Appendix A
$SU(n)$

A.1 Fundamental representation of $SU(n)$

In the following appendices we record some properties of the representations of the group $SU(n)$. First we review the construction of a complete basis set of Hermitian traceless $n \times n$ matrices, similar to the $n = 2, 3$ examples. We shall denote these matrices by λ_k, $k = 1, 2, \ldots, n^2 - 1$. The symmetric off-diagonal matrices have the form

$$(\lambda_k)_{ab} = \delta_{am}\delta_{an} + \delta_{bm}\delta_{an} \quad k \leftrightarrow \{m, n\} \tag{A.1}$$

and the antisymmetric matrices are given by

$$(\lambda_k)_{ab} = i(\delta_{am}\delta_{an} - \delta_{bm}\delta_{an}), \tag{A.2}$$

where $a, b, m, n = 1, 2, \ldots, n$, $m > n$. The non-zero elements of the diagonal matrices may be taken as

$$(\lambda_k)_{aa} = \sqrt{\frac{2}{m + m^2}} \quad a = 1, \ldots, m, \tag{A.3}$$

$$= -m\sqrt{\frac{2}{m + m^2}} \quad a = m + 1, \tag{A.4}$$

where $m = 1, 2, \ldots, n - 1$. We add the multiple of the unit matrix

$$\lambda_0 = \sqrt{\frac{2}{n}}\, \mathbb{1}, \tag{A.5}$$

such that the $k = 0, 1, \ldots, n^2 - 1$ matrices form a complete set of $n \times n$ matrices. They satisfy

$$\lambda_k = \lambda_k^\dagger, \tag{A.6}$$

$$\mathrm{Tr}\,(\lambda_k\lambda_l) = 2\delta_{kl}, \tag{A.7}$$

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and either $\lambda_k = \lambda_k^{\mathrm{T}} = \lambda_k^*$ or $\lambda_k = -\lambda_k^{\mathrm{T}} = -\lambda_k^*$. An arbitrary matrix X can be written as a superposition of the λ's,

$$X = X_k \lambda_k, \tag{A.8}$$

$$X_k = \tfrac{1}{2}\,\mathrm{Tr}\,(X\lambda_k). \tag{A.9}$$

For instance

$$\lambda_k \lambda_l = \Lambda_{klm}\lambda_m, \tag{A.10}$$

$$\Lambda_{klm} = \tfrac{1}{2}\,\mathrm{Tr}\,(\lambda_k\lambda_l\lambda_m). \tag{A.11}$$

Let

$$\Lambda_{klm} = d_{klm} + i f_{klm}, \tag{A.12}$$

where d_{klm} and f_{klm} are real. Then

$$\begin{aligned} d_{klm} &= \tfrac{1}{4}\,\mathrm{Tr}\,(\lambda_k\lambda_l\lambda_m + \lambda_k^*\lambda_l^*\lambda_m^*) = \tfrac{1}{4}\,\mathrm{Tr}\,(\lambda_k\lambda_l\lambda_m + \lambda_k^{\mathrm{T}}\lambda_l^{\mathrm{T}}\lambda_m^{\mathrm{T}}) \\ &= \tfrac{1}{4}\,\mathrm{Tr}\,(\lambda_k\lambda_l\lambda_m + \lambda_m\lambda_l\lambda_k) = \tfrac{1}{4}\,\mathrm{Tr}\,(\lambda_k\lambda_l\lambda_m + \lambda_l\lambda_k\lambda_m) \\ &= \tfrac{1}{4}\,\mathrm{Tr}\,(\{\lambda_k, \lambda_l\}\,\lambda_m), \end{aligned} \tag{A.13}$$

and similarly,

$$i f_{klm} = \tfrac{1}{4}\,\mathrm{Tr}\,([\lambda_k, \lambda_l]\,\lambda_m). \tag{A.14}$$

These representations of the d's and f's and the cyclic properties of the trace imply that d_{klm} is totally symmetric under interchange of any of its labels. Likewise f_{klm} is totally antisymmetric. Hence, (A.10) and (A.12) imply

$$[\lambda_k, \lambda_l] = 2i f_{klm}\lambda_m, \tag{A.15}$$

$$\{\lambda_k, \lambda_l\} = 2 d_{klm}\lambda_m. \tag{A.16}$$

We note in passing that

$$\lambda_0\lambda_l = \sqrt{\frac{2}{n}}\,\lambda_l \to d_{0lm} = \sqrt{\frac{2}{n}}\,\delta_{lm}, \quad f_{0lm} = 0. \tag{A.17}$$

A standard choice for the generators t_k of the group $SU(n)$ in the fundamental (defining) representation is given by

$$t_k = \tfrac{1}{2}\,\lambda_k, \quad k = 1, 2, \ldots, n^2 - 1. \tag{A.18}$$

In the exponential parameterization an arbitrary group element can be written as

$$U = \exp(i\alpha^k t_k), \tag{A.19}$$

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where the α^k are n^2-1 real parameters. From their occurence in the commutation relations

$$[t_k, t_l] = i f_{klm}\, t_m, \tag{A.20}$$

the f_{klm} are called the structure constants of the group.

Next we calculate the value C_2 of the quadratic Casimir operator $t_k t_k$ in the defining representation. For this we need a useful formula that follows from expanding the matrix $X^{(cd)}_{ab} \equiv 2\delta_{ad}\delta_{bc}$ in terms of $(\lambda_k)_{ab}$. According to (A.8) and (A.9) we have the expansion coefficients $X^{(cd)}_k = \mathrm{Tr}\,(X^{(cd)}\lambda_k)/2 = \delta_{ad}\delta_{bc}(\lambda_k)_{ba} = (\lambda_k)_{cd}$, hence,

$$(\lambda_k)_{ab}(\lambda_k)_{cd} = 2\delta_{ad}\delta_{bc}, \tag{A.21}$$

where the summation is over $k = 0, 1, \ldots, n^2-1$ on the left-hand side. It follows that

$$\begin{aligned}(t_k)_{ab}(t_k)_{cd} &= \frac{1}{4}\,(\lambda_k)_{ab}(\lambda_k)_{cd} - \frac{1}{4}\,(\lambda_0)_{ab}(\lambda_0)_{cd} \\ &= \frac{1}{2}\,\delta_{ad}\delta_{bc} - \frac{1}{2n}\,\delta_{ab}\delta_{cd}\end{aligned} \tag{A.22}$$

(note that $k=0$ is lacking for the t_k). Contraction with δ_{bc} gives

$$(t_k t_k)_{ad} = \frac{1}{2}\left(n - \frac{1}{n}\right)\delta_{ad} \equiv C_2\,\delta_{ad}, \tag{A.23}$$

or

$$C_2^{\mathrm{fund}} = \frac{1}{2}\left(n - \frac{1}{n}\right). \tag{A.24}$$

For $n=2$, $C_2^{\mathrm{fund}} = \frac{3}{4}$ which is just the usual value $j(j+1)$ for the $j=\frac{1}{2}$ representation of $SU(2)$.

A.2 Adjoint representation of $SU(n)$

The adjoint (regular) representation R is the representation carried by the generators,

$$U^\dagger t_k U = R_{kl} t_l. \quad U \in SU(n). \tag{A.25}$$

Note that $\mathrm{Tr}\,(U^\dagger t_k U) = \mathrm{Tr}\, t_k = 0$, so that $U^\dagger t_k U$ can indeed be written as a linear superposition of the t_k. By eq. (A.9) we have the explicit representation in terms of the group elements

$$R_{kl} = 2\,\mathrm{Tr}\,(U^\dagger t_k U t_l). \tag{A.26}$$

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We shall now calculate R in terms of the parameters α^k of the exponential parameterization of U. Let

$$U(y) = \exp(iy\alpha^p t_p), \quad R_{kl}(y) = 2\,\mathrm{Tr}\,(U^\dagger(y) t_k U(y) t_l). \tag{A.27}$$

Then

$$\begin{aligned}\frac{\partial}{\partial y} R_{kl}(y) &= -i\alpha^p\, 2\,\mathrm{Tr}\,(U^\dagger(y)[t_p, t_k]U(y)t_l) \\ &= \alpha^p f_{pkn} 2\,\mathrm{Tr}\,(U^\dagger(y) t_n U(y) t_l) \\ &= i\alpha^p (F_p)_{kn} R_{nl},\end{aligned} \tag{A.28}$$

where

$$(F_p)_{mn} = -if_{pmn}. \tag{A.29}$$

In matrix notation (A.28) reads

$$\frac{\partial}{\partial y} R(y) = i\alpha^p F_p R(y), \tag{A.30}$$

which differential equation is solved by

$$R(y) = \exp(iy\alpha^p F_p), \tag{A.31}$$

using the boundary condition $R(0) = 1$. Hence,

$$R = \exp(i\alpha^p F_p), \tag{A.32}$$

and we see that the F_p are the generators in the adjoint representation. By the antisymmetry of the structure constants we have

$$F_p = -F_p^* = -F_p^{\mathrm{T}}, \tag{A.33}$$

and it follows that the matrices R are real and orthogonal,

$$R = R^*, \;\; R^{\mathrm{T}} = R^{-1}. \tag{A.34}$$

Notice that the derivation of (A.28) uses only the commutation relations of the generators, so that we have for an arbitrary representation $D(U)$

$$D(U)^{-1} T_k D(U) = R_{kl} T_l, \tag{A.35}$$

where the T_k are the generators in this representation D.

Next we calculate the value of the Casimir operator in the adjoint representation, $F_p F_p$, using the results of the previous appendix:

$$\begin{aligned}(F_p F_p)_{km} &= if_{kpl} if_{lpm} \\ &= 4\,\mathrm{Tr}\,(t_k t_p t_l)\, if_{lpm} = 8\,\mathrm{Tr}\,(t_p t_l t_k)\,\mathrm{Tr}\,([t_m, t_l] t_p) \\ &= 8 (t_p)_{ab} (t_l t_k)_{ba}\; [t_m, t_l]_{dc} (t_p)_{cd}.\end{aligned} \tag{A.36}$$

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With (A.22) for $(t_p)_{ab}(t_p)_{cd}$, this gives

$$(F_p F_p)_{km} = 4\,\mathrm{Tr}\,(t_l t_k [t_m, t_l]), \tag{A.37}$$

and using (A.22) again and $t_l t_l = [(n^2-1)/2n]\mathbb{1}$ gives finally

$$F_p F_p = n\mathbb{1}, \quad C_2^{\mathrm{adj}} = n. \tag{A.38}$$

The matrix $S_k(\alpha)$ introduced in (4.41) can be calculated as follows. We write $D(U(\alpha)) = D(\alpha)$ and consider (4.42),

$$M(y) = D(y\alpha)D(y\alpha + y\epsilon)^{-1} = 1 - i\epsilon^k S_k(\alpha) + O(\epsilon^2) \tag{A.39}$$

$$= e^{iy\alpha^k T_k}\, e^{-iy(\alpha^k+\epsilon^k)T_k}. \tag{A.40}$$

Then

$$\begin{aligned}\frac{\partial}{\partial y} M(y) &= D(y\alpha)[i\alpha^k T_k - i(\alpha^k + \epsilon^k)T_k] D(y\alpha + y\epsilon)^{-1} \\ &= -i\epsilon^k D(y\alpha) T_k D(y\alpha)^{-1} + O(\epsilon^2) \\ &= -i\epsilon^k R^{-1}_{kl}(y\alpha) T_l + O(\epsilon^2).\end{aligned} \tag{A.41}$$

This differential equation can be integrated with the boundary condition $M(0) = 1$, using $R^{-1}(y\alpha) = \exp(-iy\alpha)$, $\alpha \equiv \alpha^p F_p$,

$$M(y) = 1 - i\epsilon^k \left(\frac{1 - e^{-iy\alpha}}{i\alpha}\right)_{kl} T_l + O(\epsilon^2). \tag{A.42}$$

Setting $y = 1$ we find $S_k(\alpha) = S_{kl}(\alpha) T_l$ with

$$S_{kl}(\alpha) = \left(\frac{1 - e^{-i\alpha}}{i\alpha}\right)_{kl}, \quad \alpha = \alpha^p F_p. \tag{A.43}$$

We end this appendix with an expression for $\mathrm{Tr}\, T_k T_l$ in an arbitrary representation D. The matrix

$$I_{kl} = \mathrm{Tr}\,(T_k T_l) \tag{A.44}$$

is invariant under transformations in the adjoint representation,

$$R_{kk'} R_{ll'} I_{k'l'} = \mathrm{Tr}\,(D^{-1} T_k D\, D^{-1} T_l D) = I_{kl}. \tag{A.45}$$

By Schur's lemma, I_{kl} must be a multiple of the identity matrix,

$$I_{kl} = \rho\, \delta_{kl}. \tag{A.46}$$

Putting $k = l$ and summing over k gives the relation

$$(n^2 - 1)\rho(D) = C_2(D)\,\mathrm{dimension}(D). \tag{A.47}$$

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For the fundamental and adjoint representations we have

$$\rho_{\text{fund}} = \tfrac{1}{2}, \tag{A.48}$$

$$\rho_{\text{adj}} = n. \tag{A.49}$$

A.3 Left and right translations in $SU(n)$

Let Ω and U be elements of $SU(n)$. We define left and right transformations by

$$U'(L) = \Omega U, \quad U'(R) = U\Omega, \tag{A.50}$$

respectively, which may be interpreted as translations in group space, $U \to U'$. In a parameterization $U = U(\alpha)$, $\Omega = \Omega(\varphi)$, this implies transformations of the α's,

$$\alpha'^k(L) = f^k(\alpha, \varphi, L), \tag{A.51}$$

and similarly for R. We shall first concentrate on the L case. For Ω near the identity we can write,

$$\Omega = 1 + i\varphi^m t_m + \cdots, \tag{A.52}$$

$$\alpha'^k(L) = \alpha^k + \varphi^m S^k{}_m(\alpha, L) + \cdots, \tag{A.53}$$

$$S^k{}_m(\alpha, L) = \frac{\partial}{\partial \varphi^m} f^k(\alpha, \varphi, L)_{|\varphi=0}. \tag{A.54}$$

The $S^k{}_m(\alpha, L)$ (which are analogous to the tetrad or 'Vierbein' in General Relativity) can found in terms of the $S_{km}(\alpha)$ as follows,

$$U'(L) = (1 + i\varphi^m t_m + \cdots)U, \tag{A.55}$$

$$t_m U = -i \frac{\partial}{\partial \varphi^m} U_{|\varphi=0} = -i \frac{\partial U}{\partial \alpha^k} \frac{\partial \alpha^k}{\partial \varphi^m}_{|\varphi=0}$$

$$= -i \frac{\partial U}{\partial \alpha^k} S^k{}_m(\alpha, L). \tag{A.56}$$

Differentiating $UU^\dagger = 1$ gives

$$\frac{\partial U}{\partial \alpha^k} = -U \frac{\partial U^\dagger}{\partial \alpha^k} U, \tag{A.57}$$

and using this in (A.56) we get

$$t_m U = iU \frac{\partial U^\dagger}{\partial \alpha^k} U S^k{}_m(\alpha, L), = S_k(\alpha, L) U S^k{}_m(\alpha, L), \tag{A.58}$$

where

$$S_k(\alpha, L) \equiv iU \frac{\partial U^\dagger}{\partial \alpha^k} \tag{A.59}$$

is the S_k introduced earlier in (4.41). The factor U can be canceled out from the above equation,

$$t_m = S_k(\alpha, L)\, S^k{}_m(\alpha, L). \tag{A.60}$$

We have already shown in (4.43) that S_k is a linear superposition of the generators, $S_k(\alpha, L) = S_{kn}(\alpha, L)t_n$, so we get

$$t_m = t_n S_{kn}(\alpha, L) S^k{}_m(\alpha, L) \tag{A.61}$$

or

$$\delta_{mn} = S_{kn}(\alpha, L) S^k{}_m(\alpha, L). \tag{A.62}$$

Thus $S^k{}_m(\alpha, L)$ is the inverse (in the sense of matrices) of $S_{km}(\alpha, L)$.

Introducing the differential operators

$$X_m(L) = S^k{}_m(\alpha, L)\, \frac{\partial}{i\, \partial\alpha^k} \tag{A.63}$$

we can rewrite (A.56) in the form

$$X_m(L)U = t_m U. \tag{A.64}$$

It follows from this equation that the $X_m(L)$ have the commutation relations

$$[X_m(L), X_n(L)] = -i f_{mnp} X_p(L). \tag{A.65}$$

These differential operators may be called the generators of left translations.

For the right translations we get in similar fashion

$$\begin{aligned}
U t_m &= -i \frac{\partial U}{\partial \alpha^k}\, S^k{}_m(\alpha, R) = U S_k(\alpha, R) S^k{}_m(\alpha, R), && \text{(A.66)}\\
S_k(\alpha, R) &\equiv -i U^\dagger \frac{\partial U}{\partial \alpha^k} \\
&= U^\dagger S_k(\alpha, L) U = S_{kn}(\alpha, L)\, U^\dagger t_n U \\
&= S_{kp}(\alpha, L) R_{pn} t_n, && \text{(A.67)}\\
S_k(\alpha, R) &= S_{kn}(\alpha, R) t_n, && \text{(A.68)}\\
S_{kn}(\alpha, R) &= S_{kp}(\alpha, L) R_{pn}, && \text{(A.69)}\\
\delta_{mn} &= S_{kn}(\alpha, R)\, S^k{}_m(\alpha, R), && \text{(A.70)}\\
X_m(R) &= S^k{}_m(\alpha, R) \frac{\partial}{i\, \partial \alpha^k}, && \text{(A.71)}\\
X_m(R) U &= U t_m, && \text{(A.72)}\\
[X_m(R), X_n(R)] &= +i f_{mnp} X_p(R) && \text{(A.73)}
\end{aligned}$$

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The left and right generators commute,

$$[X_m(L), X_n(R)] = 0, \tag{A.74}$$

which follows directly from (A.64) and (A.72), and their quadratic Casimir operators are equal,

$$X^2(L) = X_m(L)X_m(L), \quad X^2(R) = X_m(R)X_m(R), \tag{A.75}$$

$$X^2(R)U = Ut_mt_m = C_2U = t_mt_mU = X^2(L)U. \tag{A.76}$$

The differential operator $X^2 = X^2(L) = X^2(R)$ is invariant under coordinate transformations on group space and is also known as a Laplace–Beltrami operator.

Finally, the metric introduced in (4.91) can be expressed in terms of the analogs of the tetrads,

$$g_{kl}(\alpha) = S_{kp}(\alpha, L)S_{lp}(\alpha, L) = S_{kp}(\alpha, R)S_{lp}(\alpha, R), \tag{A.77}$$

$$S_{kp}(\alpha, L) = g_{kl}(\alpha)S^l{}_p(\alpha, L), \quad S_{kp}(\alpha, R) = g_{kl}(\alpha)S^l{}_p(\alpha, R). \tag{A.78}$$

For a parameterization that is regular near $U = 1$ (such as $\exp(i\alpha^k t_k)$),

$$U = 1 + i\alpha^k t_k + O(\alpha^2), \tag{A.79}$$

it is straightforward to derive that

$$S^k{}_p(\alpha, L) = \delta_{kp} - \tfrac{1}{2}\, f_{kpl}\alpha^l + O(\alpha^2), \tag{A.80}$$

$$S^k{}_p(\alpha, R) = \delta_{kp} + \tfrac{1}{2}\, f_{kpl}\alpha^l + O(\alpha^2), \tag{A.81}$$

$$g_{kl}(\alpha) = \delta_{kl} + O(\alpha^2). \tag{A.82}$$

A.4 Tensor method for $SU(n)$

It is sometimes useful to view the matrices U representing the fundamental representation of $SU(n)$ as tensors. Products of U's then transform as tensor products and integrals over the group reduce to invariant tensors. It will be useful to write the matrix elements with upper and lower indices, $U_{ab} \to U^a_b$. We start with the simple integral

$$\int dU\, U^a_b U^{\dagger p}_q = I^{ap}_{bq}. \tag{A.83}$$

By making the transformation of variables $U \to VUW^\dagger$, it follows that the right-hand side above is an invariant tensor in the following sense:

$$I^{ap}_{bq} = V^a_{a'} W^p_{p'} V^{\dagger q'}_q W^{\dagger b'}_b I^{a'p'}_{b'q'}. \tag{A.84}$$

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Here V and W are arbitrary elements of $SU(n)$ and similarly for their matrix elements in the fundamental representation and their complex conjugates $V^\dagger$ and $W^\dagger$. We are using a notation in which matrix indices of U are taken from the set a, b, c, d, ..., while those of $U^\dagger$ are taken from p, q, r, s, ... Upper indices in the first set transform with V, upper indices in the second set transform with W; lower indices in the first set transform with $W^\dagger$, lower indices in the second set transform with $V^\dagger$, as in

$$U^a_b \to V^a_{a'} W^{\dagger b'}_b U^{a'}_{b'}, \qquad U^{\dagger p}_q \to W^p_{p'} V^{\dagger q'}_q U^{\dagger p'}_{q'}. \tag{A.85}$$

This notation suffices for not-too-complicated expressions.

Returning to the above group integral, there is only one such invariant tensor: $I^{ap}_{bq} = c\delta^a_q \delta^p_b$, which is a simple product of Kronecker deltas. The constant c can be found by contracting the left- and right-hand sides with δ^p_b, with the result

$$\int dU\, U^a_b U^{\dagger p}_q = \frac{1}{n}\, \delta^a_q \delta^p_b. \tag{A.86}$$

Invariant tensors have to be linear combinations of products of Kronecker tensors and the Levi-Civita tensors

$$\begin{aligned} \epsilon^{a_1\cdots a_n} &= +1, \quad \text{even permutation of } 1,\ldots,n \\ &= -1, \quad \text{odd permutation of } 1,\ldots,n, \end{aligned} \tag{A.87}$$

and similarly for $\epsilon_{a_1\cdots a_n}$, etc. They are invariant because

$$V^{a_1}_{a'_1} \cdots V^{a_1}_{a'_1} \epsilon^{a'_1\cdots a'_n} = \det V\, \epsilon^{a_1\cdots a_n}. \tag{A.88}$$

These tensors appear in

$$\int dU\, U^{a_1}_{b_1} \cdots U^{a_n}_{b_n} = \frac{1}{n!}\, \epsilon^{a_1\cdots a_n} \epsilon_{b_1\cdots b_n} \tag{A.89}$$

$$= \frac{1}{n!} \sum_{\text{perm}\pi} (-1)^\pi \delta^{a_1}_{b_{\pi 1}} \cdots \delta^{a_n}_{b_{\pi n}}. \tag{A.90}$$

The coefficient can be checked by contraction with $\epsilon_{a_1\cdots a_n}$.

In writing down possible invariant tensors for group integrals we have to keep in mind that, according to (A.85), there can be only Kronecker deltas with one upper and one lower index, and furthermore one index should correspond to a U and the other index to a $U^\dagger$, i.e. they should be of the type δ^a_p or δ^p_a. It is now straightforward to derive identities for

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integrals of the next level of complication:

$$\int dU\, U^a_b U^c_d U^e_f = 0, \quad n > 3, \tag{A.91}$$

$$\int dU\, U^a_b U^c_d U^{\dagger p}_q U^{\dagger r}_s = \frac{1}{n^2-1}\left(\delta^a_q \delta^c_s \delta^p_b \delta^r_d + \delta^a_s \delta^c_q \delta^r_b \delta^p_d\right) - \frac{1}{n(n^2-1)}\left(\delta^a_s \delta^c_q \delta^p_b \delta^r_d + \delta^a_q \delta^c_s \delta^r_b \delta^p_d\right),\ n > 2. \tag{A.92}$$

Note the symmetry under $(a,b) \leftrightarrow (c,d)$ and $(p,q) \leftrightarrow (r,s)$ in (A.92). The coefficients follow, e.g. by contraction with δ^p_d. By contracting (A.92) with the generators $(t_k)^s_c (t_l)^d_r$ we get an identity needed in the main text:

$$\int dU\, U^a_b U^{\dagger p}_q R_{kl}(U) = \frac{2}{n^2-1}\,(t_k)^a_q (t_l)^p_b, \quad n > 2. \tag{A.93}$$

where $R_{kl}(U)$ is the adjoint representation of U (cf. (A.26)).

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