reader would surely be well equipped to attack the large and complex literature which has now arisen in this area.

A. GUYAN ROBERTSON

DONALDSON, S. K. and KRONHEIMER, P. B. *The geometry of four-manifolds* (Oxford Mathematical Monographs, Clarendon Press, Oxford 1990), x+440 pp. 0 19 853553 8, £35.

Analysis has played an important role in several of the great developments of geometry over the last 150 years. Complex analysis was an important motivating force behind Riemann's study of manifolds; the geometric theory of ordinary differential equations and dynamical systems led Poincaré to develop the basic ideas of algebraic topology; Hodge made fundamental use of the Laplacian in his deep study of the topology of manifolds. Ten years ago, Simon Donaldson utilised basic theorems on the space of solutions of certain non-linear partial differential equations for the resolution of what had seemed to be very intractable problems concerning the topology of four-dimensional spaces. The book under review is a polished account of Donaldson's pioneering work; also included are several of the consequent developments in which he has played the leading role.

The understanding of the topology of two-dimensional compact manifolds was complete by the end of the last century. During the present century considerable attention has been devoted to three-dimensional topology particularly through knot theory and attempts to solve the Poincaré conjecture. A recurring technical point in the study of higher-dimensional manifolds is the need for enough room to move lower-dimensional pieces around within the manifolds themselves. Two basic problems arise: the first is the need to find embedded two-dimensional discs in the manifolds, the second occurs in the "middle dimension", and is the need to separate two k-dimensional manifolds within a $2k$-dimensional one. H. Whitney and J. Milnor respectively understood how to treat these two problems in high dimensions; in four dimensions the two problems reinforce each other and how to deal with them in this case had been a puzzle for a considerable period. Donaldson's work proves that it is impossible to circumvent these problems in four dimensions because he shows that topology in four dimensions behaves in a fundamentally different way. A striking example of this contrast is the following: if $n \neq 4$, then any smooth manifold that is homeomorphic to $\mathbb{R}^n$ is diffeomorphic (i.e. smoothly homeomorphic) to $\mathbb{R}^n$ (a result proved for $n \geqslant 5$ during the considerable developments in higher-dimensional topology around 1960 and known previously for $n \leqslant 3$); however, there is an uncountable family of smooth 4-manifolds M^4 (including $\mathbb{R}^4$) all of which are homeomorphic to $\mathbb{R}^4$ but no two of which are diffeomorphic. Many of these examples can be exhibited as open sets in $\mathbb{R}^4$ itself.

Donaldson's method is to study the space of all anti-self-dual connections for a bundle over a Riemannian four-manifold; there are invariants of this space that turn out to be independent of the choice of Riemannian metric but vary for the different smooth structures on the manifold. The anti-self-dual connections correspond to absolute minima of the Yang–Mills functional and are therefore solutions of the corresponding Euler–Lagrange equations. The smooth structure on the manifold manifests itself through these differential equations; it turns out, in a later analysis, that only a Lipschitz or a quasiconformal structure is relevant. At about the same time as Donaldson made his initial breakthrough, the purely topological theory of four dimensions was clarified enormously by Michael Freedman. When the two pieces of work are put together there is a great simplification in the statements of a number of results, such as those made above about manifolds homeomorphic to $\mathbb{R}^4$.

For someone with a good working knowledge of a range of graduate level courses, this book is self-contained. The background is in algebraic and differential topology, differential geometry, algebraic geometry and global analysis; some specific topics are reviewed in the early chapters and the necessary background from analysis (e.g. Sobolev spaces) is given in the Appendix. The work depends fundamentally on the Yang–Mills equations but background from physics is not given since it is not needed explicitly for the mathematical development. The bulk of the book is about differential geometry and Chapter Six is probably the key: it is here that the crucial link between differential geometry and algebraic geometry is made. Calculations are best organised by using the Donaldson polynomials; these are introduced and their properties developed in the penultimate chapter. The final chapter is devoted entirely to making such calculations on specific examples. Considerable effort has gone into organising the material and many of the proofs are different from those in the literature. Choosing between the already large number of approaches was becoming something of a problem for anyone learning this material and the authors deserve our gratitude for providing such a clear and unified approach.

Donaldson's ideas are sure to have a considerable further impact on geometry and toplogy and this book will be the basic reference. It will undoubtedly inspire future generations of mathematicians to attack some interesting and difficult questions.

E. G. REES

KIRWAN, F., *Complex algebraic curves* (London Mathematical Society Student Texts 23, Cambridge University Press, Cambridge 1992) pp. viii+264, cloth 0 521 41251 X, £30, paper 0 521 42353 8, £13.95.

The stated aims of this book are to demonstrate to final-year undergraduates how basic ideas in pure mathematics which they have met previously are brought together in one of the great showpiece subjects of mathematics and, by adding some extra material for postgraduates, to exhibit the varied and useful nature of this core topic in algebraic geometry. In this it succeeds splendidly: the reader receives a broad and enlightening education on the many aspects of complex algebraic curves—the algebraic, the analytic, the topological—and on their rich history and wide range of application from the time of the Greeks up to the present day. The exposition has been given careful consideration; important ideas and results appear near the start of a chapter, with technical parts of proofs separated out and left to the end if necessary. Where needed, telling examples are worked out in detail, so that the reader can grasp the main point immediately. The many exercises provide a good grounding in the basic material, ranging as they do from concrete examples to theoretical questions (which can be quite testing) and applications in a wide variety of fields. The writing style is crystal clear and enthusiastic.

Chapter 1 provides motivation and historical background. Highlights include the connections between singularities and knots and links, and a discussion of the crucial role played by abelian and elliptic integrals (here, typically, applications to arc-length and to the motion of a simple pendulum are mentioned). Lots of examples of plane complex curves are given and sketched; in particular, the use made by the Greeks of certain curves in trisecting angles and in doubling the square is featured.

Chapter 2 introduces the reader to basic definitions and material. The relationship between algebraic and projective curves in the plane is given and the multiplicity at a point is defined. Chapter 3 builds on this by presenting the algebraic properties of curves (the book treats only plane curves). Highlights include Bézout's theorem (with applications) and a discussion of the additive group law on smooth cubic curves. Resultants are used to define intersection multiplicities and the calculation of these multiplicities is given algorithmically. The topological viewpoint is presented in Chapter 4 with a discussion of the degree–genus formula in the smooth case. Two treatments of this are presented; the first is in terms of deformation to a set of lines while the second involves cutting and gluing. This leads to a discussion of branched covers of the projective line and of the Riemann–Hurwitz formula. Complex analysis is added in Chapter 5 with the introduction of the central concept of a Riemann surface (the example of the projective line and of the complex torus being treated in detail). The Wierstrass $\wp$-function and its connection with cubic curves are treated comprehensively.