A TRANSFORMATION OF WHICH THE EQUATION OF TELEGRAPHY IS A DIFFERENTIAL INVARIANT

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We consider the "equation of telegraphy"

$$\partial^2 V/\partial x^2 - \{LC\partial^2 V/\partial t^2 + (RC+GL)\partial V/\partial t + GRV\} = 0, \tag{1}$$

where L, C, R, G are constants (the inductance, capacity, resistance and leakage conductance per unit length), V the potential, x the distance from the origin, and t the time.

Let

$$\left.\begin{aligned} x' &= \tfrac{1}{2}(a+1/a)x + \{\tfrac{1}{2}(a-1/a)/\sqrt{(LC)}\}t, \\ t' &= \{\tfrac{1}{2}(a-1/a)\sqrt{(LC)}\}x + \tfrac{1}{2}(a+1/a)t, \\ V &= V' \exp\left(\{(RC+GL)/2LC\}[\{\tfrac{1}{2}(a-1/a)\sqrt{(LC)}\}x + \{\tfrac{1}{2}(a+1/a)-1\}t]\right) \end{aligned}\right\} \tag{2}$$

a being an arbitrary parameter. Then by direct calculation we see that

$$\begin{aligned} &\frac{\partial^2 V}{\partial x^2} - \left\{LC\frac{\partial^2 V}{\partial t^2} + (RC+GL)\frac{\partial V}{\partial t} + GRV\right\} \\ &\quad \equiv \left[\frac{\partial^2 V'}{\partial x'^2} - \left\{LC\frac{\partial^2 V'}{\partial t'^2} + (RC+GL)\frac{\partial V'}{\partial t'} + GRV'\right\}\right] \\ &\qquad \times \exp\left(\{(RC+GL)/2LC\}[\{\tfrac{1}{2}(a-1/a)\sqrt{(LC)}\}x + \{\tfrac{1}{2}(a+1/a)-1\}t]\right) \end{aligned}$$

so that the form of (1) is unchanged by the transformation (2), x, t, V being replaced by x', t', V' respectively.

From this result we see that if $V(x, t)$ be an integral (supposed known) of (1), so also is

$$\begin{aligned} &V[\tfrac{1}{2}(a+1/a)x + \{\tfrac{1}{2}(a-1/a)/\sqrt{(LC)}\}t,\ \{\tfrac{1}{2}(a-1/a)\sqrt{(LC)}\}x + \tfrac{1}{2}(a+1/a)t] \\ &\qquad \times \exp\left(\{(RC+GL)/2LC\}[\{\tfrac{1}{2}(a-1/a)\sqrt{(LC)}\}x + \{\tfrac{1}{2}(a+1/a)-1\}t]\right), \end{aligned}$$

so that from one known integral we at once deduce an infinity of new integrals, depending upon an arbitrary parameter, a.

It is interesting to compare this transformation with corresponding transformations applicable to Laplace's equation in three dimensions (**1, 2**), Laplace's equation in two dimensions (**3**), the equation for the stream function in axially symmetric liquid flow (**4**), and the equation of heat flow (**5, 6**).

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