THEORETICAL CONSIDERATIONS OF THE WARP AND THICKENING

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1. INTRODUCTION

As a theoretical subject, the study of the bending of the gaseous planes of galaxies has not been extremely popular. It seems that this is due to an excellent and thorough paper on the subject, published by Hunter and Toomre in 1969. At that time there existed three (or four) rival theoretical mechanisms:

(1) vertical oscillations of the galactic disk, considered first by Lynden-Bell (1965);
(2) asymmetrical pressure on the disk due to intergalactic wind (Kahn and Woltjer 1959);
(3) tidal influence of the Magellanic Clouds, from
  (a) the case of Stationary Clouds, and
  (b) the case of Clouds orbiting around the Galaxy (Elwert and Hablick 1965, Avner and King 1967).

The case 3a was, of course, of no interest really, because the Clouds have to orbit somehow. Moreover, it was shown by Burke (1957) and by Kerr (1957) that the amplitude of the effect is inadequate.

Hunter and Toomre eliminated also the first two possibilities and concluded that the observed bending can be explained only by a comparatively recent and close passage of the Magellanic Clouds. This subject could not, in general, be handled analytically, but it seemed that the number of degrees of freedom inherent in the tidal picture could provide an explanation for any observed picture. So the widespread interest in the problem quickly vanished. By the way, the first more or less complete tidal models of the bending appeared only in 1976 (Fujimoto and Sofue).

What has changed now, compared to the year 1969, to justify reinvestigation of the problem? The obvious answer is the discovery of (or rather a sound belief in) the massive outer halos (or coronas) of galaxies. As far as we know the coronas become dynamically dominant just in the outer parts of galaxies, where bending occurs. The introduction of coronas has suggested a few new possibilities. These mechanisms and the re-evaluation of the old ones will be the subject of the next sections.

W. B. Burton (ed.), The Large-Scale Characteristics of the Galaxy, 513 522.

2. SPHERICAL CORONA

The simplest distribution of matter in a corona is a spherical. It reflects, of course, only the fact that we do not have a sufficient number of luminous test particles to determine the detailed density distributions. So, let us assume a spherical corona for this section. The problem is: how do the three mechanisms listed above function if a galaxy is immersed in a corona?

2.1. Vertical Oscillations of the Disk

As the corona becomes gravitationally dominant near the edge of the disk (the phenomenon of constant rotational velocities), the basic approximation is that of test particles in an external gravitational field, moving in ring orbits (cold disk). Rink orbits are stable in a spherical field (e.g. Polyachenko and Fridman 1976), and so the vertical oscillations of our disk are stable too. To obtain a distorted disk we can apply, following Hunter and Toomre (1969), a lopsided impulse vertical force

$$F_{imp} = f(r) \cdot \cos\theta \cdot \delta(t), \qquad (1)$$

where r and θ are the inertial polar coordinates in the plane of the disk and f(r) is any smooth function. As a result, the particles will populate new orbits, inclined to the old ones, and the resulting response can be written as

$$h(r,\theta,t) = \frac{f(r)}{v_{rot}(r)} \cdot r \sin \Omega t \cdot \cos(\theta - \Omega t)$$

$$= g(r) \{\sin\theta - \sin[\theta - 2\Omega(r)t]\}, \qquad (2)$$

where h is the height of the distorted disk above the old one and $\Omega = \Omega(r)$ is the angular velocity. The frequencies of oscillation form a continuous spectrum,

$$\omega \varepsilon [\Omega(r_i), \Omega(R)], \qquad (3)$$

where $r_i < R$ is the limiting radius of our approximation and R is the outer edge of the disk. This means that the initial impulse excites a definite number of modes. As the different modes have different evolution rates, the response as in eq. (2) does not approach a stationary limit with time: all the shapes of the disk are transitory. Because of the differential rotation the initial shape becomes more and more corrugated, the number of modes, N, in any radial direction growing with time. Assuming a constant linear velocity, v_{rot}, this number increases by one every interval

$$\tau = \frac{dN}{dt}^{-1} \approx \frac{\pi r_i}{v_{rot}} \approx 3 \cdot 10^8 \text{ years}, \qquad (4)$$

where the numbers are from the recent model of our Galaxy with a corona (Einasto, Joêveer, and Kaasik, 1976). Thus, during the orbital period of the Magellanic Clouds the initially smooth plane has developed about ten maxima and minima and is better described as a thickened plane.

The gaseous component of the disk is not self-gravitating, but feels the potential caused by the massive flat disk of stars. So, maybe it is the stellar disk that is bent, and maybe its self-gravitation can stabilize differential rotation? This problem was considered in detail by Hunter and Toomre (1969), and by Hunter (1969). They studied the limiting cases of totally self-gravitating disks and found that the edges of disks will almost always spoil the picture. There, the gravitational influence of the other parts of the disk is minimal, and, in order to possess only a discrete spectrum of oscillations, the surface mass-density, μ, near the edge, at R, has to behave as

$$\mu(r) \sim (R-r)^{1/2}. \qquad (5)$$

For more realistic edges, where locally $\mu(r) \sim \delta r = R-r$, there always exists a range of continuous spectra, caused by differential rotation, which cannot be neutralized near the edge (Hunter and Toomre, 1969; Polyachenko and Fridman, 1976).

As noted by Hunter and Toomre (1969), factors tending to diminish the self-gravitation of the disk only worsen the situation. Evidently a massive spherical corona works in the same sense. And, indeed, supposing the halo effects to be small, it can be shown that the region of continuous spectrum widens with the growing role of halo. Consequently, this mechanism does not function in the case of heavy halos.

2.2. Intergalactic Wind

The failure of vertical oscillations as a bending mechanism led Kahn and Woltjer (1959) to propose a gas dynamical explanation. They supposed the Galaxy to be surrounded by a gaseous halo and estimated the pressure differences at its boundary due to intergalactic gas flow around the halo. This mechanism leads to a qualitatively correct picture, if one supposes a rather "rigid" halo. It is clear that potential wells generated by massive halos can serve as containers for the gaseous halos and can provide high temperatures for the gas, leading thus to high sound velocities.

Let us disregard all the difficulties listed by Hunter and Toomre (1969) and by Binney (1977) and estimate the resulting displacement of the Galactic gas layer.

To obtain the upper limit on the displacement we shall retain the value of $\approx$ 10% for the pressure differences at the boundary (Kahn and Woltjer, 1959) and take for the pressure the maximum value for the interstellar medium, $p_m = 2 \cdot 10^4$ k, suggested by Shapiro and Field (1976). Kahn and Woltjer supposed that the total pressure difference $\Delta p = 2 \cdot 10^3$ k is applied directly to the gaseous disk of the Galaxy. This, evidently, cannot be done, because the pressure differences have to cause a pressure gradient throughout the whole corona. So the pressure difference acting on the galactic gas (we suppose it for the moment to be homogeneous) is seriously reduced and the equilibrium of pressure forces and the gravitation of the stellar component leads to the amplitude of bending

$$h = \frac{\Delta p}{R_{cor}\ \alpha\ \rho_G}\ , \qquad (6)$$

where

$$\alpha = K_z/z\big|_{z\ =\ 0}. \qquad (7)$$

Here R_{cor} is the outer radius of the gaseous corona, ρ_G is the gas density in the galactic disk and K_z the vertical acceleration. For R_{cor} = 300 kpc and our Galaxy, this height works out to $h \approx 1.3$ pc at r = 15 kpc, which is 10^{-3} times the observed value.

2.3. Tidal Forcing

Thus there remains, as the last hope to produce warped layers, tidal forces during the close passages of the companion galaxies. It turned out to be rather difficult to model the warp in our Galaxy: we saw the first results only in 1970 (Toomre). Here I can show you the most recent, and excellent, theoretical model (Fig. 1) of a warp (Spight and Grayzeck, 1977). They used an enormous number of test particles (12,000), a retrograde elliptic orbit ($\varepsilon = 0.5^5$) of the Large Magellanic Cloud, and a perigalactic distance of 20 kpc. The closest passage had to occur 4 10^8 years ago. So, all seems to be natural?

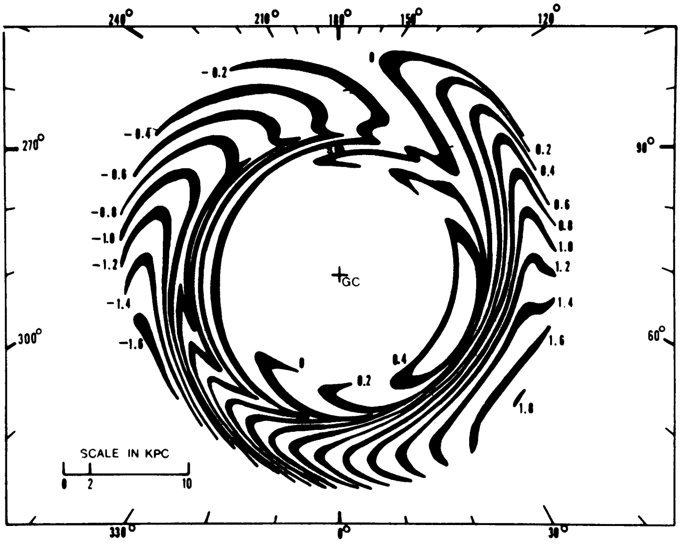

Figure 1. Relief map of the distorted disk after Spight and Grayzeck (1976). The z-distances are given in kpc with respect to the galactic plane.

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There are only some minor points to worry about, some dark clouds on the horizon.

First, the gravitational field used by Spight and Grayzeck is spherical; there is no flat component. The degree of isotropy of a force field can be described by the ratio γ of the frequency of z-oscillations Ω_z to the angular velocity of rotation:

$$\gamma = \frac{\Omega_z}{\Omega} = \left(\frac{K_z \cdot r}{K_r \cdot z}\right)^{1/2} . \tag{8}$$

It approaches unity in the case of isotropy. The model of the Galaxy by Einasto, Joêveer and Kaasik (1976) gives in the region shown in Figure 1, $\gamma \varepsilon [2.4, 1.2]$ for $z = 0$ kpc and $\gamma \varepsilon [2.0, 1.2]$ for $z = 1$ kpc.

Second, the mass of the LMC was taken to be $3 \cdot 10^{10}$ $M_\odot$--about two times larger than the observational limit. Third, for so close a passage there arises the problem of disruption of the LMC-SMC system.

Fujimoto and Sofue (1976, 1977) tried to model the Galactic warp, considering all the points made above, and their pictures are not half as nice as Figure 1. So it seems that the model warp shown here belongs rather to some other Galaxy with a more centrally condensed corona than ours.

Summing up, the only mechanism that benefits by the introduction of spherical coronas is that of tidal forcing. The required amplitudes are simpler to produce and the condition $\gamma \to 1$ assures that there remains only one oscillation frequency at every radius instead of two different ones. But we have not answered the problem posed by Sancisi (1976)--how are warps produced in galaxies without companions?

3. NONSPHERICAL HALO

Considering the formation of massive halos we see that there is no reason for them to be spherical. Both the dissipative gravitational collapse picture (Doroshkevich, Sunyaev, and Zeldovich 1974) and the non-dissipative one (Aarseth and Binney 1978) give rise to triaxial mass distributions. Such a halo can lead to pleasant surprises when considering the problem of bending. This was realized recently by Binney (1977).

3.1. Free Oscillations

Binney represented his triaxial ellipsoid, for simplicity, as a superposition of two spheroids with perpendicular rotation axes. To be more specific, let us consider one of them oblate--this would be the conventional massive galaxy, bulge and all. The other one could be an oblate cake or prolate cucumber with the major axis in the plane of the first spheroid.

Representing now the gaseous disk by test particles in circular orbit, we find that they are moving in a periodic potential; there appears a kind of periodic driving force. For free vertical oscillations Binney obtained an equation

$$\frac{d^2h}{dt^2} + \Omega^2(r)\ \{1+b(r) - q(r)\ \cos\ [2\Omega(r)t]\}\ h = 0 \qquad (9)$$

and demonstrated that it has regions of resonance, which leads to growing modes. This occurs whenever the two spheroids are both of the same shape and of comparable eccentricity. If those conditions are not satisfied, vertical oscillations cannot be coordinated, and we obtain once more a thickened disk. By assuming the "perturbing" spheroid to be rotating, the condition for resonance can be satisfied only in a specific range of radii.

So you can see yourself what can be done with two spheroids only!

One has not to possess second sights to predict that this work will open a wide field for people who love to derive dispersion relations, calculate eigenfunctions, and so on. As we have got our instability at last, there rises the nonlinear problem of final amplitudes, and, as we enter into the nonlinear domain, the problems of interaction of planar and vertical perturbations (bending and spiral structure), pumping of energy and so on, become important. It may really be a major breakthrough in the problem.

3.2. Hydrodynamical Models

The picture of halos proposed by Binney is extremely fruitful. Namely, it allows me to introduce the notion of accretion layers (Jaaniste and Saar 1976) and to divide the responsibility with Binney. Let us see: if we consider it natural that the plane of symmetry of one spheroid is populated with gas, then what about the other spheroid? We have listed a number of examples where such perpendicular gaseous planes can be suspected (Jaaniste and Saar 1976); additional examples are given by Gunn (1977). The gas in both layers has a cloudy structure, which seems to be best described by the recent model of McKee and Ostriker (1976), but, of course, the "extragalactic" gas is more rarefied, and the number of warm clouds is smaller. Bending can be produced now by the crude interaction of the two planes.

3.2a. Perpendicular rotation. It is difficult to imagine a mechanism causing rotation in perpendicular planes, but if we suppose it possible we come to the models of bending constructed by Haud (1977). He used the simple "snowplow" approximation for calculating the impulse changes of intergalactic clouds falling through the galactic plane. His models are numerical ones, as he used a realistic model of the gravitational potential and followed the clouds through many collisions. But to obtain an estimate of the amplitude of bending we can discard all the finessess and write

$$h = \frac{v_{cloud} \cdot r}{v_{rot} \cdot \phi(r)} \sin\left[\theta_o - \frac{\Omega_z(r)}{\Omega(r)}\theta\right], \qquad (10)$$

where θ_o defines the intersection region of two planes and

$$\phi_{(r)} = \frac{\sigma_{gal}}{\sigma_{cloud}} \qquad (11)$$

is the ratio of the surface densities of the (diffuse) gas in the galactic plane and of the infalling clouds. This ratio defines, in fact, the radial dependence of the amplitude of the warp. As it approaches zero in the outer regions of the disk, any bending amplitudes can be explained, in principle. In order to get a correct angular dependence one has to work near the halo region $\Omega_z \approx \Omega$, but this condition is not so strong as in the case of tidal forcing.

3.2b. Asymmetrical thickening. This variant of hydrodynamical forcing was proposed by Jaaniste (1977). He dropped the assumption of rotating accretion planes, but assumed inclined accretion layers. Now the infalling clouds will oscillate at both sides of the galactic gas, producing the thickening of the plane. But the clouds moving in retrograde orbits will lose more of their momentum than those in direct orbits. This leads to an asymmetry of the thickening. If the angle between the two planes α does not differ much from $\pi/2$ and $v_{rot} \approx v_{cloud}$, the thickness of the gas layer is

$$\Delta h = \frac{r \sin \alpha}{\sqrt{2}\ \phi} \sin (\vartheta_0 - \vartheta) \quad (12)$$

and the amplitude of the warp

$$\bar{h} = \frac{1}{2} \Delta h \cot \alpha . \quad (13)$$

A typical result of density distributions produced in this way is shown in Figure 2. We note that it is the only mechanism predicting both bending and thickening to occur simultaneously.

Both hydrodynamical mechanisms predict a cloudy, irregular picture of the outer disk. But they need more input information than the case of vertical oscillations. A crucial, but unknown, factor is the degree of energy dissipation in collisions, for example.

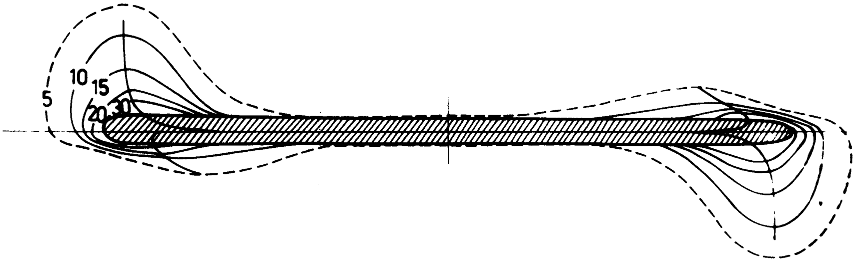

Figure 2. Total hydrogen density map for NGC 5907 after Jaaniste (1977). The densities given are in units of 10^{19} cm^{-2}, the tilt angle α = 45°, infall velocity v_{cloud} = 160 km sec^{-1}.

4. CONCLUSION

We have now once more three (or four) different possibilities to explain bending:

(1) tidal forcing has remained,
(2) instabilities of vertical oscillations in triaxial ellipsoids has emerged, as has
(3) gas-dynamical interactions with
  (a) perpendicular rotation and with
  (b) inclinded disks (or layers).

All of them can be functional given appropriate initial conditions. It is not simple to discard a priori any of them. The final words remain to be said by observers.

And, when fitting our models with observations, we have to choose with care the right things to fit. So, we do not know really the $\rho(h,\theta,r)$ dependence of hydrogen in the Galaxy to give observational plots like the theoretical one shown in Figure 1. From observations we get the functions $\rho(b,\theta,v)$, where the $r = r(v)$ dependence is fixed more by the mechanism of bending than by the undisturbed circular velocity law. The latter could be used only in case of a theory of small vertical oscillations in an axisymmetric galaxy. Both the tidal and hydrodynamic mechanisms lead to large perturbations of radial velocity, and radial oscillations are usual in disks immersed in the axial ellipsoids (Binney 1977). Thus only observed and model $\rho(b,\theta,v)$ profiles should be compared, as has been done by Fujimoto and Sofue (1976, 1977). It is worth displaying one of their figures to demonstrate the complexity of such diagrams, the velocities typical for tidal pictures, and the need for additional observations.

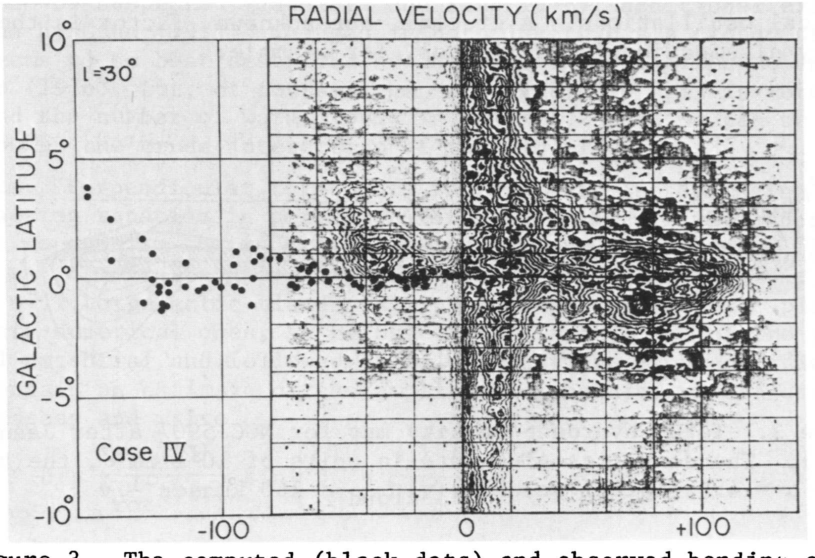

Figure 3. The computed (black dots) and observed bending of the galactic disk on the latitude-radial velocity diagram at L = 30°, after Fujimoto and Sofue (1977).

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DISCUSSION

Sanders: What is the halo-mass to disk-mass ratio in your model which accounts for the persistence of a galactic warp.

Saar: This warp is generated in a spherically symmetric potential.

van Woerden: A washboard effect is seen in the deep photograph of the SO galaxy NGC 4762, the only warped edge-on galaxy in the Hubble Atlas.

Kerr: Your point that radial displacements would occur is an important one. Have you computed the size of such displacements?

Saar: Yes. In case of the model 3A (inclined accretion layer), the systematic deviations of radial velocities were about 40-60 km s^{-1}. This would lead to displacements in radii of 4-5 kpc, depending slightly on the type of smooth rotation curve adopted.

Giovanelli: What sort of accretion rates do you need for your models 1978.3 and 1978.4?

Saar: About two or three solar masses per year. Chemical evolution theories with infall need a little more.

Basu: At large distances from the galactic center, modes of density waves become unstable. Instability of density waves is likely to thicken the disk, which may in turn initiate the warping of the disk. So warping may be associated with the propagation of density waves.

Saar: It seems likely that nonlinear planar oscillations will generate vertical oscillations. But in order to get a warp you must tune up oscillations at different radii. It cannot be done, using the total self-gravitation of the disk, and it seems that density waves also cannot manage it.

Lynden-Bell: It is a flaw in your model that the halo has been taken exactly spherically symmetrical in a problem where departure from that symmetry is a crucial matter. Do you know what would happen to a self-gravitating disk placed at an angle to a somewhat non-spherical halo? If most perturbations eventually lead to a thickening of the disk edge, do we see more galaxies with thick edges than with warped edges?

Miller: F. Hohl has constructed some computer models such as you described: a self-consistent disk and bulge combination, that remained reasonably stable. The disk was not as thin as we would like for disk galaxies (axis ratios around 1:5), but they were reasonably thin for his grid. The disk thickened a bit from the starting condition. However Hohl did not do any experiments with a warp in the disk.

Cesarsky: I wonder if primordial tilts are a likely occurrence.