SECULAR INSTABILITIES

B. Paczyński
N. Copernicus Astronomical Center,
Polish Academy of Sciences,
ul. Bartycka 18, 00-716 Warszawa, Poland.

The relation between a linear series and thermal (i.e. secular) stability of stellar models is discussed. Models of accreting degenerate dwarfs are considered as an example of transition from instability to stability. This transition occurs while the accretion rate increases. Such models are relevant for novae and symbiotic stars.

I shall limit this presentation to the secular i.e. thermal instabilities of stellar models in the hydrostatic and thermal equilibria. The time dependence of a small perturbation of an equilibrium model is assumed to be exponential, but the eigenvalues do not have to be real. In general they are complex. That means that thermal instability, if present, may either lead to an exponential growth of a perturbation, or may lead to oscillations with a growing amplitude. The complex nature of the eigenvalues makes a search for them difficult and no general prescription is available for a search to be complete and successful. However, a search for real eigenvalues is much easier. Some information about the presence of purely exponential modes may be achieved even without searching for them by means of calculating a linear series of stellar models. I shall describe this concept in this communication.

Let us consider a stellar model in the thermal and hydrostatic equilibria. The distribution of chemical composition with mass fraction is specified. For simplicity we may specify the distribution of hydrogen content only

$$X = X(M_r) \quad \text{for} \quad 0 \leqslant M_r \leqslant M,$$

where M is the total stellar mass. Now we shall vary either the total mass or the distribution of hydrogen, or both, in such a way that the variations will be described with one parameter only. For example, we may consider stellar models which are chemically homogeneous, but the total mass M is variable. Or we may consider models with a constant

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total mass and the distribution of hydrogen described with a function

$$X = \begin{cases} 0, & \text{for } 0 \leqslant M_r \leqslant M_{core}, \\ X_o = \text{const.}, & \text{for } M_{core} < M_r \leqslant M. \end{cases}$$

In the first example the total mass is the only variable parameter. In the second example the core mass is the only variable parameter. We obtain different models for different values of the variable parameter. These form a linear series of stellar models with one parameter e.g. the total mass, or the core mass varying along the series. In general this parameter passes a number of maxima and minima along the series. These extrema are called the turning points. They separate various branches of the linear series. For a fixed value of the variable parameter there may be a number of different stellar models on different branches of the linear series. However, there will be only one or none such model on any given branch.

It may be shown that at every turning point of a linear series of stellar models one of the thermal eigenvalues must be equal to zero. This eigenvalue changes its sign at this point. It is negative in models on one branch, and it is positive in models on the second branch, the two branches being separated by the turning point. Therefore, the models on the second branch must be thermally unstable, while the models on the first branch may be stable if they do not have other positive eigenvalues, or eigenvalues with a positive real part.

The connection between the concept of a linear series of stellar models and thermal stability was noticed by Gabriel and Ledoux (1967) and Gabriel and Noels-Grotsch (1968), and discussed in detail by Paczyński (1972). Many examples of various linear series may be found in the papers of Gabriel and Noels (1973), Gabriel, Refsdal and Ritter (1974), Kozłowski and Paczyński (1973, 1975), Kozłowski, Paczyński and Popova (1973), Paczynski and Kozłowski (1972), Paczyński and Różyczka (1977). Some additional references may be found in the recent review by Hansen (1978). The catastrophe theory may be consulted for deeper understanding. Even though the concept of a linear series cannot be applied to all possible stellar models, it is perhaps the most efficient and simple tool to be used in a search for thermal instabilities, and for the understanding of the nature of these instabilities.
I shall give here just one example.

The process of mass accretion onto degenerate dwarfs has received a lot of attention recently. It is believed that low accretion rate gives rise to cyclic thermonuclear explosions observed as novae (Gallagher and Starrfield, 1978). High accretion rates were studied in very few papers. Sion, Acierno and Turnshek (1978) claimed that all accreting models were thermally stable. Paczyński and Żytkow (1978) studied accretion onto a degenerate dwarf of 0.8 solar masses. When the accretion rate exceeded 2.7×10^{-7} solar masses per year the matter could not be burnt fast enough in the shell source, and gradually a red giant envelope accumulated around a degenerate core. When the accretion rate was below 1.1 x 10^{-7} solar masses per year the models underwent cyclic hydrogen shell flashes. For intermediate accretion rates the hydrogen shell burning was stable and a steady state envelope was formed. Recently, Sion, Acierno and Tomczyk (1979) obtained new results which are consistent with those of Paczyński and Żytkow. The transition from stable to unstable models may be readily understood within the realm of linear series. Sienkiewicz (1975) calculated a large number of static models of accreting degenerate dwarfs. Their envelope mass had a minimum at a certain accretion rate. Recently, Sienkiewicz (1979) found with a linear stability analysis that models with an accretion rate above that corresponding to the minimum envelope mass were thermally stable, while models with a lower accretion rate were unstable, in qualitative agreement with Paczyński and Żytkow. The minimum envelope mass clearly corresponds to a turning point of a linear series. Therefore, the instability of low accretion rate models should be exponential rather than oscillatory. This could explain why Paczyński and Żytkow found large amplitude relaxation oscillations developing immediately in models accreting just under the critical rate.

The stability of models with degenerate cores accreting at a high rate is important for the understanding of the nature of symbiotic stars, Pacsyński and Rudak (1979).

REFERENCES

Gabriel, M., and Ledoux, P., 1967. Ann. Astrophys., 30, 975.
Gabriel, M., and Noels, A., 1973. Astron. Astrophys., 25, 313.
Gabriel, M., and Noels-Grotsch, A., 1968. Ann. Astrophys., 31, 167.
Gabriel, M., Refsdal, S., and Ritter, H., 1974. Astron. Astrophys., 32, 217.
Gallagher, J.S., and Starrfield, S., 1978. Ann. Review Astron. Astrophys., 16, 171.
Hansen, C.J., 1978. Ann. Review Astron. Astrophys., 16, 15.
Kozłowski, M., and Paczyński, B., 1973. Acta Astron., 23, 65.
Kozłowski, M., and Paczyński, B., 1975. Acta Astron., 25, 321.
Kozłowski, M., Paczyński, B., and Popova, K., 1973. Acta Astron., 23, 263.
Lauterborn, D., and Siquig, R.A., 1974. Astron. Astrophys., 30, 385.
Paczyński, B., 1972. Acta Astron., 22, 163.
Paczyński, B., 1973. Astron. Astrophys., 26, 291.
Paczyński, B., and Kozłowski, M., 1972. Acta Astron., 22, 315.
Paczyński, B., and Różyczka, M., 1977. Acta Astron., 27, 213.
Paczyński, B., and Rudak, B., 1979. Astron. Astrophys., in publication.
Paczyński, B., Żytkow, A., 1978. Astrophys. J., 222, 604.
Sienkiewicz, R., 1975. Astron. Astrophys., 45, 411.
Sienkiewicz, R., 1979. submitted to Astron. Astrophys.
Sion, E.M., Acierno, M.J., and Tomczyk, S., 1979. Astrophys. J., 230, 832
Sion, E.M., Acierno, M.J., and Turnshek, D.A., 1978, Astrophys. J., 220, 636.