

THE INTEREST QUESTION.

To the Editor of the Assurance Magazine.

SIR,—Will you allow me to add, as a *postscript* to my letter of December last, the following few lines on the *interest* question?

No one will dispute that the progressive series in the capital and interest is of a geometrical character; wherefore the amount of the 1st year bears the same proportion to that of the 2nd year, as does that of the latter to that of the 3rd year. In like manner, 2nd : 3rd = 3rd : 4th, and so on.

Example, at 3 per cent. compound interest:—

$$\begin{array}{l} \text{1st year} \dots\dots\dots \left\{ \begin{array}{l} 1\cdot03^1 : 103^2 = 103^2 : 103^3 \\ \text{2nd ,,} \dots\dots\dots \left\{ \begin{array}{l} 1\cdot03^2 : 103^3 = 103^3 : 103^4 \\ \text{3rd ,,} \dots\dots\dots \left\{ \begin{array}{l} 1\cdot03^3 : 103^4 = 103^4 : 103^5 \\ \text{4th ,,} \dots\dots\dots \left\{ \begin{array}{l} 1\cdot03^4 : 103^5 = 103^5 : 103^6 \\ \text{nth ,,} \dots\dots\dots \left\{ \begin{array}{l} 1\cdot03^n : 103^{n+1} = 103^{n+1} : 103^{n+2} \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right. \end{array}$$

Now the entire question may be reduced to this: Is the amount at half year to be considered the geometrical mean in relation to the respective amounts of the two years between which it lies? If so, how is that mean to be obtained? Is it by halving the difference of the two terms, and adding the same to the lesser, as is pointed out by Mr. Farren and “A Young Associate”; or, by taking, as the mean required, the square root of the product of the two given terms?

In the former case:—

$$1\cdot03 : \left\{ \frac{1\cdot03^2 - 1\cdot03}{2} + 1\cdot03 \right\} = \left\{ \frac{1\cdot03^2 - 1\cdot03}{2} + 1\cdot03 \right\} : 1\cdot03^2;$$

or, (a) $103 : 1\cdot04545 = 1\cdot04545 : 1\cdot0609$; or, (b) $1\cdot03^2 = 1\cdot04545^2$; or, (c) $1\cdot092727 = 1\cdot0929657025!$

While in the latter case, $103 : \sqrt{1\cdot03^3} = \sqrt{1\cdot03^3} : 103^2$; by which equation we correctly obtain, $\sqrt{103^3} \times \sqrt{1\cdot03^3} = 1\cdot03 \times 1\cdot03^2 = 1\cdot03^3$.

To dwell any further on the subject, would be as much as an endeavour to prove that 3 times 4 are not 13, but 12; or the like.

March 1, 1854.

H. F.

We insert this second letter of Mr. Filipowski at his request, although it contains little else than a reiteration of the notions expressed in the first. The following communication, we think, will serve to show him that the entire question is not quite as he would state it.—Ed. A. M.

To the Editor of the Assurance Magazine.

SIR,—As it would appear, from the slip prefixed to the last Number of the *Assurance Magazine*, that you are to insert in the forthcoming Number a communication from Mr. Filipowski, with reference, it is presumed, to Mr. Sang’s method of determining the value of an assurance payable at the moment of death, perhaps you will allow another correspondent to make a few remarks on the same subject.

Notwithstanding all the learned dust which the discussion of this *voxata questio* has raised, the whole point at issue seems really to be neither more nor less than—how to ascertain the interest of £1 from the day of death till the end of the year then current. Once this is determined, we have only to find what the sum assured with interest from the day of death will amount to at the end of the year, and then to calculate in the ordinary way the value of an assurance of that amount; the result thus obtained being the value of the sum assured payable on the day of death. It must, indeed, be evident that, after the death has occurred, the only elements affecting the calculation are, the rate of interest and the period for which it has to run.

Holding then, according to the ordinary hypothesis, that the period between the day of death and the end of the current assurance year may on an average be reckoned as half a year, we have, in order to ascertain the value of an assurance of £1 payable at death, to find the amount of £1 with a half year's interest thereon, and then to calculate by the ordinary rule the value of an assurance for that amount. Now according to ordinary practice and understanding, the interest of £1 for half a year at 3 per cent. per annum is $\cdot 015$; so that the premium for, or value of, an assurance of £1 payable at death, will be the same with the premium for, or value of, an assurance of £1 $\cdot 015$, ascertained in the ordinary way, and payable at the end of the year in which death takes place, or at the end of six months from the death.

According to Mr. Sang, on the other hand, the interest of £1 for half a year, at the rate of 3 per cent. per annum, is $\cdot 01488916$; and consequently the sum to be assured is $1\cdot 01488916$ in place of $1\cdot 015$. In other words, he proceeds on the assumption that interest is accumulated momentarily, at such a rate as to produce $\cdot 01488916$ per £1 at the end of half a year, and £ $\cdot 03$ at the end of a year.*

It may be questioned however, whether, even if otherwise admissible, the application of this extreme mathematical notion of the principle of compound interest is consistent with the conditions on which in practice the returns on actual investments are made and calculated. We hear of interest being receivable half yearly and quarterly at certain rates; and we know that when interest is charged for periods shorter than a year, it is universally taken at a proportion for the time of the ordinary simple interest for a year. There is no instance however of an engagement to allow interest to be *accumulated de die in diem*. But even admitting that interest ought to be dealt with on the principle of momentarily accumulations, the application of that principle must be extended to the whole period of the assurance, and ought not to be confined merely to the period subsequent to the death. In like manner, to be consistent, a similar principle ought to be adopted with reference to the probabilities of life; for there is no reason for the adoption of momentarily accumulations of interest which does

* Half yearly or quarterly accumulations at the rates respectively of $\cdot 0148891$ and $\cdot 00741707$, and daily accumulations at the rate of $\cdot 00008099$, would produce the same result. But it is evident that the principle of *momently* accumulations must be adopted as the general rule, otherwise the number of accumulations per annum would have to be varied, whenever the period for payment of the sum assured was altered either from the date of death or the end of the year of death. See Baily's *Doctrine of Interest and Annuities*, where the difference between the true and the nominal rate of interest, and the effect of compound interest when fractional portions of a year have to be taken into account, are fully discussed; and a Table is given, showing the amount of £1 in any number of days at various rates of compound interest.

not equally apply to the introduction *momently* of the risk of death. Accordingly, different formulæ are given in the books, for determining the value of an assurance for every *half year*, quarter year, &c. of existence, interest being accumulated at same periods; and in chap. x. § 360 of Mr. Baily's *Treatise on Life Annuities*, we are presented with the following formula for finding the value of a "sum to be received *immediately* on the extinction of the given life":—

$$\frac{s \cdot \rho}{\text{Nap. L. } (1 + \rho)} \times \frac{1 - \rho A}{1 + \rho},$$

where ρ represents the true annual rate of interest, which, even on Mr. Sang's principle, is here 3 per cent. It is somewhat remarkable that Mr. Baily gives, as a near approximation to the above, but as "more convenient for practice," the following formula:—

$$s \left(1 + \frac{\rho}{2} \right) \times \frac{1 - \rho A}{1 + \rho},$$

which on examination will be found to be identical with the rule usually adopted, viz., the addition of half a year's simple interest to the sum assured.

I have thus shown, as I think satisfactorily, that the common method of deducing the value of an assurance payable at death, from that of an assurance payable at the end of the year in which death takes place (by adding six months' simple interest to the amount assured), is preferable to that adopted by Mr. Sang,—whether we assume the ordinary commercial principle of calculating at simple interest for fractional portions of a year, or adopt the strict mathematical idea that the effects of interest and mortality are to be dealt with as arising *momently*.

I shall now, before closing, notice shortly the challenge thrown out by Mr. Filipowski, in a communication inserted in the 12th Number of this *Journal*. He there asks, how Mr. Farren would solve the problem—"In what time will the sum s amount to a , at d compound interest?" Now this might be answered, Scotch fashion, by putting another question, viz., Whether by d is meant the rate of interest *per annum*, or for what other period? and, if it is the rate per annum, *how often* interest is supposed to be accumulated in the course of the year? In point of fact, as a mathematical problem, the question should have been stated thus:—"How frequently will interest require to be accumulated in order to produce d per cent. per annum, and at the same time to make s amount to a ?" As the question is put by Mr. Filipowski, it is obvious that, unless he holds that the principle of *momently* accumulation is essential to the notion of compound interest, it is quite possible that no definite answer can be given to it.

The puzzle here, as in many other disputes, arises from the parties not having defined precisely the meaning of the terms employed by them. Let it be distinctly understood what is meant by "compound interest," and there is no fear but the mathematical conclusion will be accurately arrived at by all.

I am your most obedient Servant,

I.

P. S. It would certainly appear, as you lately remarked, that the Northern actuaries have a special delight in the calculation of tables; otherwise I

cannot conceive what could have induced Mr. Chisholm to frame D and N columns with reference to this question, when the whole difference arises from an addition made to the sum assured.

MEDICAL STATISTICS OF LIFE ASSURANCE COMPANIES.

To the Editor of the Assurance Magazine.

SIR—I have recently (through the courtesy of the managers of the respective Companies) had an opportunity of perusing two reports,* the one by Dr. Christison, of the Standard Assurance Company, and the other by Dr. Begbie, of the Scottish Widows' Fund, on the subject of the mortality experienced in these two Offices.

As I believe these papers are not in general circulation, having been published originally in the *Monthly Journal of Medical Science*, and subsequently printed only for the private information of the members of the two Companies, probably some remarks upon them may not be unacceptable to your readers.

The papers in question are prepared by the medical officers of the two Companies, and form a valuable contribution to life assurance statistics.

With the exception of an account given by one of the present writers, Dr. Begbie, in the year 1847, of the experience in respect of mortality of the Scottish Widows' Fund, little or nothing of the kind now under consideration seems to have been published up to the present time. The reason given for this is, that very few Companies of sufficient standing can afford data on which such investigations could be satisfactorily based, owing to the meagre nature of the documents on which, until lately, Assurance Companies have been contented to accept their risks; and probably the documentary evidence adduced in proof of death has not been, prior to the organization of the Registrar General's department in the year 1837, of a more satisfactory nature.

There can be no doubt, I think, that the formation of the Institute of Actuaries is likely to give a great impetus to scientific inquiries of this nature. Their extreme importance to Assurance Companies is manifest; and the directors of these Societies, however unaware at present they may be of the fact, are in many ways reaping the fruits of an institution of the very existence of which they may not perhaps even be cognizant. It is from such inquiries as these, however, that the real practical use of the Institute will become apparent to them; for both the papers under discussion show, *inter alia*, how very important a careful selection of lives becomes: and this is particularly the province of the directors, of course under the advice of their physician.

The Standard Life Assurance Company was founded in the year 1825, and the Scottish Widows' Fund ten years previously.

I find, from a table given in the 2nd volume of the *Assurance Magazine*, page 360, that the total annual income of the first mentioned Company is about £180,000; and, from the same source, that the income of the latter Company is as much as £306,500 annually. I mention this to show the important character of these two Societies; and that, from the

* Noticed in No. XIII. of this *Journal* (see page 76).—ED. A. M.