SOME STATISTICS ON LONG-PERIOD COMETS

I. Hasegawa and T. Shimizu
2-3-11, Saidaijinogami, Nara 631, and
Bukkyo University, Kyoto 603, Japan

Distributions of the original reciprocal semimajor axes 1/a and the perihelion distances q of eighty "new" comets (Marsden et al, 1978) are discussed from the view point of a hypothesis of the interstellar origin (Hasegawa, 1976).

Consider a sphere of the radius R centered at the sun which moves with a velocity V_0 and assume that comets outside the sphere are distributed uniformly in space and their velocities obey the Maxwellian law, but if they happen to enter into the sphere they move along Keplerian orbits around the sun. Then, the frequency distribution of both the perihelion distance and the eccentricity of the comet inside the sphere is derived from equation (1) given in the preceding article by Shimizu as follows:

$$dN(e, X_0) = \frac{1}{4}\sqrt{\frac{\pi}{2}}\, n R^2 \sigma \frac{\alpha_1^3}{Y_0} \left\{ \mathrm{Exp}\left[-\left(Y_0 - \alpha_1\sqrt{1+\frac{e-1}{2X_0}}\right)^2\right] - \mathrm{Exp}\left[-\left(Y_0 + \alpha_1\sqrt{1+\frac{e-1}{2X_0}}\right)^2\right] \right\} \left(1+\frac{e-1}{2X_0}\right)^{-\frac{1}{2}} de^2\, d(\log X_0), \quad (1)$$

where X_0 and $\sqrt{3}\sigma$ denote q/R and the dispersion of comets' velocities outside the sphere respectively, while $Y_0 = V_0/(\sqrt{2}\sigma)$ and $\alpha_1^2 = GM/(\sigma^2 R)$ (G: gravitational constant, M: the sun's mass) are the parameters. The notations used here are identical with those of Shimizu. Rejecting the imaginary radial velocity at a point on the assumed sphere, the permissible values of X_0 should be in such a range as

$$\text{for } e \leqq 1,\ 1 \geqq X_0 \geqq \frac{1-e}{1+e} \geqq 0;$$

$$\text{for } e \geqq 1,\ 1 \geqq X_0 \geqq 0. \quad (2)$$

It is concluded therefore that for $e = \varepsilon\ (0 < \varepsilon \leqq 1)$ the value of X_0 is limited to a narrow range between $1 - 2\varepsilon$ and 1, while for $e > 1$ any value of X_0 in the whole range between 0 and 1 is admissible. It is noted here that when e = 1 it follows that $\sin^2\theta = X_0$, but for $e = 1 \mp \varepsilon$ we have $\sin^2\theta$

R. L. Duncombe (ed.), Dynamics of the Solar System, 319–322.

https://doi.org/10.1017/S007418090001295X Published online by Cambridge University Press

$= X_0(1\mp\varepsilon/2)/(1+\varepsilon/2X_0)$, and for an assigned $X_0 \leqq 1$ such elliptic orbits as with the eccentricities less than $1-2X_0$ could not be found.

Then, with the aid of Shimizu's equation (1) above mentioned and the energy equation for the two body problem, the frequency distribution of R/2a with respect to small range of eccentricities from e to e'(>e), excluding e=1 in this interval, is derived as follows:

$$dN\left(\frac{R}{2a}\middle| e,e'\right)=\frac{1}{4}\sqrt{\frac{\pi}{2}}\,n R^2 \sigma \alpha_1^3 \frac{e'^2-e^2}{Y_0}\left\{\mathrm{Exp}\left[-\left(Y_0-\alpha_1\sqrt{1-\frac{R}{2a}}\right)^2\right]-\mathrm{Exp}\left[-\left(Y_0+\alpha_1\sqrt{1-\frac{R}{2a}}\right)^2\right]\right\}\left(1-\frac{R}{2a}\right)^{-\frac{1}{2}}\frac{d\left(\frac{R}{2a}\right)}{\frac{R}{2a}}. \quad (3)$$

With an assumed value of 1 km/s for both the mean relative velocity of the sun and the r.m.s. of comets' velocities, and some values for R between 10^3 and 10^5 AU, the theoretical distributions of 1/a and q are calculated. The observed data are taken from Table III of Marsden et al. (1978). Among both the first- and the second-class orbits in this table, we have eighty comets with the original 1/a values smaller than $+100 \times 10^{-6}$ AU^{-1}. These can be considered to be "new" comets which come near the sun probably for the first time.

For all of these eighty "new" comets and some additional ones, the frequency distribution of the original 1/a is obtained as shown by the shaded areas in Fig.1. The dotted lines in the figure represent the theoretical frequency distributions calculated from equation (3) with

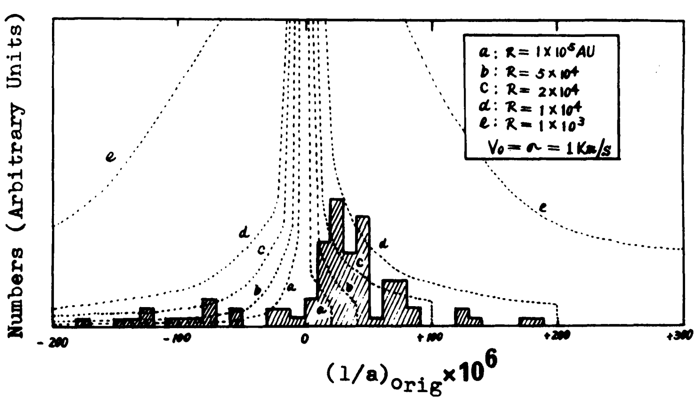

Fig.1 Distribution of 1/a

different parameters entered therein, These theoretical distributions seem to resemble closely the observed one in general trend, though a deviation is found around the zero value of 1/a.

In Fig.2 the frequency distribution of the perihelion distance q from the same data is shown together with the theoretical ones from equation (1) with different parameters written in. A general similarity is also discernible here, though it fails for small values of q.

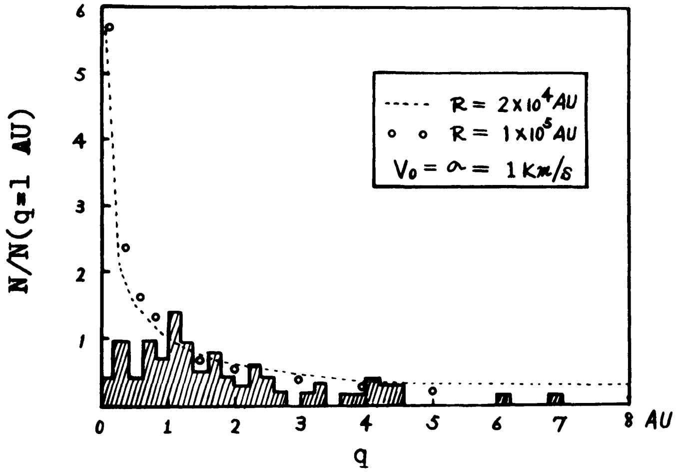

Fig.2 Distribuion of q

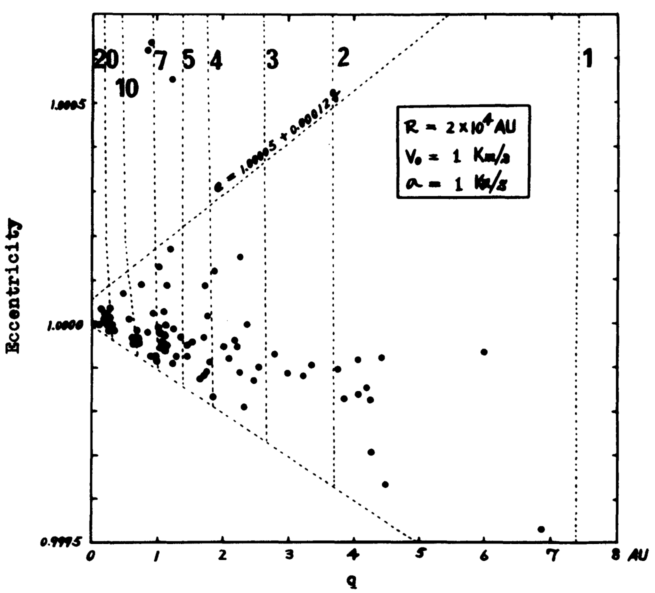

Fig.3 Correlations between e and q

https://doi.org/10.1017/S007418090001295X Published online by Cambridge University Press

On the other hand, Fig.3 is a correlation diagram between q and e (= 1-q/a), where the numbers entered between pairs of successive nearly vertical lines indicate the corresponding number of comets calculated from equation (1). It is seen there that almost all of the available data are included in a triangular region wedged between two dotted lines specified as follows: the lower one represents $e = 1-2X_0$, namely, the existence limit of comets with elliptic orbits and the upper one represents

$$e = 1.00005 + 0.00012q \text{ (AU)}. \qquad (4)$$

Assuming $R = 2 \times 10^4$ AU, the numerical values in equation (4) correspond to $\theta = 0^\circ.6$, V=0.50 km/s for q=1 AU, and $\theta = 1^\circ.5$, V=0.46 km/s for q=10 AU.

If the distribution of the comet's space velocity is not approximated well by the Maxwellian type assumed here, some deviations, as seen in Figs. 1 and 2, may be caused. This is because θ and V should vary in a narrow range such as above mentioned at the assumed distance R from the sun. Furthermore the selection effect sets in as for the observed data adopted here, especially for those with small perihelion distances, so that some deformations in the distributions of 1/a and q may be produced.

Therefore, our hypothesis that the comets originated in interstellar space seems to explain, at least in a general way, their observed distributions of 1/a and q by merely assuming a Maxwellian velocity distribution.

References

Hasegawa, I.; 1976, "Publ. Astron. Soc. Japan", 28, pp. 259-276.
Marsden, B. G., Sekanina, Z., and Everhart, E.; 1978, "Astron. J.", 83, pp. 64-71.