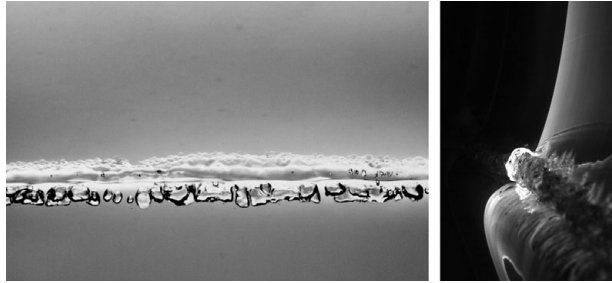


Plunging cavities

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When a wave breaks, the tip forms a liquid sheet which impinges the base and creates an air cavity which breaks into bubbles. Gomez-Ledesma, Kiger & Duncan (*J. Fluid Mech.*, this issue, vol. 680, 2011, pp. 5–30) have conducted a nice experiment on this problem, enabling them to discuss both the inclination of the jet and the effect of its translation. This work has interesting links with other transient cavities.

Key words: bubble dynamics, free-surface flows, jets, wave breaking

1. Introduction

Water waves break in a wide range of conditions (Battjes 1988; Banner & Peregrine 1993). In recent years, surf-zone breaking waves (Kimmoun & Branger 2007) and bow waves (Shakeri, Tavakolinejad & Duncan 2009; Noblesse *et al.* 2008) have received considerable attention due to their connection to coastal and naval applications. Mathematically, this breaking is an example of singularity formation: the equations are derived in the small slope limit but their solution develops beyond the vertical (Pomeau *et al.* 2008*a,b*). Physically, breaking waves play a major role in energy dissipation (Jessup *et al.* 1997) and air entrainment across the interface (Deane & Stokes 2002). Examples of breaking waves are shown in figure 1, and in top and sideview in the figure by the title. We observe in figure 1(*a*) that beyond the vertical, the tip of the wave plunges towards the front. The jet impact is visualised in figure 1(*b*). The article by Gomez-Ledesma *et al.* (2011) is dedicated to a precise experimental and theoretical study of the resulting cavity. We discuss here the link between their nice study and other types of transient cavities.

2. Overview

Three examples of cavities are presented in figure 2. In the first sequence (figure 2*a*), extracted from Zhu, Oguz & Prosperetti (2000), a vertical cylindrical water jet of diameter $D_j = 5.4$ mm and velocity $U_0 = 1.65$ m s⁻¹, initially connects to the pool without any air entrainment. Then, a bulge is generated on the liquid jet. The first image of the sequence presents the arrival of the bulge at the surface of the pool. Subsequent images show that the impact of the bulge creates a cavity. Zhu *et al.* (2000) have characterized this type of cavity and found that the crater size D_c scales as $D_c/D_j \sim Fr^{1/4}$, where $Fr = U_0^2/gD_j$ is the Froude number. What we

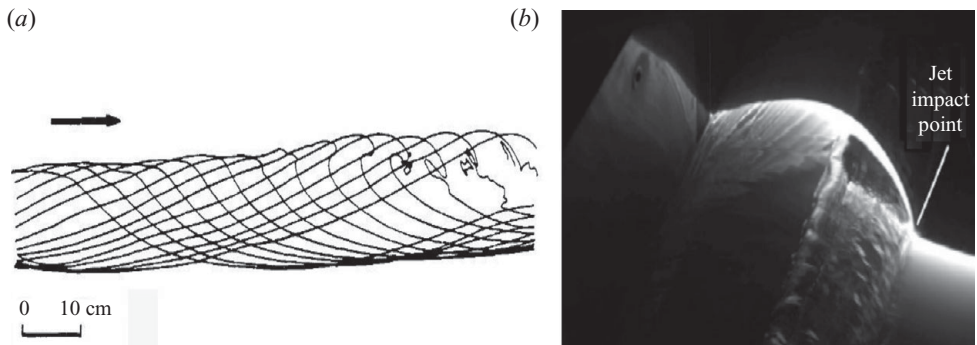


FIGURE 1. (a) Observed crest deformation of a plunging breaking wave extracted from Bonmarin (1989). (b) Image of wave breaking from Shakeri *et al.* (2009).

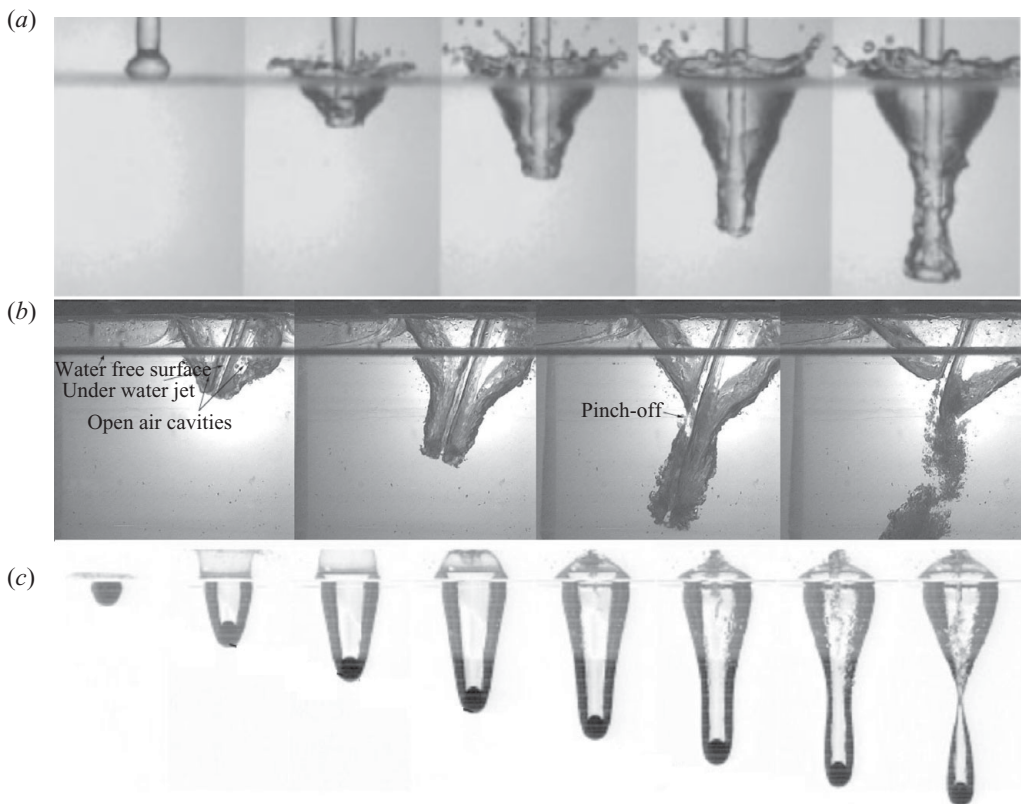


FIGURE 2. (a) Cavity created by a vertical cylindrical water jet of diameter $D_j = 5.4$ mm and velocity $U_0 = 1.65$ m s⁻¹ (from Zhu *et al.* 2000). (b) Cavity created by the impact of an inclined water sheet of thickness $T_j = 2.9$ mm and velocity $V_j = 3.32$ m s⁻¹. The liquid sheet is translating from right to left at velocity $V_l = 0.15$ m s⁻¹ (from Gomez-Ledesma *et al.* 2011). (c) Cavity created during the impact of a solid sphere of diameter $D_0 = 15.6$ mm and velocity $U_0 = 3.7$ m s⁻¹ (from Duclaux *et al.* 2007).

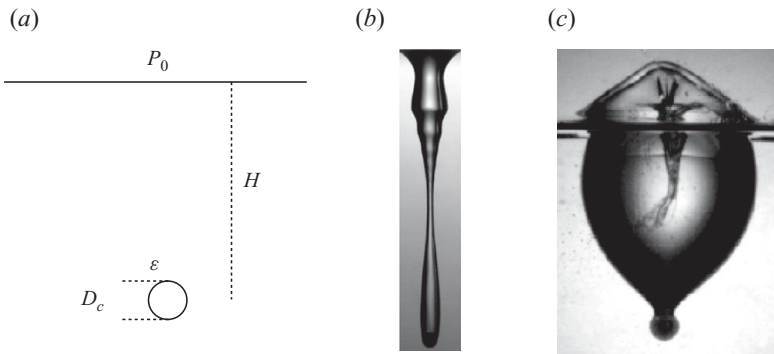


FIGURE 3. (a) Basic model developed to capture the crater size. (b) Impact of a steel sphere in water ($R_0 = 0.79$ mm and $U_0 = 3.1$ m s $^{-1}$). (c) Impact of a polypropylene sphere ($R_0 = 2.5$ mm) in a viscous pool ($\eta = 1$ Pa s) with velocity $U_0 = 28$ m s $^{-1}$.

retain here is that a bulge is needed to generate the cavity. Without the bulge, the jet connects directly to the pool. In the second sequence (figure 2*b*), we present a cavity as observed by Gomez-Ledesma *et al.* (2011). The cavity is created by an inclined liquid sheet of thickness $T_j = 2.9$ mm and velocity $V_j = 3.32$ m s $^{-1}$. In this case, the liquid sheet is also translating from right to left with velocity $V_t = 0.15$ m s $^{-1}$. The translation induces a lateral displacement of the order of $V_t \tau_t$ after the time τ_t . This displacement becomes of the order of T_j at the time $\tau_t \sim T_j / V_t$, of the order of 50 ms. One does not expect to observe an effect of the translation before τ_t . Indeed, in the first two images, the cavity is symmetrical with respect to the liquid sheet. The symmetry breaks in the third image and the pinch-off occurs on the ‘front’ side of the cavity. Even though the general shape of the cavity is close to that observed with cylindrical jets, Gomez-Ledesma *et al.* (2011) report and characterize the sensitivity of the shape of the cavity to both the tilt angle and the translation velocity. Finally, in the last sequence extracted from Duclaux *et al.* (2007) (figure 2*c*), we present the cavity created by the vertical impact of a solid sphere of diameter $D_0 = 15.6$ mm and velocity $U_0 = 3.7$ m s $^{-1}$. There again, the cavity is wider at the interface and pinches off for some critical elongation. The authors have shown that the size of the crater also scales as $D_c / D_0 \sim Fr^{1/4}$. A nice discussion of this type of cavity can also be found in Bergmann *et al.* (2009).

A common feature of the above three cavities is that they are all created by a local input of energy: the impact of the bulge in figure 2(*a*), the impact of the tip of the liquid sheet in figure 2(*b*) and the impact of the solid sphere in figure 2(*c*). We use this observation to develop a small argument on the size of such plunging cavities. The model is presented in figure 3(*a*). The energy \mathcal{E} is deposited below the interface at the depth H . The size of the crater D_c which results from this ‘explosion’ results from the balance $\mathcal{E} \sim \rho g H D_c^3$. If the energy \mathcal{E} is deposited by an impactor (liquid jet or solid) of velocity U_0 and size D_0 , we evaluate $\mathcal{E} \sim \rho U_0^2 D_0^3$. The cavity size can finally be found by taking the limit $D_c \sim H$. The energy balance leads to the scaling $D_c / D_0 \sim (U_0^2 / g D_0)^{1/4}$, which has been reported by Zhu *et al.* (2000) and Duclaux *et al.* (2007). A more elaborate model is developed by Gomez-Ledesma *et al.* (2011), which captures all the features of the experiment up to pinching.

3. Future

In the case of solid spheres, Aristoff & Bush (2009) have shown that some surprises occur with sizes smaller than the capillary length. In this limit, surface tension can no longer be neglected and the cavity shape undergoes spectacular changes, as shown in figure 3(b). When the viscosity of the pool is markedly increased, Le Goff, Quéré & Clanet (2011) report a new type of cavity where the sphere stops on a time scale shorter than the cavity collapse. In this limit, the cavity is no longer elongated as shown in figure 3(c). Both small size jets and flows with viscous pools are possible future avenues of research for liquid sheet cavities. However, in the context of a bow wave and plunging jets, a more probable future research direction is the characterization of the bubble size which results from the cavity collapse and the depth to which these bubbles are entrained by the jet.

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