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EDITED BY P. PINKERTON, M.A., D.Sc.

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On the Equation $c^2=a^2+b^2$.—*If* $c^2=a^2+b^2$ *and* a, b, c *have no common factor, then* c *exceeds* a *or* b *by an odd square.*

No two of a, b, c must have common factor, or all three would.

$$b^2=(c^2-a^2)=(c+a)(c-a)$$

$\therefore$ $c+a$, $c-a$ are both exact squares or have common factor.

(1) $c+a$, $c-a$ exact squares: $c+a$, $c-a$ are both odd or both even,

$$\therefore \text{ squares are both odd or both even,}$$

but the latter is impossible, for if $c+a=4\lambda^2$, $c-a=4\mu^2$, c, a have the common factor 2,

$$\therefore c-a \text{ is an odd square.}$$

(2) $c+a$, $c-a$ have a common factor, say ω.

$$\therefore c+a=m\omega,\ c-a=n\omega \quad \therefore c=(m+n)\frac{\omega}{2},\quad a=(m-n)\frac{\omega}{2},$$

$\therefore$ if $\omega>2$, c, a have common factor, $\therefore$ only possible value of ω is 2.

$$\therefore c+a=2m,\ c-a=2n \quad\text{and}\quad c=m+n,\ a=m-n.$$

And $b^2=c^2-a^2=4mn$ $\therefore$ mn is an exact square,

$\therefore$ m, n are both exact squares or have a common factor, the latter impossible, for then c, a have a common factor,

$\therefore$ m, n are exact squares, and both can't be even or odd, since then $m+n$, $m-n$ would be even and c, a would have a common factor,

$\therefore$ of m, n one is even, the other odd.

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Hence $c - b = m + n - \sqrt{4mn} = (\sqrt{m} - \sqrt{n})^2$
$=$ (diff. of odd and even numbers)^2 = (odd number)^2 = odd square.

So in all cases c exceeds a or b by odd square.

LAWRENCE CRAWFORD.

New Clock Problems.—The following problem was suggested by a difficulty in distinguishing between the two hands of a clock.

"The hour and minute hands of one clock A are parallel respectively to the minute and hour hands of another clock B. The time on A is between 7 and 8 o'clock, that on B between 10 and 11 o'clock; find the time on each clock."

I. *Algebraic Solution.*—Let the times registered on the clocks be x minutes past seven, and y minutes past ten. We thus have

$$\left.\begin{aligned} &(1) \quad x/12 + 35 = y \\ &(2) \quad y/12 + 50 = x \end{aligned}\right\}$$

Multiplying (1) by $^1/_{12}$ and adding (2), we have

$$\tfrac{143}{144}x = 50 + \tfrac{35}{12} \text{ or } 143x = 7620,$$

Similarly $$\tfrac{143}{144}y = 35 + \tfrac{50}{12} \text{ or } 143y = 5640,$$

II. *Arithmetical Solution.*—Imagine two clocks one at 7 o'clock and the other at 10.35, and imagine the first clock to go twelve times as fast as the second. The hour hand of the first clock thus keeps parallel to the minute hand of the second, while the minute hand of the first goes 144 times as fast as the hour hand of the second. Hence we have at once

$$\tfrac{143}{144}x = 50 + \tfrac{35}{12}. \quad i.e., \ 143x = 7620.$$

Similarly we find y.

It is to be noticed that the idea involved in making the one clock go twelve times as fast as the other is identical with that used in eliminating one of the variables from equations (1) and (2) above.

If we are given only that the hour and minute hands of the one clock are parallel respectively to the minute and hour hands of the other, the problem of finding the time on each clock is indeterminate, although the number of solutions is finite. We can easily find the number of solutions by supposing two clocks to start from 12 o'clock