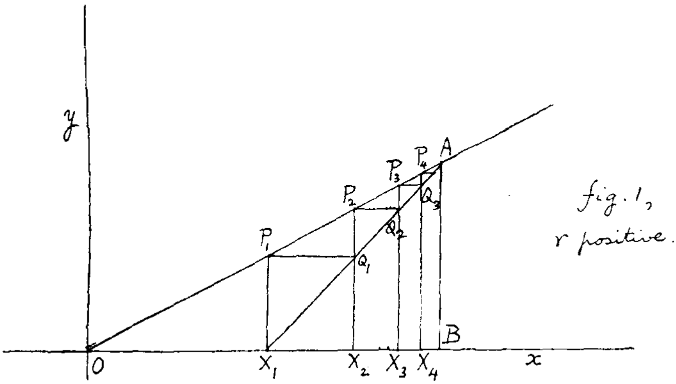
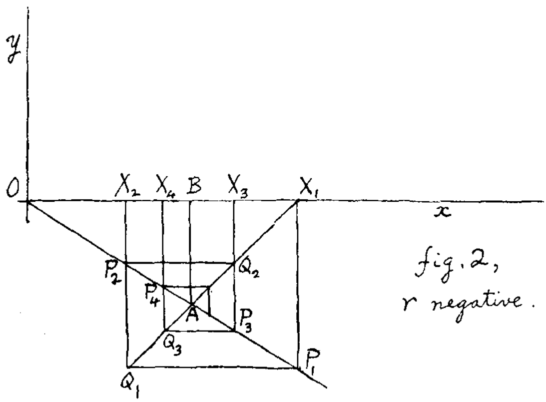
We have thus a proof by intuition that if we fix on any number ϵ, however small, we can choose the integer n so that the sum of n or any greater number of terms of the G.P. (with r numerically less than 1) differs from $a/(1-r)$ by less than ϵ.





JOHN DOUGALL.

Oral and Written Work in Arithmetic.—The following notes deal mainly with the Arithmetic of the primary school, but the subject under consideration is only a single aspect of a very wide question.

It is a common remark that Oral (or Mental) and Written Arithmetic should be as nearly as possible identical; that pupils ought to be able to do mentally with small numbers whatever they can do in writing with larger numbers, and *vice versa*. Unfortunately, experience shows that, as a rule, they are not able to do so; that the child fails to identify written calculation completely with mental work. It is here proposed to try to discover some of the reasons why this is so.

It may be remarked in the first place, that in Arithmetic, more perhaps than in any other subject of the primary curriculum, it is hard to graduate the difficulties; in fact, young pupils have to deal with matters more or less abstruse, and often, perhaps, beyond their comprehension—though an incomplete understanding is usually possible. Take, for example, "long division"; for practical reasons it is necessary that children of nine or ten learn to carry out this process, but it is doubtful whether all of them fully grasp the meaning of each step until they are considerably older.

In such cases the practice of the teacher varies; some give such explanation as they think the pupil can grasp; others, thinking that the child is unaware that any explanation is required, pass on and leave explanations till the pupil is old enough to understand the process fully. It is not here intended to express any opinion as to which course is the better one to pursue, but it may be noticed that each has its peculiar dangers. In trying to water down abstruse ideas to the strength suitable for a small child, it sometimes happens that one manages to invent a specious explanation that sounds all right, but which, on closer examination, proves to be all wrong; while if explanations are postponed they sometimes fail to be given at all, especially in large schools where the pupils pass on from teacher to teacher.

This is a digression; but the point to be noticed here is that, theories to the contrary notwithstanding, a child is bound sometimes to be carrying out on slate or paper processes it understands but imperfectly—unless, indeed, we are prepared to allow it to make no progress in some directions for several years. This being so, it is clear that complete unity of mental and written work is, to say the least of it, hard to secure.

But not only is it hard to graduate the difficulties; it is also hard to take them singly. We may, roughly, distinguish between difficulties of calculation and difficulties of interpretation. In order to keep these as far as possible from clashing, it is usual to introduce all new principles first mentally, and then in writing, using small numbers; and it is not until the pupils have grasped the principle and the processes it involves that they are asked to apply them to such quantities as could possibly present any serious difficulty of calculation.

Now this procedure meets the case so far, but not entirely. For however familiar a child may be with the application of a principle to small numbers, it has great difficulty in realising that the same principle applies equally to large ones. That is to say, the transition from small numbers to large ones is attended by special difficulties independent of mere difficulties of calculation; and by *large* numbers it is not intended to imply tens of thousands, but any numbers too large for clear visual conception.

This seems to be one of the most important causes that militate against the success of our efforts to assimilate Oral and Written work involving the same principle, and it is worthy of closer examination.

First, we must remember that it is not only children that find it hard to generalise arithmetical, or indeed, any other conceptions, and that the transition to large numbers involves something of the kind. Working with small enough numbers, one can actually picture to oneself what goes on; not that one always does so, but one can do so if necessary. To take a simple example, it is easy to imagine the process of dividing six oranges among three boys—easy for us, and quite possible even for young children. But suppose we have to divide 320 oranges amongst 80 boys; knowing the answer, and being familiar with the factors of 80, it is not difficult for us to picture ten rows of eight boys each, each boy with four oranges; but to a child just introduced to such problems both the boys and the oranges must appear innumerable, and no visual conception of the circumstances is possible at first; and if the oranges number 2104, and the boys 263, few people could form any picture more definite than that of a crowd of boys each happy in the possession of eight oranges. In short, the step from small numbers to large involves a generalisation similar in kind to that involved in passing from Arithmetical calculations with numbers, to what are called Algebraic calculations with generalised symbols, and even if the generalisation be not so complete in the former case, the children who have to make it are much younger, and the process is probably not less difficult.

Secondly, we have to take account of the fact that the various steps in calculation wear a different appearance when the numbers are large. Suppose, for example, that it is necessary to multiply two numbers together. If the numbers be four and three it is, at most, an effort of memory to obtain the product; but should the numbers be, say, 266 and 73, the result is obtained by means of what is to the pupils a somewhat complex and more or less mysterious process. One is, indeed, inclined to think that few young children are, in their hearts, quite sure that the processes are really identical in the two cases, and this incertitude is partly to be attributed to the fact that one case involves a complicated piece of work which is absent in the other case.

But only *partly*, for there is another factor in the situation—that of vocabulary. In Arithmetic, as in all sciences, we have many technical terms which are also in use as ordinary words; but their everyday meaning is always less precise than, and sometimes considerably different from, their technical definition; and while we should not trouble children unnecessarily with technical terminology, we cannot avoid it entirely. Now, the conflict between the technical and the popular use of the same word is a source of endless confusion in all branches of knowledge, but its application to the matter in hand lies in the fact that in ordinary speech we habitually use one vocabulary to denote operations with small numbers, and another to denote the same operations when performed with large ones. Thus we commonly talk of "three times four" or "three fours," rarely of "three multiplied by four": while, on the other hand, we are more likely to speak of "256 multiplied by 73" than of "73 256's." With us this is merely a habit of speech, but it is not surprising that amongst children, who must always get a large part of their ideas at second-hand from what their elders say, there is a corresponding habit of thought. Time and again one has found children who know their multiplication table well and can perform any ordinary multiplication sum on paper, but who have certainly not realised that each item of the table is a multiplication sum. They can tell without hesitation how much nine times eight or nine eights make, but do not know what to do when asked to multiply nine by eight, and again, they can readily multiply, say, 287 by 96, but are quite at a loss to discover the value of 96 times 287. The same trouble arises in connexion with the other "simple rules," the usual

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tendency being to associate the longer technical words with large numbers, and their short every-day equivalents with small ones. If, in addition to this, we remember that these words are used in ordinary life with no strictly defined meaning, we may find it possible to sympathise with *Punch's* little girl, who wanted to take her pet rabbit to school because she had heard that rabbits multiply quickly.

Thirdly, let us turn to another difference between mental and written solutions of the same question. The latter involve conscious performance of each step in the work, while, in the former, steps may be omitted or only semi-consciously performed. Not that some easy steps may not be omitted even in a written exercise, but it must be sufficiently explicit to be comprehended by all who read it, whereas no one need make a mental process intelligible to any but oneself. And herein, it may be remarked, lies a danger into which excessive devotion to mental work may lead us, viz., that a reasonable proportion of written work is necessary as a salutary check on slovenliness of thought.

There is, to be sure, a means of ensuring the performance of each step without having recourse to writing, by getting the pupil to give orally an account of the steps by which he arrives at his answer; but this method cannot, of course, replace writing as a medium for the working of arithmetical problems. This oral account is in a sense intermediate between the purely mental process and its exhibition in writing, and a child who can give a clear account of his solution of a question has made some advance towards overcoming the main difficulty of attacking similar problems in writing.

But he has by no means overcome it all; for, besides the fact that he is still confined to small numbers, and has not yet overcome obstacles to which reference has already been made, there seem to be other difficulties. In giving an oral account of the process each step is described, or may be described, by name; but in the corresponding written solution it would be hopelessly tedious and gratuitously pedantic not to avail ourselves of the recognised symbols for the various operations involved, such as +, −, =, etc. These symbols are few in number, but it appears not to be easy for the pupil to recognise that, *e.g.*, two little parallel lines are merely an abbreviation for the words, "is (*or* are) equal to"; and he seemingly finds it still harder to believe that, if the symbol can mean all that, it may not be used to mean a host of other things of a somewhat similar nature, such as: "is the price of," "is the time taken to walk," etc. Only the very greatest care in using symbols can save disaster in this connexion.

And the symbols are not the only conventional abbreviations. We do not necessarily write 27 – 13 to denote the subtraction of 13 from 27; we may simply write 13 below 27 and subtract in the usual way, and this the children often learn to do before they have learned to use the symbol –.

W. G. FRASER.

Dynamics as a School Subject.—The inclusion of Dynamics in a school curriculum is a topic that affords always much scope for discussion among interested teachers, and widely opposed opinions are held. I venture to state my own opinion, and to suggest a somewhat fuller treatment in dealing with one or two sections of the subject than is usual in the text-book.

I think that the subject of Theoretical Dynamics (including Statics and Hydrostatics) is one that every boy who has reached the post-intermediate stage of his school career, and who intends to remain at school for one or two years longer, ought to study.

Provided that the school is equipped with a physical laboratory, the subject ought to be introduced to pupils at the stage I have suggested as a course of Experimental Dynamics and Statics extending in time to not less than six months. I do not mean that this course should be a course of "Practical Science" in the sense that the Chemistry and Physics of the modern school is. It must be largely a course of Experimental Demonstrations, supplementing the theoretical development, and must never become—as it is unfortunately too apt to become—a mere tabulation of results, excellent or otherwise, by pupils who have difficulty in seeing beyond their hands. The content of this introductory course will be that of the elementary text-book, while to the second year of study—almost entirely theoretical, of necessity—shall be relegated those sections of the subject that deal with circular motion, parabolic motion, simple harmonic motion, impact, centre of mass, couples, the general conditions of equilibrium among forces, and in addition there might be included some of the fundamental notions of Rigid Dynamics.