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DIRICHLET PROBLEM WITH L^P-BOUNDARY DATA

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In this work the Dirichlet problem

$$Lu + \lambda u = f \quad \text{in} \quad Q \,, \tag{1}$$

$$u \quad = \phi \quad \text{on} \quad Q \tag{2}$$

is considered where Q is an open set in the Euclidean n-space, with C^2-boundary ∂Q, $n > 2$, ϕ an element of the Lebesgue space $L^p(\partial Q)$, $1 < p < \infty$, λ a real number and L an uniformly elliptic and linear operator in divergence form. The equation (1) is understood in the weak sense, that is, the function u is in the Sobolev space $W^{1,p}_{\text{loc}}(Q)$. The boundary condition (2) is defined by

$$\lim_{\delta \to 0} u o\, x_\delta = \phi \quad \text{in} \quad L^p(\partial Q) \,,$$

where x_δ is given by $x_\delta(x) = x - \delta v(x)$ and $v(x)$ is the outward normal to Q at x.

In [1], Chabrowski showed that the problem (1), (2) has a unique solution, provided Q is bounded, $p = 2$, the leading coefficients of L are differentiable and λ is sufficiently large. Here the lower coefficients may tend to infinity at the boundary ∂Q in a certain way. Mikhailov [3] obtained solutions to the Dirichlet problem (1), (2) for the case $1 < p < \infty$, with assumptions on λ and the leading coefficients of

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L similar to those in [1], but with considerably more restrictive assumptions on the growth at the boundary of Q. The case that Q is a half space is considered in [2] by Chabrowski and Lieberman. Here $p = 2$, the leading coefficients of L are Dini continuous and the lower coefficients of L may tend to infinity at the boundary of Q.

In the first two parts of this work existence of weak solutions to the Dirichlet problem (1), (2) is established. In the first part (see also [4]) Q is assumed to be bounded, while in the second part Q is assumed to be a half space. In both cases the leading coefficients of L are assumed to be Hoelder continuous if $1 < p < 2$, Dini continuous if $p = 2$ and differentiable if $p > 2$, the lower coefficients of L may tend to infinity at the boundary of Q, and λ is sufficiently large.

For the case that Q is bounded the existence of solutions to (1), (2) is shown by approximating ϕ, f and Q in a suitable way. Doing this, a sequence of solutions of the corresponding problems is obtained, from which, by the help of some energy estimate, a unique solution of (1), (2) is deduced. For a half space of sequence of bounded domains, which exhaust the space, is considered, and the result of the first part is applied.

In the third and final part of this work the Dirichlet problem for elliptic systems in a half space is discussed. Here the leading coefficients are assumed to be differentiable and the lower coefficients to be bounded. Vector-valued boundary data with components in L^2, is considered.

References

[1] J. Chabrowski, "On the Dirichlet problem for a linear elliptic equation with unbounded coefficients", *Boll. Un. Mat. Ital B (6)* 5 (1986), 71-91.

[2] J. Chabrowski and G.M. Lieberman, "On the Dirichlet problem with L^2 boundary values in a half space", *Indiana Univ. Math. J.* (to appear).

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[3] Y. Mikhailov, "Boundary values in L_p, $p > 1$, of solutions of second-order linear elliptic equations", *Differential Equations* 19 (1983), 243-258.

[4] T. Hoffmann-Walbeck, "On the Dirichlet problem for linear elliptic equations and L^p-boundary data, $p > 1$", *Boll. Un. Mat. Ital.* (to appear).

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