

# Robust Design Optimization of Mechatronics Systems: Parallel Electric Drivetrain Application

A. Rosich , C. López, P. Dewangan and G. Abedrabbo

Flanders Make, Belgium

 albert.rosich@flandersmake.be

## Abstract

This paper addresses the problem of finding a robust optimal design when uncertain parameters in the form of crisp or interval sets are present in the optimization. Furthermore, in order to make the approach as general as possible, direct search methods with the help of sensitivity analysis techniques are employed to optimize the design. Consequently, the presented approach is suitable for black box models for which no, or very little, information of the equations governing the model is available. The design of an electric drivetrain is used to illustrate the benefits of the proposed method.

*Keywords: robust design, design optimisation, uncertainty, mechatronics*

## 1. Introduction

The design engineer is typically confronted in making a set of choices in the early stage of the design when not all the required information is yet available. However, the result of these choices might have a significant impact on the subsequent development of the product and ultimately on the performance of the resulting design. Therefore, handling this uncertain information correctly at the beginning of the design phase is paramount to avoid extra costs and times in redesigning the product at the end of the development life cycle. For this reason, methods and tools to aid the design engineer in this regard are needed.

The design process of a mechatronic system can be divided in several level or phases being: i) topology generation, ii) topology optimization, iii) technology & size optimization and, iv) optimal control (Silvas et al., 2016). Although all the levels are important, this paper focus on the forth level, on the sizing of the components for a given technology and an already-fixed topology, with the aim to equip the design engineer with a methodology to optimize the design when uncertainty is present, also known as optimal robust design (Zang et al., 2005). In particular, robust optimization with deterministic uncertainty (Goerigk and Schöbel, 2016) will be considered, instead of the more conventional approaches using stochastic uncertainty (Sahinidis, 2004; Hamzaçebi, 2020), to make the method more realistic and better suited for the design of new product for which stochastic information is typically not yet available. It is well know that robust optimization with deterministic uncertainty suffer from computational tractability for which reformulations are required (Gorissen et al., 2015). In addition, these methods can also benefit from sensitivity analysis (Hamel, 2010) to reduce the complexity of the problem. Both techniques, problem reformulation and sensitivity analysis, will be employed in the first part of this paper to develop a new method with the aim to efficiently optimize the design when the worst-case scenario is implicitly or explicitly considered (Beyer and Sendhoff, 2007).

The second part of the paper will focus on a practical case based on the design of a drivetrain for an electrical vehicle and how the proposed method can be applied to it. The design of drivetrains for

vehicles involving electrical motors is a well-researched topic (Bayrak *et al.*, 2015; Silvas *et al.*, 2016; Son *et al.*, 2017). Nevertheless, little attention has been paid to the robust aspects of the design optimization.

## 2. Problem statement

In the proposed formulation, parametric models describing the behaviour of the system are assumed to be available and to be used to compute key design performance attributes for which the designer would like to optimize. Besides, the model will be also used to impose design constraints which are typically given in form of design specifications. Therefore, the goal is to choose the optimal value of those model parameters that define the size of the components to be considered in the design exercise.

In this type of problem formulations, two classes of model parameters are distinguished: i) design parameters,  $d$ , for which the values can be freely chosen, and ii) uncertain parameters,  $u$ , where the exact values are unknown and cannot be freely chosen (Schuëller and Jensen, 2008). Lower and upper bounds for both types of parameters must be provided beforehand. These bounds will define the design space,  $D$ , in the case of the design parameters and the uncertain space,  $U$ , in the case of the uncertain parameters (Klir and Filger, 1998).

Let  $f(d, u)$  be a function to compute the performance of the system for a given set of design and uncertain parameters,  $d$  and  $u$ , respectively. And, let  $h(d, u)$  be a set of functions to compute certain attributes that must fulfil given design specifications. Then the robust optimization problem can be mathematically formulated as below (Equation 1).

$$\min_{d \in D} f(d, u) \quad \text{such that} \quad h(d, u) \geq 0; \quad \forall u \in U \quad (1)$$

Solving such optimization problems is not trivial in general. This is because the resulting optimal design,  $d^*$ , must fulfil the constraints  $h(d^*, u) \geq 0$  for all the possible values of  $u$  (i.e., all the values in  $U$ ). In the literature, there are several well developed techniques to solve this problem for a particular subclass of optimization problem such as linear programming, quadratic programming or semidefinite programming (Ben-Tal *et al.*, 2009). However, the general case implies to evaluate the design for all values in the continuous uncertain space  $U$  which is obviously not possible. In practice, there are two approaches to circumvent this difficulty and limit the number of evaluations in the uncertain space: i) provide the worst case scenario beforehand and ii) discretize the uncertain space.

### 2.1. Worst-case scenario

In some occasions, the designer might already know for which values of the uncertain parameters, the design will underperform the most. In these cases, there is no need to consider the full uncertainty space. More formally, let  $u_{wc} \in U$  be a known value such as  $f(d, u) < f(d, u_{wc})$  and  $h(d, u) > h(d, u_{wc})$  for all  $d \in D$  and for all  $u \in U \setminus \{u_{wc}\}$ , then the following formulation in Equation 2 is equivalent to the formulation in Equation 1.

$$\min_{d \in D} f(d, u_{wc}) \quad \text{such that} \quad h(d, u_{wc}) \geq 0; \quad u_{wc} \in U \quad (2)$$

Note that this is a conventional non-linear optimization problem and  $u_{wc}$  is typically referred as the worst case scenario (Gorissen *et al.*, 2015). Although finding the (global) optimal design might not be straightforward, the complexity nature of this optimization problem is significantly lower than the one described in Equation 1. In fact, the optimization problem in Equation 2 can be directly solved by using out-of-the-shelf non-linear optimization solvers (Martins and Ning, 2021).

Employing the worst-case scenario approach is a common practice in design system engineering, whether optimal design is sought or not. However, assuming the worst case scenario to be known beforehand has its own risks. While it is true that sometime the worst case scenario can be inferred for simple models in one domain, this might not be the case with multidomain complex models typically developed by different specialists. In these latter cases, the designer might overlook the behaviour of the model and might not be able to deduce the correct worst case scenario leading to a design which is not robust.

## 2.2. Uncertain space discretization

An alternative approach, especially when the worst case scenario is assumed to be unknown, is to discretize the uncertain space with a number of samples. As a result, the initial problem (Equation 1) is transformed in a new one (Equation 3),

$$\min_{d \in D} f(d, u) \quad \text{such that} \quad h(d, u) \geq 0; \quad \forall u \in \{u_1, u_2, \dots, u_n\} \quad (3)$$

where now, the continuous uncertain space in Equation 1,  $U = [\underline{u} \quad \bar{u}]$  with  $\underline{u}$  and  $\bar{u}$  being the lower and upper bounds of the uncertain parameters, is replaced by its discrete counterpart  $U = \{u_1, u_2, \dots, u_n\}$  with  $n$  samples.

It should be noted that in this case, the formulation is not equivalent to the one in Equation 1 since the solution will be optimal and fulfil the constraints only for a subset of values. Consequently, there is no guarantee that the solution will be robust for the entire uncertain space nor it will be globally optimal. The number of samples and their distribution is critical and play an important role between solution robustness/optimality and computational efficiency. Obviously, the more samples are used, the better the uncertain space is covered, and hence more guarantees to find the robust and global optimal solution. On the other hand, as the number of samples is increased, longer time are needed to compute the solution.

## 3. Proposed optimal robust design method

In this section, a novel method that combines the two previously introduced approaches is presented. The proposed method leverages the available computational resources by providing relevant information to the design engineer. In this sense, he or she can make educated guesses on whether an uncertain parameter can be fixed as a worst case scenario, must be fully sampled or can simply be disregarded. The method consists of three main steps as depicted in the Figure 1.

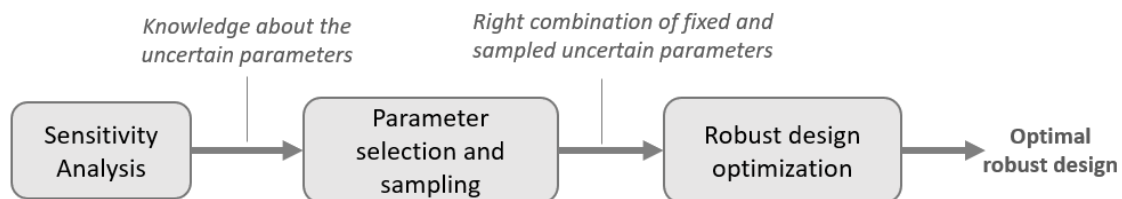


Figure 1. Three steps of the proposed Robust Design Optimization method

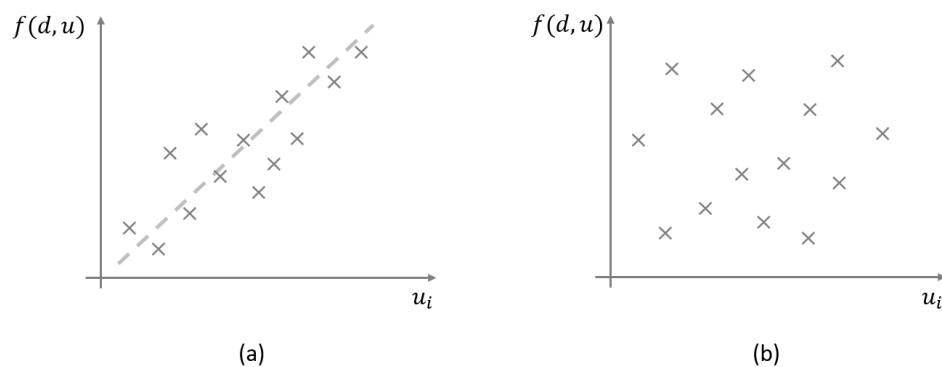
The first step of the proposed method is to gain knowledge of the uncertain parameters, in particular about their influence on the attributes, i.e., the design performance  $f(d, u)$  and the design constraints  $h(d, u)$ . To this end, global Sensitivity Analysis (SA) is employed over the entire design and uncertain spaces,  $D$  and  $U$ . Based on the SA outcome and considering the available computational resources and desired computational time, some uncertain parameters can be effectively fixed to a single value, while others need to be sampled during the optimization. The next subsections provide further details about this three steps.

### 3.1. Sensitivity analysis

Global sensitivity analysis is the collection of methods and techniques (Saltelli, 2008) that allow to study how the uncertainty in the model influences the output variability of the model, in our case, the variability of the design performance,  $\partial f$ , and the variability of design constraints,  $\partial h$ . The contribution of each parameter to the output is typically evaluated by means of the sensitivity index which provides a quantifiable metric. As a result, the set of uncertain parameters can be ranked based on their sensitivity index. Then, those parameters with a higher contribution (i.e., high sensitivity index values) can be identified and properly handled in the robust optimization. On the other hand, uncertain parameters with a low sensitivity index can be fixed and should be disregarded if computational time constraints are not met (Iooss *et al.*, 2015). In the proposed methodology, it is upon the designer, based on how quick a solution is desired, to set the sensitivity index threshold for which parameters are need to be considered or can be disregarded.

To perform the sensitivity analysis, the user must predefine the sampling method that is going to be used to generate the samples, as well as the type of index to be computed for each sample. In the literature, there are multiple sampling techniques (Glynn and Iglehart, 1989) and types of sensitivity indices (Cotter, 1979; Kucherenko *et al.*, 2012; Morris, 1991; Sobol, 1993) that can be used. In our case, an adaptation of the Latin Hypercube sampling method (McKay *et al.*, 1979) (where the bounds of all the uncertain parameters are also sampled) and Kucherenko sensitivity index (Kucherenko *et al.*, 2012) have been selected for the parallel electric use case detailed in the Section 4.

Besides the sensitivity indexes, extra knowledge can be inferred from the evaluations performed during the sensitivity analysis. Note that the set of samples is extracted from the combined design and uncertain spaces, i.e.,  $\{s_1, s_2, \dots, s_n\} \in D \times U$ . Each sample, containing design and uncertain parameters, is used to evaluate the performance and constraints, i.e.,  $f(s_i)$  and  $h(s_i)$  for  $i = \{1, 2, \dots, n\}$ , and in turn, to compute the sensitivity indices. If the set of evaluations are projected in one dimension of the uncertain set of parameters, it is possible to see if some values of the uncertain parameter under inspection have a major impact on the degradation of the performance and constraints, independently of the values of the other design and uncertain parameters. Figure 2 shows two possible scenarios of projections of the design performance evaluations onto the uncertain parameter  $u_i \in u$ . In the scenario (a), there is a clear trend indicating that larger values of the uncertain parameter  $u_i$  degrade the performance (in case of performance minimization), hence the user might consider to fix the uncertain parameter to its upper bound,  $\bar{u}_i$ . On the other side, scenario (b) shows no trend and therefore all values are equally relevant, suggesting that the full interval must be considered during the robust optimization.



**Figure 2. Projection with a clear trend (a); No trend in the projection (b)**

Sensitivity indexes and trends in the projections are used as extra knowledge to assist the designer in the task of selecting which uncertain parameters will be considered in the robust optimizations, and among the selected ones, which ones need to be sampled and which ones can be fixed. It should be stressed that this extra information is meant as a guide for the designer to make an educated guess without guarantees, which is in general better than making the selection randomly, or based on wrong assumptions.

### 3.2. Sampling of uncertain parameters

As in the sensitivity analysis, uncertain parameters that are not fixed, can be sampled in multiple ways. If no a priori information is assumed on how the values of the uncertain parameters influence the design performance and constraints, then a logical sampling strategy is to uniformly grid each uncertain interval with  $N$  samples. However, the number of samples with this strategy exponentially grows with the number of uncertain parameters to be sampled as indicated in Equation 4 where  $|w|$ , with  $w \subseteq u$ , denotes the number of uncertain parameters to be sampled.

$$\text{Number of samples with gridding} = N^{|w|} \quad (4)$$

An alternative strategy is the use of Latin Hypercube Sampling techniques (McKay *et al.*, 1979) which can efficiently cover the uncertain space to be sampled with a predefined number of samples,  $N_{LHS}$ . The handicap of this strategy is that upper and lower bound values are not added in the sampling set by default. Based on our experience, the extreme values of the interval are critical values that should be always

considered. Consequently, these values are added to the sample set after generating the samples with the LHS method yielding to the total number of samples indicated in the following expression (Equation 5).

$$\text{Number of samples with LHS plus extreme values} = N_{LHS} + 2^{|w|} \quad (5)$$

After testing both sampling strategies with the parallel electrical vehicle case, similar results were obtained between using gridding strategies with large number of samples and LHS plus extreme values with a significant smaller set of samples. Therefore, based on our experience, we advocate the use of the LHS with the inclusion of extreme values as a sampling method during the robust optimization.

### 3.3. Robust design optimization

Once the set of parameters to be sampled is identified, the next step is to launch the robust optimization algorithm that computes the solution of the problem in Equation 1. For convenience, the original problem is reformulated as follow (Equation 6),

$$\min_{d \in D} f(d, v, w) \quad \text{such that} \quad h(d, v, w) \geq 0; \quad \forall w \in \{w_1, w_2, \dots, w_n\} \quad (6)$$

where  $v \subseteq u$  and  $w \subseteq u$ , with  $\{v, w\} = u$ , are the subset of uncertain parameters that are fixed and the subset of uncertain parameters that must be sampled, respectively.

The optimization problem in Equation 6 is in the category of non-linear non-differentiable constrained optimization problem which can be solved with free derivative solvers that treat the function and the constraints as black boxes. Metaheuristic optimization algorithms such as Genetic Algorithms and Particle Swarm, were initially applied to solve this problem. However, we have finally opted for direct search solvers (Abramson *et al.*, 2009) that adapt the search direction, iteration after iteration, with the aim to converge to the optimal solution, offering in this way a better performance than the metaheuristics approaches. In particular, NOMAD solver (Abramson *et al.*, n.d.; Digabel, 2011) has been used to obtain the results of all the approaches of the parallel electric drivetrain case detailed in the next section.

## 4. Parallel electric drivetrain use case

The design of drivetrains for electrical vehicles is a typical application in which some design parameters need to be decided at the early stages of the design since they will determine the feasibility and viability of the design. However, also at the early stage of the design, the values of some system parameters are not exactly known, which means that the designer must consider them as uncertainties and consequently assume the worst case scenario. In this section, the design of a drivetrain for an electrical vehicle will be used as an example to first show the current state of practice and its potential risks when dealing with uncertainties, and secondly to demonstrate the advantages of the proposed methodology.

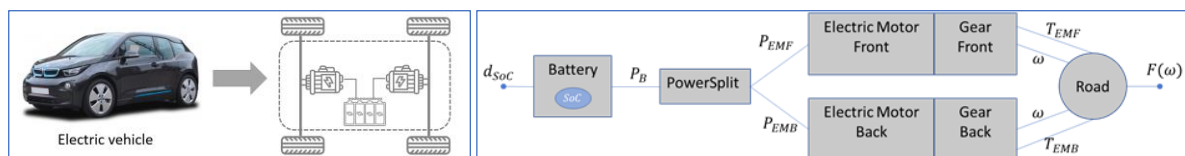


Figure 3. Parallel electric drivetrain modelling

### 4.1. Introduction of the design problem

In this example, the design a drivetrain consisting of two electrical motors, front and rear axles, is considered. The two motors are powered with a single unique battery as depicted in Figure 3. The parallel topology of the drivetrain is fixed and not subject to be changed. However, the power of the two motors and the capacity of the battery are not decided yet. The goal is to select the motor powers and battery capacity such that the normalized total cost of ownership (TCO) is minimized while a set of performance specifications are guaranteed for a specific driving cycle. More specifically, the follow specifications are regarded:



1. the maximum torque ( $maxTrq$ ) of the vehicle at slow speed (i.e., when the vehicle starts) must be 1600Nm,
2. the vehicle must be able to reach a maximum speed ( $maxSpeed$ ) of 220km/h,
3. the distance ( $range$ ) that the vehicle must be able to drive, should be at least 250km.

The designer is equipped with a model of the vehicle from which after running a simulation, the aforementioned attributes can be computed to verify if the design specifications are met. The equations governing the model are unknown to the designers and their behaviour is difficult to be inferred without a deep knowledge. For instance, the computation of the TCO depends, amongst others, on the cost of the battery in which aging, operability and the number of charging/discharging cycles are taken into account in a non-linear manner. Besides, the model is parametrized with 19 parameters that can be tuned by the designer to produce different attribute values. For the sake of space and clarity only eight parameters are considered uncertain. The design and uncertain parameters are detailed in Tables 1 and 2. The rest of the parameters have a default nominal value assigned to them and are not relevant for this design exercise.

**Table 1. Design parameters.**

Name	Description	Range
<i>batt_capacity</i>	Battery capacity	[20 80] kWh
<i>frontEM_maxpower</i>	Max. power frontal electro motor	[30 100] kW
<i>rearEM_maxpower</i>	Max. power rear electro motor	[30 100] kW

**Table 2. Uncertain parameters.**

Name	Description	Nominal value	Range
<i>base_mass</i>	Base mass	1000 kg	[800 1100] kg
<i>payload_mass</i>	Payload mass	500 kg	[300 800] kg
<i>emCostUncFact</i>	Uncertainty factor cost EM	1	[0.8 1.2]
<i>batCostUncFact</i>	Uncertainty factor cost battery	1	[0.7 1.3]
<i>batSpEngyNom</i>	Nominal specific energy battery	100 Wh/kg	[75 150] Wh/kg
<i>batSocMax</i>	Max state of charge battery	0.9	[0.85 0.98]
<i>batSocMin</i>	Min state of charge battery	0.1	[0.02 0.15]
<i>elecPriceUncFact</i>	Uncertainty factor price electricity	1	[0.5 2]

## 4.2. Modelling approach

The model used in this study is based on the methodology proposed by (Vandenhove et al., 2020). It consists of a set of components, each of which is characterized separately in an object-oriented component library. The component library contains all the information to describe the individual components, but also describes how the components can be connected to each other.

Figure 3 shows the main components of Electric vehicle model, where  $SoC$  represents the state of charge,  $d_{SoC}$  represent the maximum battery charge,  $P$  represents the output power, and finally,  $T$  and  $\omega$  represent the torque and speed. The model uses the HEV library presented by (Vandenhove, 2020) to establish the constitutive equations of each component, e.g., Battery Power ( $P$ ) is function of its capacity ( $d_{SoC}$ ) and state of charge ( $SoC$ ).

## 4.3. Solving approaches

In the subsequent sections, the optimization problem will be solved in four different approaches with the aim to demonstrate the advantage of the proposed methodology presented in Section 3. The solution of each approach is validated by sampling the uncertain space with 1000 random samples, plus the 256 extreme values of the space. It should be noted that the validation is high time consuming and should not be carried out in a real exercise. Here, it is done in order to demonstrate the consequences of pursuing each approach.

#### 4.3.1. Nominal optimization

The first approach considers the case in which the designer neglects the uncertainty and assumes the nominal values on the uncertain parameters as real ones. In this case, the optimization problem is defined with all the parameters fixed at their nominal values except for the design parameters which must be optimally determined by the optimization routine. Results of the optimization and validation are shown in Table 3 (second column).

The results in Table 3 shows that, at first glance, the optimal design fulfils all the design specifications (all constraint are fulfilled) and a good TCO is achieved, if compared with the one in the subsequent approaches. However, the validation exercise unveils the lack of robustness of the design. In fact, for some values of the uncertain parameters the TCO is worsened respect the wrongly assumed optimal, leading to an overestimation of -61%. Also, constraints are not fulfilled for all the scenarios, being the worst one the range of the vehicle.

#### 4.3.2. Robust optimization with prefixed worst case scenario

The second approach solves the problem assuming that the designer knows the worst case scenario beforehand as described in Section 2.1. Consequently, for solving this approach, the uncertain parameters are fixed as follows: *base\_mass*, *payload\_mass*, *batSocMin* and *elecPriceUncFact* are fixed to the upper bound, and *emCostUncFact*, *batCostUncFact*, *batSpEngyNom* and *batSocMax* are fixed to their lower bound of their corresponding ranges (see Table 2).

This optimization problem is in the same category of the previous approach in Section 4.2.1. Only the uncertain parameters are fixed to different values. Therefore, the same computational time is expected. The results of the optimization are shown in Table 3 (third column).

In this case, the optimal design also fulfils the design specifications. However, the resulting TCO is significant worse than in the previous case. This is because the solution found in this approach is more robust than the previous one, as it can be noticed in the validation results. Now, the optimal TCO is slightly overestimated (-2.7%) and only 4 samples do not fulfil the constraints. Although this seems a good result, the fact that for some uncertain scenarios the vehicle can only cover a distance of 226 km, instead of the specified 250 km, can be a reason to invalidate the design. Note that in the case of using this approach, the issue with the shorter range would have been detected once the vehicle is produced, with the subsequent negative impact for the manufacturer.

#### 4.3.3. Robust optimization with full sampling of the uncertain space

This approach refers to the alternative approach introduced in Section 2.2 in which the designer does not want to assume any worst case scenario and opt for fully sampling all the uncertain parameters. Table 3 (forth column) shows the result of this robust optimization.

In this case, a fully robust optimal design is found according to the validation results where no sample invalidates the design. Besides, the same worst TCO is found in the optimization and the validation, indicating that no overestimation is done. Note however, that this result is obtained with a high computational cost for which 22 minutes were needed in each iteration to evaluate the design during the optimization routine. In fact, several days were needed to let the solver to converge which makes this approach impractical.

#### 4.3.4. Robust optimization with the new methodology

The new methodology introduced in Section 3 is here applied to the parallel electrical drivetrain. First the sensitivity analysis is performed in order to obtain the sensitivity indexes and trends as described in Section 3.1. The results of the sensitivity analysis are shown in Figure 4. The four attributes are presented in rows, and plots are sorted according to the sensitivity index rank from left to right. The corresponding Kucherenko sensitivity index is displayed below each plot for completeness. In addition, trends are computed by fitting a line (in red) and a second degree polynomial (in green).

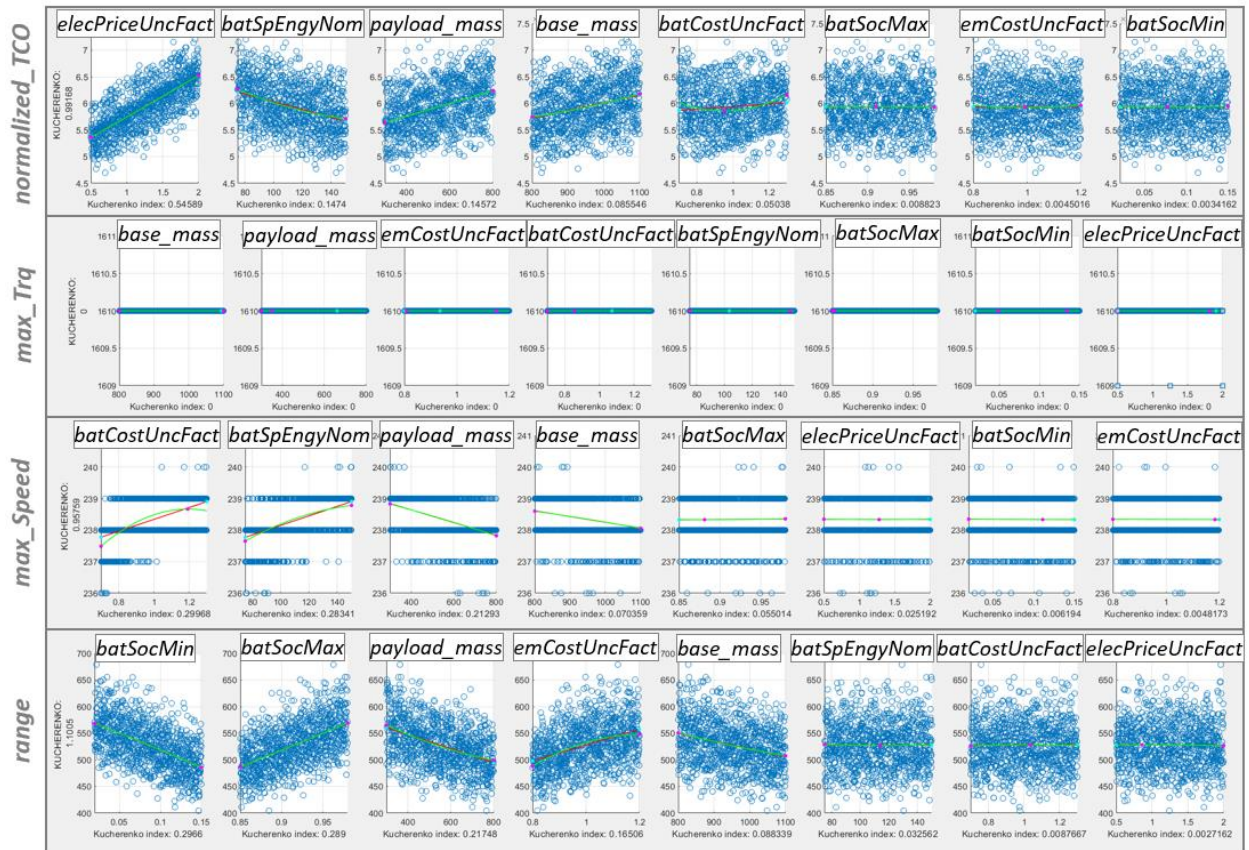


Figure 4. Sensitivity analysis for the parallel electrical drive train case.

From the sensitivity analysis, the designer can draw some knowledge on how the uncertain parameters influence the attributes. For instance, looking at the index and trend of the *elecPriceUncFact*, it is clear that this is an influential parameter for the *normalized\_TCO* and considering the upper bound is enough. This is not the case for the *base\_mass* on the *normalized\_TCO* which has a less pronounced trend and hence it is not clear whether an extreme value can cover all the scenarios. Also, it is obvious that none of the uncertain parameters influence the *max\_Trq* attribute. Consequently, uncertainty can be disregarded for this attribute. Similar reasoning is done for each parameter-attribute combination and the following selection on the uncertain parameters is decided:

- *base\_mass*: fully sample,
- *payload\_mass*: fix at the upper bound,
- *emCostUncFact*: fully sample,
- *batCostUncFact*: fix at the lower bound,
- *batSpEngyNom*: fix at the lower bound,
- *batSocMax*: fully sample,
- *batSocMin*: fix at the upper bound,
- *elecPriceUncFact*: fix at the upper bound.

The optimization problem corresponding to this mix of sampled and fixed uncertain parameters is solved with the same solver as in the previous approaches. The results are indicated in Table 3 (last column). In this case, a very similar design as in the fully sampling case (Section 4.2.3) is found. However, now the computational time is significantly reduced to only 160 seconds per iteration since only three parameters are sampled. The validation results confirm that the design is robust with all the specifications (constraints) fulfilled for all the samples. The TCO is slightly overestimated by -1.5% which should not be an issue to make the final decision on whether to pursue the manufacturing of this design.



Table 3. Optimization results for the different approaches.

Optimal design	Nominal optimization	Prefixed worst case optimization	Full sampling optimization	New method optimization
<i>batt_capacity</i>	32.82 kWh	59.57 kWh	67.02 kWh	66.87 kWh
<i>frontEM_maxpower</i>	55.94 kW	65.47 kW	81.47 kW	81.00 kW
<i>rearEM_maxpower</i>	79.52 kW	83.14 kW	87.50 kW	88.94 kW
<b>Optimized attributes</b>				
<i>normalized_TCO (minimization)</i>	2.69	4.42	4.98	4.91
<i>maxTrq (≥ 1600)</i>	1600	1670	1760	1790
<i>maxSpeed (≥ 220)</i>	239	240	244	245
<i>range (≥ 250)</i>	250	250	250	250
<b>Computation performance</b>				
<i>computation time per iteration</i>	8 sec/iter.	8 sec/iter.	22 min/iter.	160 sec/iter.
<b>Validation with 1000+256 samples</b>				
<i>worst normalized_TCO found</i>	4.35	4.54	4.98	4.98
number of unfeasible samples	186	4	0	0
largest constraint violation	<i>range = 140</i>	<i>range = 226</i>	-	-

## 5. Conclusions and future work

A new method for optimizing the design when uncertainty is present has been presented. The method does not aim to guarantee optimality nor robustness but to help the designer to make the correct trade-off between computation time and solution reliability. If computational time becomes an issue, the designer is forced to simplify the problem by omitting some uncertain parameters or fixing them to certain values. On the other side, if more reliability is needed, then the number of samples must be increased by sampling more densely and/or adding more parameters. With the use of the new method, the designer is better positioned to make the right choices when selecting which parameter to omit and which one to consider.

The effectiveness of the method has been tested by solving a realistic case, the parallel electric drivetrain, with four different approaches. It has been shown how the proposed method provides same results as sampling all the uncertain parameters but with a significantly lower time. Besides, solving the parallel electric drivetrain with the different approaches has also motivated why considering uncertainty is important and the risks to overlook to it.

The proposed method presented here is still in an early stage and should be seen as a first step toward a more elaborated approach. In this regard, there are some open issues than have not been properly addressed in this paper due to the lack of space. The authors would like to highlight two of them as the most promising in views of future research. In the current method, the sensitivity analysis is done before the optimization and fully decoupled. However, there is room for improvement by embedding the sensitivity analysis into the optimization routine where local sensitivity analysis could be used instead, and hence increasing the efficiency. Another improvement is on how the samples are distributed among the uncertain parameters. Non uniform distributions where influential regions of the uncertain space are more densely sampled could lead to significant improvements as well.

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