being the capital share in production and A the technology parameter. Capital is rented from households through an intermediary at the riskless rate r and depreciates over time with rate δ. Labor inputs are paid the competitive wage w. Factor prices are then determined by marginal productivity conditions, i.e.:

$$w = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha} \tag{13}$$

$$r = \alpha A\left(\frac{K}{L}\right)^{\alpha-1} - \delta. \tag{14}$$

3.7 Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the household decision problems (6), (7), (9), and (10) reflect competitive factor prices (13) and (14) and balance aggregate inheritances with unintended bequests

$$\int b(x)\mathrm{d}\Phi(x) = \int \frac{1-\psi_{j+1}(h)}{1+n}(1+r)a^{+}(x)\mathrm{d}\Phi(x).$$

Furthermore, in the closed economy aggregation holds,

$$L = \int z(x)l(x)\mathrm{d}\Phi(x),$$
$$C = \int c(x)\mathrm{d}\Phi(x),$$
$$K = \int a^{+}(x)\mathrm{d}\Phi(x),$$

the budgets of the general government (11) and the pension system (12) are balanced and the goods market clears in every period, i.e.:

$$Y = C + G + (n+\delta)K.$$

The computational method to solve the model numerically follows the Gauss–Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for aggregate variables, bequest distribution, and policy parameters. Then we compute factor prices, individual decision rules, and value functions which involve discretization of the state space and interpolation, see Fehr and Kindermann (2018). Next, we obtain the distribution of households and aggregate assets, labor supply and consumption as well as payroll and consumption taxes to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values of macro variables and policy parameters have sufficiently converged.

4. Calibration of the initial equilibrium

In what follows, we describe the parametrization of the model. This is a two-stage process. We first specify parameter values that are estimated outside the model. Some of these parameters are taken directly from the literature. Then we calibrate further parameters by matching the moments of our model to the data.

4.1 Demographics, health, and productivity

In our model, one period covers one year. Agents therefore start their economic life at age 20 ($j = 1$) and may receive health shocks after the initial work year. They may start to retire early with OAP at age 63 ($j_B = 44$) or work until age 70 ($j_E = 51$) and face a maximum possible life span of 99 years ($J = 80$). The specification of the retirement window reduces the state space significantly, but this restriction is not very binding. In 2019 only 0.5 percent of retirees retired after age 69, see Deutsche Rentenversicherung (2021). The growth rate of the population is set at $n = 0.0065$ which reflects the average population growth from 2012 until 2017 and generates a fairly realistic old-age dependency ratio. The ratio of cohorts aged 65+ relative to cohorts at ages 20–64 is 31.6 percent in the model while it was at roughly 34 percent around 2012, see DRV (2021, 288). Finally, the share of college-educated workers $\omega_s = 0.2373$ as well as the share of workers with a stable career $\omega_m = 0.5$ is taken from Kindermann and Püschel (2021, 24) who base their estimates on administrative data from the German pension insurance (Versichertenkontenstichprobe, 2017).

4.1.1 Health transitions and survival probabilities

Since labor productivity is affected by the health state h, we need to describe the health process over the life cycle first. The model distinguishes three life-cycle phases for health transitions. Appendix 1A shows that disability applications only increase slightly between ages 20 and 44. Here we assume that agents may receive only a bad health shock with probability $\pi^h_{j,s}$ from which they potentially recover, but not until they become 45 and face a new health transition process. Consequently

$$\pi_h(h^+|j,s,h) = \begin{cases} \pi^h_{j,s} & h = 0, h^+ = 2 \\ 1 - \pi^h_{j,s} & h = 0, h^+ = 0 \\ 1.0 & h = 2, h^+ = 2 \\ 0.0 & \text{otherwise,} \end{cases}$$

where the probabilities $\pi^h_{j,s}$ are derived as in Fehr *et al.* (2013) from an exponential function

$$\pi^h_{j,s} = \varrho_s \times \exp(\upsilon_s \times j)$$

with $\varrho_s = (0.0018, 0.0005)$ and $\upsilon_s = (0.062, 0.048)$. This procedure generates bad health probabilities between 0.2 percent and 0.9 percent which in turn leads to realistic disability applications and disability rates close to those reported in Hagen *et al.* (2010) for these two groups.

Appendix 1A documents that in 2019 more than 80 percent of disability applications were submitted by individuals older than age 44. Following Jürges *et al.* (2015), we applied a principal component analysis to estimate two health transition matrices for each age group from the German sub-sample of the Survey of Health, Ageing and Retirement in Europe. The data selection and more details on our

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Table 4. Health transition matrix for age group 45–64 ($j = 26, \ldots, 45$)

	High school ($s = 0$)			College ($s = 1$)		
	$h = 0$	$h = 1$	$h = 2$	$h = 0$	$h = 1$	$h = 2$
$h = 0$	0.8738	0.1195	0.0066	0.9140	0.0813	0.0047
$h = 1$	0.1346	0.7849	0.0806	0.1349	0.8117	0.0534
$h = 2$	0.0050	0.1649	0.8302	0.0090	0.1829	0.8081

Table 5. Health transition matrix for age group 65+ ($j = 46, \ldots, 80$)

	High school ($s = 0$)			College ($s = 1$)		
	$h = 0$	$h = 1$	$h = 2$	$h = 0$	$h = 1$	$h = 2$
$h = 0$	0.8340	0.1557	0.0103	0.8691	0.1309	0.0000
$h = 1$	0.0934	0.8117	0.0948	0.1131	0.7960	0.0910
$h = 2$	0.0005	0.1028	0.8968	0.0000	0.1287	0.8713

estimation approach are explained in Appendix 1C. The resulting health transition probabilities for the both age and education groups are reported in Tables 4 and 5.

Table 6 in the upper part compares the model results for the two educational groups in three phases of their life cycle. Note that health deteriorates with rising age for both education groups. In each age group considered, college graduates are significantly healthier than high-school graduates. This will determine the difference in life expectancy discussed below. In the lower part of Table 6 the two educational groups are aggregated in each age and health cell considered and the resulting distribution of health states generated by the transition matrices is compared with the respective self-assessed health data for Germany during the period 2008–19 from Eurostat (2021). The model seems to slightly overstate bad health for the oldest group, but overall the calibrated health transition seems to be quite realistic.[19]

Given the health transition matrices, we can determine the age- and health-dependent survival probabilities $\psi_j(h)$. The calibration starts with the average male survival probabilities $\bar{\psi}_j$ taken from the Human Mortality Database (HMD, 2020). We follow the approach of Kindermann and Püschel (2021) and compute the health-dependent probabilities according to

$$\psi_j(h) = \frac{1}{1 + \exp\left(-\iota_h \times \bar{x}_j\right)} \quad \text{with } \bar{x}_j = -\log\left(\frac{1}{\bar{\psi}_j} - 1\right).$$

The three parameters ι_h are calibrated to match exactly the average life expectancy of newborn and 65-year-old men in Germany as well as the difference in life expectancy at age 65 for the two education groups. Table 7 reports these targeted data moments and the resulting education-specific life expectancies.

The HMD (2020) reports an average life expectancy of newborn men in Germany of 79.5 years and 83 years for those who have reached age 65. In addition, Luy *et al.* (2015) report that 65-year-old college graduates in Germany face a 2.5-year higher life expectancy than high-school graduates. We determine the three parameters ι_h by solving a non-linear equation system in life expectancies. As Table 8 shows, the resulting education-specific difference in life expectancy even amounts to 4.5 years at birth, but then decreases to 2.5 years for those who have reached retirement age. Table 8 reports the values of the exogenously specified and internally calibrated parameters.

[19]The comparison of the education-specific match is not possible due to the different classifications of educational groups.

Table 6. Distribution of health status in the data and the model (in %)

	Age cohorts					
	45–54		55–64		65+	
	s = 0	s = 1	s = 0	s = 1	s = 0	s = 1
Good health	64.0	75.7	47.2	59.8	35.7	47.2
Medium health	25.9	20.1	37.9	32.4	43.5	39.1
Bad health	10.1	4.2	14.9	7.8	20.8	13.7
	Data*	Model	Data*	Model	Data*	Model
Good health	65.1	66.8	52.0	50.3	40.0	38.8
Medium health	26.4	24.5	35.3	36.5	45.4	42.3
Bad health	8.5	8.7	12.7	13.2	14.6	18.9

*Source: Eurostat (2021), average between 2008 and 2019.

Table 7. Education-specific and targeted life expectancies

	High school s = 0	College s = 1	Targeted average	Data source
Life expectancy at birth	78.4	82.9	79.5	HMD (2020)
Life expectancy at age 65	82.5	85.0	83.0	HMD (2020)
Difference at age 65			2.5	Luy *et al.* (2015)

4.1.2 Productivity and employment over the life cycle

Given health transitions and survival probabilities, the productivity process over the life cycle can be calibrated with our simulation model to match empirical targets taken from Kindermann and Püschel (2021).[20] As explained before, individual labor income is defined by $y = w \times z(j, s, m, \eta, h) \times l$. We normalize the wage rate to unity in the initial equilibrium and specify the productivity term

$$z(j, s, m, \eta, h) = \begin{cases} \exp(e_{j,s} + \eta) \times \theta_{h,s} & \text{employed normal earners } (1 - \pi_h^{nw}) \\ \exp(\eta_0) \times \theta_{h,s} & \text{employed low earners } (1 - \pi_h^{nw}) \\ 0 & \text{unemployed } (\pi_h^{nw}), \end{cases}$$

which is similar to the one in Kindermann and Püschel (2021), but allows for health-related productivity shocks $\theta_{h,s}$ and temporary non-working π_h^{nw}.

The calibration starts with a guess for the innovation variance $\sigma^2_{\epsilon,s}$ and then proceeds in two steps. In the first step, given the guess for the innovation variance, we calibrate the age-productivity profile $e_{j,s}$. Following Kindermann and Püschel (2021) we assume the following functional form

$$e_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3, \tag{15}$$

which captures both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < 60$) life-cycle labor productivity profile where productivity is constant from age $j_{M,s}$ onward. The parameters $b_{i,s}$ and $j_{M,s}$ of the polynomials are selected to match specific incomes $y_{j,s}$ for the normal earner group. More specifically, the profile is combined with health-related productivity effects from Capatina (2015) and a productivity shock η_s that follows an education-specific AR(1) process with autocorrelation parameters $\hat{\rho}_s$ taken from Kindermann and Püschel (2021) and innovation variances $\sigma^2_{\epsilon,s}$ specified before. The processes for the two education levels are discretized using a Rouwenhorst method as described in Kopecky and Suen

[20] Appendix 1D describes in more detail their approach using administrative data from German pension insurance to study the dynamics of labor earnings over the life cycle.

Table 8. Parameter values: demographics, health, and productivity

Parameter	Value	Source/target
Externally set		
Maximum life span (J), working age (j_E))	80, 51	Maximum age 99, latest retirement at 70
Education shares (ω_s)	0.7627, 0.2373	Kindermann and Püschel (2021)
Stable career share (ω_m)	0.50	
Autocorrelation normal earnings ($\hat{\rho}_s$)	0.9869, 0.9900	Kindermann and Püschel (2021)
Health-related productivity ($\theta_{h,0}$)	1.00, 0.81, 0.64	Capatina (2015)
($\theta_{h,1}$)	1.00, 0.86, 0.72	
Low-earning productivity (exp (η_0))	0.10	Kindermann and Püschel (2021)
-initial share (ω^s_{low})	0.2040, 0.8136	
-inflow probability ($\pi^s_{low,0}$)	0.0063, 0.0051	
-probability to stay ($\pi^s_{low,1}$)	0.8399, 0.7324	
Internally calibrated		
Population growth (n)	0.0065	Dependency ratio (65+/20–64) 31.6%
Bad health probability (ages 20–44)		$\pi^h_{j,s} = 0.2–0.9\%$
-absolute term (ϱ_s)	0.0018, 0.0005	DP application/rates in Hagen *et al.* (2010)
-exponent (υ_s)	0.062, 0.048	
Survival probabilities (ι_h)	1.775, 1.405, 0.554	Life expectancy in Germany
Age-productivity profile ($e_{j,s}$)		
-intercept ($b_{0,s}$)	−1.3006, −5.7498	Age fixed effects $\hat{\theta}_{j,s}$
-linear age term ($b_{1,s}$)	0.8411, 3.9905	From Kindermann and Püschel (2021)
-quadratic age term ($b_{2,s}$)	−0.0767, −0.7503	(See Appendix 1D)
-cubic age term ($b_{3,s}$)	0.0000, 0.0481	
-stagnation threshold ($j_{M,s}$)	∞, 52	
Innovation variance ($\sigma^2_{\epsilon,s}$)	0.00445, 0.00397	Unconditional variance of earnings (0.178, 0.198)
Non-working probabilities (π^{nw}_h)	(0.04), 0.59, 0.8	Long- and very long-term insured

(2010) with seven approximation points.[21] The resulting incomes are then fit to the fixed effects derived by Kindermann and Püschel (2021).[22] In the second step, we use the parametrization for the low-earner group provided by Kindermann and Püschel (2021) in combination with the computed incomes of normal earners to simulate the earnings processes of the complete model and generate an unconditional variance of earnings. If the latter does not match with the targeted values, the iteration starts again with a new guess for the innovation variances. The final unconditional variances exactly match the values reported in Table 8. Table 8 summarizes the resulting parameter values and Appendix 1D shows the match of the age-productivity profiles. Table 8 also reports the exogenous transition probabilities for stable and unstable careers of the two education groups. A fairly high fraction of college graduates start in the low-earner group, but afterward, the chance to transition into a low-earnings episode is very small (<1% for both education groups). Being in a low-income state, however, has quite some persistence. The average duration of a low-earning episode is 6.24 and 3.7 years for high school and for college-educated, respectively.

Finally, the non-working probabilities π^{nw}_h are specified as follows: assuming a non-working probability of healthy individuals of 4 percent, the respective probabilities of individuals with medium and bad health are selected to match realistic fractions of long-term (i.e., those with at least 35 years of contributions) and very long-term insured (i.e., those with at least 45 years of contributions) in the data. Appendix 2A shows that the resulting fractions of non-working households increase up to 40 percent for elderly cohorts.

4.2 Preference technology and government parameters

To calibrate the parameters of the utility function we first set the IES γ at 0.5, which is in the range of commonly used parameters in these types of models, see Conesa *et al.* (2009, 33). The time

[21]This also provides the initial distribution of the productivity shocks for the two education groups.

[22]More specifically, we applied a minimization routine to the sum of the squared distances $\sum_j (y_{j,s} - \hat{\theta}_{j,s})^2$ for each skill level. The considered household group is the same as in Kindermann and Püschel (2021), but the latter takes labor supply at the intensive margin into account. For that reason, we interpret the fixed effects as income.

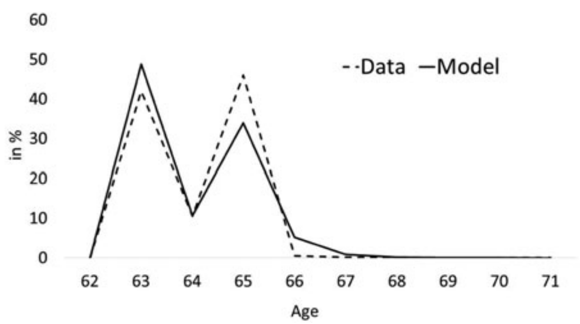

Figure 4. Old-age retirement inflow pattern in model and data.
Source: Computed from https://statistik-rente.de for 2014.

preference rate β is set to 0.98 to calibrate a realistic capital-output share of 330 percent. Labor disutility

$$\chi_{j,s} = \zeta_{1,s} j + \zeta_2 \max\left[0, j - (j_N - 1)\right]$$

increases with age, but differently in the two age groups. There is an additional utility cost of ζ_2 if households would still work after the NRA. Consequently, as in Seibold (2021) or Dolls and Krolage (2023), the NRA acts as an RP that changes with the reforms and affects retirement decisions directly. The parameter ζ_2 reduces employment after j_N to a minimum and $\zeta_{1,s}$ are calibrated to match the retirement pattern of the two education types for OAP. As Figure 4 shows, the specification of labor market disutility helps to generate with the model the two peaks in retirement observed in the data (when we abstract from retirement before j_B).

Similarly, j_N also acts as an RP for the stigma cost $\xi_{j,s} \sim LN(\mu_{j,s}, \sigma_\xi^2)$ that are associated with DP claiming. The latter now decreases when the claiming age approaches j_N. Assuming a polynomial form for the expected value

$$\mu_{j,s} = \varphi_{0,s}(j_N - j) + \varphi_{1,s}(j_N - j)^2 \quad \text{with } \varphi_{0,s}, \varphi_{1,s} \geqslant 0,$$

$\varphi_{0,s}$ are calibrated to match the respective retirement ages for DP. Concerning the two remaining values we set $\varphi_{1,1}$ to zero and calibrate the value $\varphi_{1,0}$ to match the shares of the two skill classes in DP inflows. High-skilled then have lower stigma costs when applying for DP, which can be motivated by the fact that they have lower stress to apply. Finally, the variance σ_ξ^2 of the stigma cost is independent of age and education and specified to match the relative share of DP retirees in the data. The resulting age pattern of the DP inflow is discussed in Figure 5.

The capital share in production is set at $\alpha = 0.30$ to match the capital income share. In addition, we choose a value for the technology parameter A to normalize the wage rate for effective labor to unity. The depreciation rate δ on capital is set at 5 percent which guarantees together with the population growth rate a realistic investment-to-GDP ratio of 21.8 percent for Germany.

Government tax policy in our model reflects the main features of the German tax system. We neglect public debt and corporate taxes and fix the public goods expenditures to 20 percent of output. The German income tax code of 2016 is applied to labor and pension income, that is, the marginal tax rate schedule rises after a basic allowance from 15.8 to 44.3 percent. We assume that the transition toward deferred pension taxation is already finished. Consequently, pension contributions are fully deductible from the tax base while pension benefits are fully taxable.[23] Concerning the tax base we apply the

[23]In reality the transition period until all pension benefits are fully taxable lasts until 2040.

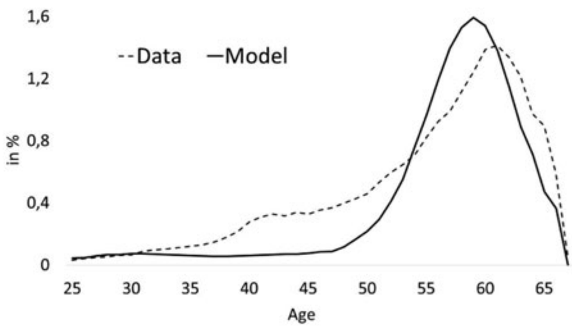

Figure 5. DP inflows in the model and the data 2014.
Source: Computed from https://statistik-rente.de for 2014.

German income splitting method to compute individual labor tax revenue. Returns from savings are taxed linearly at the rate of 15 percent. This reflects the quasi-dual income taxation in Germany. The resulting income tax revenue in the initial equilibrium is 10.5 percent of GDP. The budget is balanced by the consumption tax, where the equilibrium tax rate of 16 percent generates a revenue of 9.5 percent of GDP.

Concerning the pension system we consider an initial equilibrium which reflects the situation in Germany before the implementation of the retirement reform 2007, see Table 2. Consequently, j_B and j_N for OAP benefits are set to 44 and 46, respectively and the j_D for receiving DP benefits without deductions is set at 44. Finally, j_{MA} is set at 41. We assume that all DP applicants in good health are rejected if they file a DP application, while the rejection rate for individuals in medium or bad health shock is set to the average rejection rate, see Deutsche Rentenversicherung (2021). Finally, replacement rates are identical for both pension types and are set at $\kappa = 0.56$ which yields a realistic contribution rate. Table 9 summarizes the parameter values and the respective calibration targets or data sources.

4.3 Initial equilibrium

Given the dynamics of health and productivity over the life cycle, our chosen preference, technology, and policy parameters generate an initial equilibrium that reflects some key indicators of the German pension system in the years when the NRA increase was phased in. Table 10 compares some model results with respective microdata in 2016 provided by the research center of the German pension insurance, see FDZ-RV (2019) and Appendix 1E.[24] More than 75 percent of cohorts aged 60–65 were long-term insured, while only 1.4 percent qualify for very long-term insured. Table 10 also shows that our model captures important empirical facts concerning retirement behavior. On the one hand and quite surprisingly, the average retirement age for OAP is (more or less) independent of skill level in our data. On the other, the retirement age for DP benefits rises significantly with the skill class. As in the data, the average retirement age in the model rises significantly with the educational background, since high-skilled retire later with DP, and high-skilled have a lower share of DP. As already shown in Figure 4, our modeling of labor disutility allows us to match not only the average OAP retirement ages but also to capture the twin peaks of retirement at the ERA and NRA.

The mapping of the DP retirement is much more complicated since the decision to file a DP application depends on many different factors and parameters, such as current health and employment conditions, retirement incentives, and the stigma cost associated with the application. Nevertheless, our model replicates DP inflow and DP retirement ages quite well. Figure 5 compares the age pattern of DP inflows in the model benchmark and the year 2014. For younger ages, the model generates a

[24]This was the earliest micro data set available, but there is no reason to expect too dramatic differences in earlier years.

Table 9. Parameter values: preferences, technology, and government

Parameter	Value	Source/target
Externally set		
Inter-temporal elasticity of substitution (γ)	0.50	Conesa *et al.* (2009)
Public goods level (G/Y in % of GDP)	0.20	Tax revenue to GDP in Germany
Retirement ages (j_B, j_N)	44, 46	ERA at 63, NRA at 65 (in 2011)
(j_D, j_{MA})	44, 41	DRA at 63, MAA at 60
Rejection rate ($q(h)$)	1.00, 0.49, 0.49	DRV (2021)
Internally calibrated		
Time discount factor (β)	0.98	Capital-output ratio 320%
Work disutility ($\zeta_{1,s}$)	0.041, 0.018	Average OAP retirement ages
(ζ_2)	2.8	No work after NRA
Stigma costs ($\varphi_{0,s}$)	0.75, 0.77	Average DP retirement ages
($\varphi_{1,s}$)	0.02, (0.00)	High-/low-skilled share in inflows
(σ_ξ^2)	3.0	DP/OAP pensioners
Capital share in production (α)	0.30	Capital income share in GDP
Technology parameter (A)		Wage rate normalized to unity
Depreciation of capital (δ)	0.05	Investment to GDP ratio 21.8%
Income tax code ($T_{16}(\cdot)$)		German tax law of 2016
Capital income tax rate (τ_r)	0.15	German income tax revenue
Pension replacement rate (κ)	0.56	Pension contribution rate 18.9%

Table 10. Long-term insured (in %) and average retirement ages in model and data

		High school ($s = 0$)	College ($s = 1$)	Average
Long-term insured (⩾35)	Model	78.7	68.2	76.0
	Data*	77.8	71.6	76.4
(⩾45)	Model	1.8	0.0	1.4
	Data*	3.0	1.5	2.7
Retirement ages OAP	Model	63.9	64.3	64.0
	Data*	63.8	64.3	63.9
Retirement ages DP	Model	52.7	54.6	52.9
	Data*	52.6	54.4	52.8
Retirement ages average	Model	61.4	63.2	61.8
	Data*	61.3	63.4	61.7

**Source*: FDZ-RV(2019): SUFVVL2016.

lower DP inflow as in the data. This is mostly due to two reasons: first, bad health is very unlikely in that period, and (more importantly) stigma costs at young ages are prohibitively high. However, the shape and the peak of the inflow are captured quite well.

As shown in Table 1, the effective retirement age has increased significantly since 2011. Consequently, the shift in the profile can be easily explained and even improves the calibration. Besides that, the two curves are very similar, especially with the steep increase after age 45.

The first column in Table 11 reports again the average retirement ages from Table 10 as well as some key aspects of the initial equilibrium. Total outlays of the pension system amount to 12.6 percent of GDP, about 7 percent of total pension expenditure is spent for DP benefits and DP pensioners receive on average about 70 percent of average OAP benefits. These results from the model match the budget figure reported in Table 1 quite well.[25] The numbers describe quite well the situation of the German pension system in the period 2012–13 when Reform 2007 was implemented. Note that only 1.9 and 7.8 percent of 50–65-year-old individuals with a medium and bad health state respectively apply for DP benefits. These rates seem quite low but they reflect economic and psychological factors in the model such as expectations about future recovery, benefits, existing wealth, and the

[25] Note that statutory pensions in Table 1 do not include civil servant pensions which are included here.

Table 11. Economic effects of recent pension reforms

		Reform 2007			Reform 2018	
		PE				
	Initial equilibrium	Const. RP	Adj. RP	GE effect	Mechanical effect	GE effect
Retirement ages (avg.)	61.8	61.5	62.7	62.8	62.8	61.8
OAP	64.0	63.8	64.7	64.7	64.7	64.6
DP	52.9	56.4	54.2	54.6	54.6	53.6
Pension budget						
Total budget (in % of GDP)	12.6	12.6/11.9	12.7/11.6	11.5	11.8	12.5
DP (in % of pension budget)	7.1	9.8	7.8	7.4	8.5	11.6
DP (in % of OAP benefit)	69.6	76.7	71.1	71.1	82.6	82.6
DP applications (50–65)	1.9/7.8	3.8/10.7	1.8/8.3	1.7/8.3	1.7/8.3	2.8/9.5
Labor supply	–	−0.3	1.3	1.5	1.6	0.5
Capital	–	3.2/1.0	2.2/0.7	3.5	3.0	0.1
Wage rate	–	0.0		0.6	0.4	−0.1
Interest rate (in pp)	3.1	0.0		−0.1	−0.1	−0.1
Contribution rate (in pp)	18.9	0.0		−1.7	−1.2	−0.2
Consumption tax rate (in pp)	16.2	0.0		−1.1	−1.0	−0.2
Welfare	–	–		3.6	–	0.3

stigma cost of applications.[26] In combination with the exogenously set rejection rates they determine the pension structure.

Concerning some key macroeconomic statistics, the capital-output ratio of 330 percent is close to the target value of 320 percent which we derived from official data. Of course, the consumption and investment fractions of GDP are somewhat higher than in the official data, reflecting the fact that Germany is a net exporter and we model a closed economy. The endogenously determined interest rate is at 3.1 percent and the unintended bequests which are distributed lump sum to working cohorts amount to 3.2 percent of GDP. Finally, the budget of the general government is financed by income and consumption taxes and closed by the consumption tax rate which is at 16.2 percent. This rate is not unrealistic, given the very stylized modeling of the income tax system.

5. Simulation of pension reforms

This section presents the long-run macroeconomic and welfare consequences of alternative pension arrangements for workers in Germany. The baseline simulations start from the initial equilibrium in the first column of Table 11 described in more detail in the previous section. Then various policy parameters are adjusted and a new long-run equilibrium is computed. The reported welfare effects are computed as a Hick'sian equivalent variation, that is, the required relative change in lifetime consumption in the initial equilibrium that yields the after-reform welfare level.

5.1 Impact of pension Reforms 2007 and 2018

In this section, we implement successively the reform packages of 2007 and 2018 in the partial (PE) and the GE. Columns 2–4 of Table 11 report the consequences of an increase in the NRAs for OAP and DP benefits by two years. To isolate the impact of the RPs and the tax and price adjustment, the second column shows the effects when the RPs for labor disutility and stigma cost as well as prices and tax rates remain unchanged. As in Li (2018) this results in a strong shift toward DP retirement especially at older ages, so that DP applications, budget share, and DP retirement age increase significantly. While relative contributions remain almost constant, increased deductions reduce the aggregate benefits resulting in a surplus of the pension system. Due to lower benefits in old age, people save more, but higher savings are not balanced by higher capital demand. In the third column the RPs for labor

[26]Further disaggregation shows that the fraction of applicants rises with educational background since college graduates have accumulated more wealth.

disutility and stigma cost are adjusted, so that households now retire later and shift back to OAP retirement. Employment increases significantly compared to the initial equilibrium, but the pension budget as well as the capital market are still not balanced. This highlights the fact that in our model RPs are important for retirement behavior as was already pointed out by Seibold (2021) or Dolls and Krolage (2023). Finally, column 'GE effect' shows the consequences of the reform, when all budgets and markets are balanced by tax and price adjustments. The higher capital stock increases the wage rate and the pension budget is balanced by a reduction in the contribution rate by 1.7 percentage points. Note that the increase in the effective retirement age for OAP benefits only by 0.7 years up to 64.7 years is mainly due to the adjusted RPs.[27] Similarly, the DP retirement age increases by 1.7 years up to age 54.6, again mainly due to the adjusted labor disutility and stigma cost.[28] Later retirement of DP pensioners (relative to OAP pensioners who retire now with higher deductions) also increases the relative DP benefit level and the DP budget share slightly. Mainly due to later retirement and higher deductions, the total pension budget decreases significantly to 11.5 percent of GDP and the contribution rate is reduced by 1.7 percentage points. At the macro level, the delay of retirement (and the fall in contributions) increases labor supply, employment, and savings significantly. Although households retire – but less than two years – later, they receive lower pensions compared to before, and hence they save more. Higher wages and employment increase income tax revenues which induces a fall in the consumption tax rate by 1.1 percentage points. Not surprisingly, the reform increases the long-run welfare of all households by about 3.6 percent of initial consumption.

The remaining columns of Table 11 consider Reform 2018 which adds to the previous one the increase in the MAA from 60 to 67 (i.e., j_{MA} increases from 41 to 48). To better understand the economic effects, we first simulate the resulting increase in benefits while fixing the retirement behavior of DP pensioners at the Reform 2007 level. Consequently, the non-behavioral 'mechanical effect' of the reform keeps DP and OAP retirement ages as in the previous simulation but increases the DP benefit ratio as well as the DP share in the pension budget significantly.[29] Due to expected higher DP benefits the capital accumulation is dampened, the contribution rate increases slightly and factor prices remain roughly constant relative to the previous simulation.

Allowing individuals to adjust their retirement behavior in the column 'GE effect' induces a strong shift toward DP retirement. Households now retire earlier so that the OAP retirement age falls slightly and the DP retirement age falls significantly by one year. Not surprisingly, the DP share of the pension budget rises further up to 11.6 percent, which in turn raises the total budget to 12.5 percent of GDP and the contribution rate by one percentage point (relative to the Reform 2007). At the macro level, labor supply now decreases as well as capital accumulation, further reducing the wage by 0.5 percent. The higher DP generosity, therefore, induces a massive increase in DP applications, especially in the medium health state, and (with constant rejection rates) an inflow of DP pensioners at all ages, so that the whole economy suffers from the policy reform. As it turns out, higher contribution and tax rates as well as the lower long-run wage reduce now long-run welfare by 3.3 percent of initial consumption (relative to Reform 2007). At first sight, these consequences of Reform 2018 in the model seem to be very strong. However, they are in line with observed reactions from the empirical studies mentioned above. Closer inspection shows that the reform increases average DP benefits by about 17 percent and applications increase by 36 percent. Consequently, the resulting elasticity of 2.1 is higher than the application elasticity computed by Mullen and Staubli (2016) for Austria, but it is not completely out of range.

[27]Etgeton (2018) derives a similar figure for the Reform 2007 with an estimated dynamic discrete choice model of work, unemployment, and retirement. Appendix 1B also provides the observed changes in DP retirement between 2014 and 2022.

[28]If we altered only j_N for OAP, especially individuals in bad health will switch toward DP so that DP applications in the age group 50–65 increase by 16 percent. This figure is strikingly close to the relative rise in the DP enrollment rate projected by Duggan *et al.* (2007) for the same reform in the United States.

[29]Note that we report here the increase in the average DP/OAP benefit ratio, while specific ages might experience much stronger benefit increases.

5.2 Sensitivity analysis

The results of Table 11 suggest that without further adjustments Reform 2018 will most likely induce a dramatic shift toward disability retirement which reduces the average retirement age, raises the pension budget, and hurts the whole economy significantly. Note, however, that the simulation assumed unaltered rejection rates for DP applicants after the full implementation of Reform 2018. This seems quite unrealistic since on the one hand households in better health will start to apply for DP benefits and on the other one can expect a stricter screening process after the full implementation of the reform. Consequently, the second column of Table 12 shows the consequences when the rejection rate increases from 49 to 60 percent for all applicants (in bad and medium health). Comparing this simulation with the last column of Table 11 (replicated in the first column of Table 12) shows that households now apply later when their health has deteriorated. As a consequence, average DP retirement age increases significantly, DP benefits in the pension budget as well as pension expenditure as a share of GDP decrease. Higher labor supply and lower contribution rates have a positive effect on the overall economy so long-run welfare increases compared to the previous simulation.

Of course, one might also argue that our benchmark assumption that rejection rates are the same in the bad and the medium health states is somewhat unrealistic. In practice, it is more likely that rejection rates increase with the better health condition of the applicants in all scenarios considered. To analyze the impact of this conjecture, the right part of Table 12 assumes differentiated rejection rates of 55 and 45 percent for the medium and bad health states, respectively. Of course, now fewer (more) households with medium (bad) health apply in the 'No reform equilibrium' compared to the initial equilibrium reported in Table 11, but the average rate remains at 49 percent. The average DP retirement age is now a little bit lower, but overall the equilibrium without reforms has hardly changed. Differentiated DP rejection rates also have only minor consequences for the effects of Reforms 2007 and 2018. Comparing the first and the last columns of Table 12 only shows a slightly lower average DP retirement age and DP benefit level, but at the aggregate level there is hardly any difference.

The simulations reported so far did not take into account population aging which might affect the results significantly. According to the main variant of the most recent population projection (Statistisches Bundesamt, 2022), average life expectancy at birth will increase in Germany from

Table 12. Sensitivity analysis: alternative rejection rates for DP applications

			RR differentiated by health		
	Rejection rate 49%	Rejection rate 60%	No reform equilibrium	Reform 2007*	Reform 2018*
Retirement ages	61.8	62.9	61.8	62.7	61.8
OAP	64.6	64.7	64.0	64.7	64.6
DP	53.6	55.2	52.5	54.0	53.2
Pension budget					
Total budget (in % of GDP)	12.5	11.7	12.7	11.5	12.4
DP (in % of pension budget)	11.6	7.9	7.2	7.6	11.8
DP (in % of OAP benefit)	82.6	82.2	69.6	71.1	80.4
DP applications (50–65)	2.8/9.5	2.5/10.4	1.7/7.9	1.5/8.4	2.4/9.7
Rejection rate (average)	49	60	49	49	49
Labor supply	0.5	1.5	–	1.5	0.6
Capital	0.1	2.9	–	3.5	0.3
Wage rate	−0.1	0.4	–	0.6	−0.1
Interest rate (in pp)	−0.1	0.0	3.2	−0.2	0.0
Contribution rate (in pp)	−0.2	−1.3	19.0	−1.8	−0.3
Consumption tax rate (in pp)	−0.2	−1.0	16.2	−1.0	−0.2
Welfare	0.3	1.8	–	3.3	0.4

Changes are reported in % of initial equilibrium if not stated otherwise.
*Changes relative to 'No reform equilibrium'.

Table 13. Sensitivity analysis: aging and fixed DP inflows

	Aging scenario			Fixed inflow scenario		
	No reform equilibrium	Reform 2007*	Reform 2018*	No reform equilibrium	Reform 2007*	Reform 2018*
Retirement ages (avg.)	61.1	62.5	60.9	60.8	61.5	60.8
OAP	64.2	65.2	65.0	64.0	64.7	64.6
DP	51.4	52.8	52.1	50.0	50.7	51.2
Pension budget						
Total budget (in % of GDP)	19.3	17.5	19.8	13.0	11.9	12.9
DP (in % of pension budget)	8.6	8.5	14.3	9.0	10.0	14.0
DP (in % of OAP benefit)	71.7	71.7	83.0	60.9	63.6	75.6
DP applications (50–65)	2.7/8.0	2.2/8.6	4.0/9.8	1.9/7.2	1.7/7.4	2.7/8.7
Labor supply	–	2.4	0.3	–	1.3	0.6
Capital	–	5.6	−1.6	–	3.1	−0.5
Wage rate	–	0.9	−0.6	–	0.5	−0.3
Interest rate (in pp)	2.9	−0.3	0.1	3.1	−0.1	0.1
Contribution rate (in pp)	29.0	26.2	28.4	19.4	−1.6	−0.1
Consumption tax rate (in pp)	19.2	17.5	19.3	16.4	−1.0	−0.1
Welfare	–	5.4	−1.0	–	3.1	0.6

Changes are reported in % of initial equilibrium if not stated otherwise.
*Changes relative to 'No reform equilibrium'.

currently 79.5 to 84.6 years by 2070. In combination with a modest fertility and migration scenario, this implies an increase in the dependency ratio from currently 31 percent up to more than 46 percent in 2070. To analyze the impact of the older population structure in future years, the left part of Table 13 simulates the economy without and with the two pension reforms with the projected future life expectancy and dependency ratio. The first column ('No reform equilibrium') illustrates an economy in 2070 with initial NRAs and unchanged MAA. The contribution rate would then rise to 29 percent and the pension budget would increase to almost 20 percent of GDP.[30] The two reforms now induce a stronger increase in DP applications and higher inflows. Consequently, the average DP retirement age is lower compared to the benchmark and the DP share in the pension budget is higher. Overall the two reforms have now a stronger impact than in the previous simulations. The contribution rates and the consumption tax rates fall stronger (in Reform 2007) and rise further (in Reform 2018) than in Table 11. Consequently, the welfare effects of the two reforms are also more significant.

The right part of Table 13 shows a scenario without aging but with a reduced application elasticity. More specifically, it is now assumed that all households in bad health below age 45 (i.e., $j < 26$) file a DP application, and 10 percent of them are accepted. Individual decisions on whether to apply or not to apply for DP are restricted to ages 46 and older. This somewhat improves the replication of the data in Figure 5 but due to the higher DP inflow at younger ages, the average DP retirement age decreases to 50 years in the equilibrium without the reforms. Earlier DP retirement increases the share of DP in the pension budget – despite the slightly lower relative DP benefit level – and increases the contribution rate. The two pension reforms considered now only affect DP retirement after age 45. Reform 2018 now increases the average DP retirement age, because there is no reaction of households younger than age 45. Nevertheless, the impact of both reforms on the pension budget as a share of GDP as well as the share of DP in the pension budget are quite similar as in the respective benchmark simulations of Table 11.

As already discussed, the demographic pressure on the pension budget will continuously rise, so further reforms that increase the retirement age will be required.[31] The final sensitivity analysis

[30]For more details see Fehr *et al.* (2013) who derive quite similar figures for such a scenario.

[31]There is already an ongoing discussion to increase the normal retirement age beyond age 67 after 2030, see Deutsche Bundesbank (2022).

Table 14. Sensitivity analysis: alternative OAP reforms

	Eligibility rules		Benefit calculation	
j_B	46	44	44	
Contribution years	⩾35	⩾37	⩾35	
Adjustment factor	Both 3.6%		OAP 6.3%	Both 6.3%
Retirement ages	62.4	61.8	62.6	63.3
OAP	65.7	64.8	65.6	65.6
DP	54.0	54.1	54.4	55.0
Pension budget				
Total budget (in % of GDP)	12.5	12.6	11.8	11.2
DP (in % of pension budget)	12.6	13.1	12.2	9.1
DP (in % of OAP benefit)	79.2	82.6	82.6	75.6
DP applications (50–65)	3.3/10.2	3.4/10.4	3.0/10.0	2.3/9.1
Labor supply	1.8	0.5	1.7	2.3
Capital	−0.6	−0.5	1.5	3.7
Wage rate	−0.7	−0.3	−0.1	0.4
Interest rate (in pp)	0.2	0.1	0.0	−0.1
Contribution rate (in pp)	−0.1	0.1	−1.2	−2.2
Consumption tax rate (in pp)	−0.8	−0.1	−0.9	−1.5
Welfare	0.1	−0.4	2.0	4.1

Changes are reported in % of initial equilibrium if not stated otherwise.

reported in Table 14 therefore analyzes two modest and two more radical reforms of the OAP system intended to induce households to retire later in the future. The two modest reforms adjust the eligibility criteria for early OAP retirement and are shown in the first and second columns of Table 14. The two remaining policy reforms in Table 14 implement an isolated and a combined increase of the adjustment factors for early retirement from 3.6 to 6.3 percent, which reflects the average actuarial value in Europe and the United States, see Börsch-Supan *et al.* (2018).

Quite surprisingly, Reform 2007 has not adjusted the ERA and/or the required number of contribution years for early OAP benefits. Households who have contributed 35 years could still retire at age 63 when they accept a benefit reduction of 14.4 percent, which partly explains the widespread early retirement in Germany and the very modest increase in the effective OAP retirement age shown in Table 11. The first column of Table 14 increases, therefore, increases the ERA from currently 63 to 65 (i.e., j_B from 44 to 46), while the second column keeps the ERA at 63, but increases the required number of contribution years for early retirement from 35 to 37.

As they are supplements to Reforms 2007 and 2018, the considered scenarios need to be compared with the last column of Table 11. The increase in j_B has a significant effect on retirement behavior. On the one hand there is a shift toward DP applications and DP retirement, while on the other households now retire significantly later with OAP benefits. As a consequence, both the OAP and DP retirement ages increase significantly by 1.1 and 0.4 years, respectively. The total pension budget (and the contribution rate) is hardly affected, but the share of DP benefits in the pension budget rises. Later retirement increases employment and the revenues from income taxes, so that the consumption tax rate could be reduced. The slight reduction in long-run welfare also reflects the lower wage rate implied by the reform.

Keeping the current ERA, but increasing the required number of contribution years for early retirement even induces a stronger shift toward DP retirement. As the second column shows, the OAP retirement age rises in this case only slightly and employment is hardly affected. As a consequence, the pension budget increases slightly as well as the contribution rate. Slightly lower wages and higher consumption taxes now result in a reduction of long-run welfare by 0.4 percent.

The phased increase of the adjustment factor from currently 3.6 up to 6.3 percent in the two remaining columns of Table 14 has a very strong effect. When the increase only applies to OAP benefits, it induces again a strong shift toward DP applications/retirement and later OAP retirement. Consequently, the effective retirement age increases for both groups significantly, while at the same time, the pension budget and contribution rate are reduced. People now work longer and save more, which in turn also increases income tax revenues and lowers the consumption tax rate, so that long-run welfare increases now by 2 percent.

Increasing the adjustment rate for both pension types has the strongest effects on long-run retirement, the pension budget, and the aggregate economy. The effective retirement ages for OAP and DP benefits increase by 1.0 and 1.4 years, respectively. Lower benefits increase aggregate savings and the capital stock and reduce the contribution rate by 2 percentage points. Higher employment and wages boost income tax revenues so that the consumption tax rate decreases further by 1.3 percentage points. Consequently, long-run welfare now increases by even 4.1 percent.

Summing up, three major conclusions emerge from the sensitivity analysis of this subsection: first, the qualitative (and major quantitative) results for the reforms in 2007 and 2018 are quite robust concerning differentiated rejection rates, aging, and lower flexibility concerning DP claiming.[32] Second, future reforms of OAP eligibility rules and/or benefit calculation may improve the fiscal situation and dampen the long-run pressure on the whole economy, but these OAP reforms typically induce unintended substitution toward DP pensions. Third, a better future response to dampen the negative consequences of Reform 2018 might be an improved 'activation policy' for sick elderly with enhanced medical rehabilitation options. In the end, this would affect retirement behavior quite similar to an increase in DP rejection rates.

6. Conclusion

This paper provided a further step to model retirement decisions and behavior in a more realistic way by considering the interlinkages between OAP and DP benefits. Since it is not possible to monitor perfectly health-related work limitations (i.e., burn-out, etc.), households may try to substitute OAP for early DP retirement even if they are still able to work. Given that late retirement is very unpopular around the world, the increase in NRAs will induce such a substitution, which dampens the rise in the effective retirement age. In Germany, the substitution toward DP retirement is even reinforced by the recent improvement of DP benefits. While these DP reforms have gained surprisingly little attention in the public debate, our simulations indicate a strong increase in DP applications and inflow after 2034, when the reform is fully implemented. With current eligibility rules, this may even completely offset and neutralize the fiscal savings and economic benefits of the previous increase in the NRA. These results are fairly robust concerning various parameter settings. Consequently, future actions such as increasing rejection rates and tightening eligibility rules are much more likely required.

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[32]In Appendix 2B we also report the consequences when households are not allowed to work during the DP application.

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