

Flight Testing of Helicopters

By F. O'HARA, M.A.

A paper presented to the Helicopter Association of Great Britain on Saturday, 25th March, 1950, in the Library of the Royal Aeronautical Society, London, W.

H. A. MARSH, A.F.C., A.F.R.Ae.S., in the Chair

INTRODUCTION BY THE CHAIRMAN.

Ladies and Gentlemen,

On behalf of our Association, I have pleasure in extending the warmest welcome to our guests, and in particular to our guest of honour who is to lecture to us on the Flight Testing of Helicopters.

MR. FRANK O'HARA is an M.A. of both Cambridge and Edinburgh, and is a Principal Scientific Officer of the Ministry of Supply stationed at the Airborne Forces Experimental Establishment at Beaulieu.

He joined A.F.E.E. in 1942 on leaving his previous appointment at the Marine Aircraft Experimental Establishment at Helensburgh. During the last 8 years the A.F.E. have increasingly looked to him for the solution of their research problems and as a fertile source of inspiration in overcoming the many difficult practical problems which the Establishment has found.

In the course of its work the A.F.E.E. has been responsible for developing the technique, and the necessary background of theoretical and practical knowledge, required for the flight testing of helicopters for the Ministry of Supply. MR. O'HARA has taken a leading part in this work with the results which he will present in his excellent paper.

Our lecturer is no zealot of the rotary wing movement. His investigations have been carried out with no more and no less skill and care than he would apply to any other subject giving comparative scope for scientific investigation. I think this consideration will be of particular interest to those of us who, through long association with rotary wings, sometimes confuse familiarity with our problems with intelligence in thinking about them.

His paper is not confined to a theoretical treatment, and it deals with many practical problems which I believe should lead to a lively discussion.

Figs. 1, 8, 9 and 10 are Crown copyright reserved and are reproduced with the permission of the Controllers, H.M. Stationery Office.

Introductory.

The aspects of flight testing with which I propose to deal in the present paper are, firstly (and mainly), the type-testing of helicopters, and secondly, the investigation of operating techniques. For fixed wing aircraft, the testing methods in both these branches of the subject are now fairly well established, although they are of course subject to constant revision and extension in the light of further practical and theoretical developments. Helicopters are at a less advanced stage of development than fixed wing aircraft, and it will be readily understood that the testing methods are in a correspondingly less developed form also.

A point that sometimes arises in considering certain developments for the firs time in relation to helicopters is how far the procedure to be followed should be based on fixed wing practice and experience. It is sometimes argued that to treat helicopters as other than entirely different from fixed wing aircraft can be misleading and that a new approach must be adopted; there exists also the opposed view that helicopters, being aircraft, should be treated in exactly the same way as fixed wing aircraft. From the point of view of type-testing the truth appears to lie somewhere between these opinions; there is much in common between the measurement of performance in forward flight in helicopters with that on fixed wing aircraft, but a somewhat different approach appears necessary in assessing the stability and control characteristics of helicopters. When one considers operating techniques, however, the question of acceptable standards of performance and comfort and safety arise, and here it is not so easy to decide whether the helicopter should be judged by established fixed wing standards or by new and special standards. If the former policy is adopted the operation of helicopters may be in certain respects handicapped, but before the latter may be considered much more information is required on the operating techniques possible with helicopters now, or shortly to become available.

To those engaged on flight testing work there is a great deal obviously requires doing in the way of developing testing techniques and much about helicopter performance and handling that we should like to find out. At the same time it is clear from comparison of the state of knowledge now with the position a few years ago that considerable progress has been made, and the rate of improvement will increase as experience is obtained on more types of helicopter.

The notes on testing methods, and the suggested developments in testing and operating techniques described in the following sections are derived mainly from work done at the Airborne Forces Experimental Establishment. In parts I have found it necessary, however, to complete the picture, to refer to, and to quote from, the work of other bodies.

TYPE TESTING OF HELICOPTERS.

General.

A schedule of tests has been drawn up at A.F.E.E. to cover in a general way the tests to be made on helicopters undergoing type trials. Many of the aspects of type testing, such as cabin comfort, radio trials, and so on, remain the same for helicopters as for fixed wing aircraft. The methods of making set tests on performance and handling do, however, differ in some essentials and it is with these aspects that I shall concern myself here.

Measurement of Airspeed.

The orthodox type of pitot-static installation can be developed to measure fairly low airspeeds; instruments are in fact already available indicating down to 10 knots, but an operational requirement is now being suggested for an instrument to indicate speeds down to 5 m.p.h. in blind approaches. For test purposes it is also necessary to have an indicator for vertical flight (vertical, that is, relative to the air).

Vertical Flight.

The problem in vertical flight has been, to some extent, solved by the dragpendulum method in which the bowing of a long cable (150-200 ft.) with a weight

at the end, indicates slight forwards or sideways motions of the helicopter. It is not possible on some helicopters, however, to hang the pendulum so that the pilot can see it. It appears possible in some steady flight conditions (not the vortex-ring state) for the pilot with practice to control the aircraft by reference to fuselage attitude alone and to make adjustments to approximate to vertical flight by means of guidance from an observer viewing the pendulum. This arrangement is obviously not entirely satisfactory and an attempt is being made to develop a system giving the pilot the indication of deviation from vertical flight.

Near the ground it is possible in low winds to gauge vertical flight by reference to an object moving with the wind. This method can also be applied to determining the vertical flight performance at altitude by making the tests near suitable high ground (without coming within the ground cushion).

It is sometimes suggested that in view of the difficulty of attaining vertical flight the performance of this state is of academic interest only, particularly as the performance improves rapidly with forward speed. There are evident technical reasons for measuring it however. Helicopters will undoubtedly be required to operate in a near-hovering state, with a definite likelihood, due to uncertainty about airspeed and the flight state generally, of approaching the vertical flight conditions. Pilots will not operate at slow speed with confidence unless the performance in the vertical state is at least adequate for hovering.

Forward Flight.

The use of the normal type of pitot-static system in forward flight is complicated not only by the necessity for measuring low airspeeds but also by the difficulty of finding positions for the system which are not affected in a complex manner by the flow from a rotor. The arrangement adopted on Sikorsky helicopters, with the pitot situated between the top of the fuselage and the rotor, and the static vent on the rear fuselage, gives a reasonable position error correction in level flight but rather different values in climbing and autorotative descent, as shown for the Sikorsky S.51 in Fig. 1. Apart from the variation with flight conditions the main point to



note is the large change occurring in climbing flight on the speed range from 40 to 60 knots; it is not in fact possible to determine speed satisfactorily by the A.S.I. in this region (covering an E.A.S. range of about 35 to 65 knots).

Position error corrections are measured using a trailing air-log, and also a trailing static. It is important to ensure that the trailing instrument is not within the rotor downwash. Estimates made of the downwash angle for a typical helicopter in level flight are shown in Fig. 2. It has been confirmed in flight that the instruments are clear of the downwash if trailed from a forward point on the aircraft at speeds down

The Journal of the Helicopter

It may also be noted that on the current types of helicopter an instruto 20 knots. ment on a strut two feet forward of the nose of the aircraft is outside the downwash at forward speeds of 15 knots and over. Measurements in flight have shown that the position error correction for the S.51 consists of a fairly regular static error but a widely varying dynamic error.

There is a need for the development of low reading airspeed instruments for flight testing. Various schemes have been considered and tried out without great success. It seems that an airlog of more sensitive form than the one at present



Fig. 2. Angle of Rotor Downwash for a typical Helicopter in level flight.

available-originally designed for speeds of not much less than 40 m.p.h.-might fulfil a useful function and perhaps record for speeds down to 3-4 m.p.h. For very low speeds in the range 0-10 m.p.h. we have considered a possible extension of the drag-pendulum scheme in which the curvature of the cable and displacement of the weight might be used as a measure of the airspeed. Estimates made for a 150 ft. cable by the method in Ref. 1 are shown in Fig. 3. So far no positive flight results have been obtained and it is probable that it will be useful only as a qualitative indication of speed.

Indicated Airspeed in Steep Flight Paths. The helicopter is capable of flying in a very wide range of climbing and gliding angles. It is now well known (because of its significance in blind flying) that the fuselage attitude is practically a function of speed only and does not vary with flight path angle. One is led then to consider how the airspeed indicator operates in steep flight paths. Only small errors will arise for the pitot system at angles up to 10 deg. to the airflow, but at greater angles the errors will increase until no real reading is obtained for vertical flight. This effect can account in part for the difference in P.E.C. in level, climbing, and gliding flight, and suggests that it might be more helpful to assume the airspeed installation to measure the horizontal flight speed, with appropriate position error corrections. Representation in this manner of the S.51 figures does not however appear to lead to any greater uniformity in the P.E.C

This uncertainty about the airspeed indication could be reduced by fitting a



swivelling pitot system. It might also be obviated by use of a flight path angle indicator which, in conjunction with knowledge of rate of climb or descent, gives the required picture of the performance of the aircraft. The development of the flight path indicator is, however, fraught with as many, if not more, difficulties than the development of the airspeed measurement installation.

Fig. 3. Bowing of cable with speed.

CLIMB AND LEVEL SPEED PERFORMANCE.

Measuring Technique.

Measurements of rate of climb in forward flight and of level speed performance are made on helicopters in the orthodox manner. At low airspeeds the problems are essentially those described in connection with the measurement of airspeed. The airspeed indication becomes less satisfactory at low speed and in addition stability and control generally deteriorate at lower speeds, so that the pilot usually finds it less easy to maintain a steady flight condition.

At operational loads helicopters do not generally have a very large rate of climb and are sensitive in consequence to atmospheric disturbances. At one time we proposed using a flight path indicator for more accurate measurement of performance, but difficulty in instrument-development led to a, perhaps temporary, abandonment of the scheme. Some interesting results were, however, obtained with an existing instrument at R.A.E.; these showed that the scatter in measured performance results could be greatly reduced by use of such an instrument. We find that sufficiently consistent results can be obtained in good weather and early in the morning.

Presentation of Results.

The manner in which the climb results should be presented merits some consideration. Following normal practice with fixed wing aircraft, rate of climb would be plotted against flight speed. This leads to some difficulty, however, as the vertical flight region is approached; in vertical flight, for example, the flight speed is not strictly zero but is in fact equal to the rate of climb or descent. An illustrative curve

of rate of climb plotted against the horizontal component of flight speed is shown in Fig. 4a; an enlarged version of the low speed part of the curve is given in (1) in Fig. 4b, while the curve (2) in this figure shows the same data plotted against flight speed. It will be noted that at X the helicopter is in vertical flight.



Fig. 4a. Illustrating curve of rate of climb against speed.

Plotting against flight speed does not, therefore, give a satisfactory pictorial representation of the performance at the low speed end, and this is the reason for following rotating wing design practice and plotting rate of climb (or vertical speed) against horizontal speed, VH. In the neighbourhood of the best climbing speed there is effectively no difference in the form of the curves so that the A.S.I. for the best climbing speed can equally well be decided from the second form of presentation.





Fig. 5. Diagram of Forces on Helicopter.

Fig. 4b. Enlarged view of low speed section of rate of climb curve.

PERFORMANCE ANALYSIS.

General.

The accepted methods of theoretical analysis of helicopter performance are based on different equations from those of fixed wing aircraft. It follows in consequence that the methods for reducing measured performance data to standard conditions are also to some extent different. It was seen from the first consideration of reduction methods for helicopters that existing methods of performance estimation were not directly applicable for reduction purposes, because quantities not normally observed, and in particular the disc incidence i, occur in the final performance equations. It was found, however, that the equations can be derived in a modified form which is suitable for reduction purposes by eliminating the disc incidence i. It will be seen that the form obtained for the equations provides also a simple method of performance estimation.

The basic performance equations used are (the list of symbols is given at the end of the paper) :---

- - V', the resultant airflow at the disc, is given by
 - $V'^2 = V^2 + v^2 + 2Vv \sin i$
- (B) the energy equation given in R. & M. 1830 (Ref. 2),

$$EP = WV_c + Wv + \frac{\nu}{8} C_D \ bc \ R \ \Omega^3 R^3 \ (1+3\mu^2) + V'D \qquad \dots \dots \dots (3)$$

(C) the rotor power equation given by WALD (Ref. 3), in which the rate of work done by the thrust is equated to the effective power at the rotor, less that expended in rotating the blades,

$$EP - \frac{r}{2} C_D \ bc \ R\Omega^3 R^3 \ (1 + \mu^2) = Wu \qquad \dots \dots \dots (4)$$

Equations (3) and (4) are evidently related to each other; the connection can be shown simply as follows.

From (3)

 $EP - \frac{\rho}{8} CD$ bc $R\Omega^3 R^3 (1 + \mu^2) = WVc + Wv + \frac{\rho}{4} CD$ bc $R\Omega RV^2 \cos^2 i + V'D$ Of the terms on the right hand side of this equation it may be shown that

 $Vc = V \sin \chi$ where χ is the angle of climb,

 $\frac{\rho}{4}$ C_D bc R Ω RV² cos ²i = HV cos i where H is the rotor drag force.

Hence

 $EP - P_R = Wv + V(W \sin \chi + H \cos i + D).$

Considering the forces on the helicopter, as shown in Fig. 5, by resolving along the flight path

 $W \sin \chi + H \cos i + D = T \sin i \qquad(5)$ Then with T = W

 $\mathbf{EP} - \frac{\rho}{\mathbf{v}} \operatorname{CD} \operatorname{bc} R\Omega^3 R^3 (1 + \mu^2) = W(\mathbf{v} + V \sin \mathbf{i}) = W \mathbf{u}.$

Thus equation (4) is derived from (3). It would evidently be possible to use with (3) the force equation (5), instead of (4), as the basis for performance analysis. For analysis of measured performance it is convenient to work in terms of

For analysis of measured performance it is convenient to work in terms of equivalent airspeeds which are denoted by the suffix i. Then putting $x = V_i/v_o$ and $y = v_i/v_o$ it follows from (1) and (2) that

x sin i =
$$\frac{1}{2}\left(\frac{1}{y^3} - y - \frac{x^2}{y}\right)$$
(6)

Also the energy and power equations may be combined and reduced to the form

$$z + \frac{d_c}{y^3} = \frac{1}{2} \left(\frac{1}{y^3} - y - \frac{x^2}{y} \right) - r_c \left\{ x^2 - \frac{1}{4} \left(\frac{1}{y^3} - y - \frac{x^2}{y} \right)^2 \right\} \qquad \dots \dots \dots (7)$$

where $z = Vc \sqrt{\sigma}/v_0$; d_c and r_c are fuselage drag and rotor coefficients respectively,

8

The Journal of the Helicopter

Similarly, with $\mathbf{p} = 8\text{Eqc}/\text{CD} - 1$, and $\mathbf{w} = \Omega i R/v_0$ the power equation becomes $\mathbf{w}^2 \mathbf{p} = \frac{1}{r_c} \left\{ \frac{1}{y^3} + y - \frac{x^2}{y} \right\} + \left\{ x^2 - \frac{1}{4} \left(\frac{1}{y^3} - y - \frac{x^2}{y} \right)^2 \right\} \dots (8)$

Equations (7) and (8) are in convenient form for general performance analysis. Knowing the fixed constants of the helicopter, y is determined by (8) for chosen values of q_c , x, w and σ . The corresponding value of z (and therefore of V_c) follows from (7). To use the equations it is necessary to chart the functions in the manner shown in Ref. 4.

Further simplification is possible in level flight since (7), with z = 0, can be solved to give x^2 in terms of y, and so with (8) a numerical relationship can be obtained between x, w^2p , rc and dc. This relationship can be plotted in lattice form to give the level speed in terms of power, for known fuselage drag and rotor characteristics.

In vertical flight, it can be shown, neglecting the drag of the fuselage, that the performance is determined by the single equation

$$z = \frac{1}{2} r_c w^2 p - \frac{z}{r_c w^2 p}$$

This formula is based on the simple momentum theory for the rotor thrust and is known to give optimistic results for performance; this point is discussed in a later section of the paper.

Performance Reduction Methods.

Current methods of performance reduction for fixed wing (propeller driven) aircraft are derived from the performance equation assuming laws of variation of engine power and propeller efficiency based on experimental data. Corrections to observed performance data are usually made at constant propeller speed and, in the case of climb, at constant airspeed. Helicopter reduction methods are obtained similarly from the performance equations assuming a law of power variation and that the rotor thrust equals the aircraft weight.

The performance of the helicopter is determined by the power and energy equations (7) and (8). The induced velocity term y, may be eliminated, and the parameters of performance are q_c , w, x and z. These may be used to develop a height change method of reduction; the law of power variation determines the height in the standard atmosphere at which the measured performance specified by w, x, z, applies. The disadvantage of this method is that a change in the true rotor speed is involved whereas in practice rotor speed is one of the fixed flight conditions. The height change method also introduces difficulties when the range covered in the measured performance is limited and so the methods for constant pressure height and constant rotor speed have been adopted.

The method used can be illustrated for climbing flight. Effectively it depends on finding the effect of temperature on the induced term y. The value of y in observed conditions is found from a chart of equation (7), at known x and z. The variation of y with temperature is found from (8), and the corrected value of z found with the new value of y (for the same x) from the chart of (8).

Vortex Ring Operating State.

It will be realized that the theoretical analysis of performance above is strictly only applicable to conditions for which the simple momentum theory for thrust obtains. The rotor operating conditions are more complex in the vortex ring state in which the airflow is partly upward through the disc. This can only occur when the disc incidence i is negative and when the component of speed normal to the disc, V sin i, exceeds the downwash at some part of the disc. An upper envelope to the region is given by i = 0, and then from (6),

$$\mathbf{x}^2 = \frac{1}{\mathbf{y}^2} - \mathbf{y}^2.$$

Further, from (7),

$$z = -\frac{d_c}{v^3} - r_c x^2.$$

A curve of z against $x_{\rm H}$ is drawn from these equations at (a) in Fig. 6; it is taken to a low value of $x_{\rm H}$ and continued to the point z = 0 at $x_{\rm H} = 0$.

Flight experience has shown that control of the helicopter becomes less satisfactory in the vortex ring condition; the control becomes rough and slight changes

may lead to fairly violent disturbances in flight condition. Little flight data is available on the extent of the region in which this is likely, but recently DREES (Ref. 5) has produced a theoretical boundary (with some experimental basis). A similar result can be produced by the above analysis using flight data available for vertical flight. In the first place it appears that disturbance of control is not likely at lower rates of descent than correspond to z = 0.4. Using this as a pointer to the amount of upward flow required before serious disturbances are likely, a new curve (b) has been drawn in Fig. 6 parallel to the i = 0 line to give a possible indication of the upper envelope to the so-called "loss of control" region.

The lower envelope to this region is related to the boundary between the vortex ring and the windmill brake states of operation. In the windmill brake condition the airflow is upwards through the disc, and at the boundary, $u = v + V \sin i = 0$. This boundary may be derived in a manner similar to the upper one, but because of the breakdown of the momentum theory at u = 0 it is necessary at low speeds to determine the curve as the mean of those for finite positive, and negative values of u. The line terminates at the ideal autorotative point on the vertical flight axis.



Fig. 6. Vortex ring operating region.

Alternatively, this boundary may be determined by the rotor velocity charts of the type proposed by HAFNER (these are discussed in the section on low speed performance.) These charts provide a relationship between $x \sin i$ and $x \cos i$ which, in conjunction with the energy equation in the form

$$z = \frac{\alpha_c}{x^3 \sin^3 i} + x \sin i - r_c x^2 \cos^2 i$$

provides the lower boundary curve. This is the line (c) plotted in Fig. 6.

This curve determines the rate of descent required to offset the mean induced velocity at the rotor. In practice, because of the variation of induced flow across the disc, some downwards flow will persist to greater rates of descent. On the other hand it may be said, by comparison with the upper boundary, that disturbance of control is not likely without some appreciable downwards flow; the u = 0 curve (c) may therefore be a reasonable approximation to the lower boundary to the "loss of control" region.

TAKE-OFF AND LANDING PERFORMANCE.

General.

Near the ground the performance of a helicopter is improved by a ground cushion effect. This effect decreases with height, extending in all to a height of about one rotor diameter. A fair amount of information, both experimental and theoretical (Ref. 6), is available on this effect for zero airspeed, but nothing seems to be known

The Journal of the Helicopter

https://doi.org/10.1017/S2753447200004984 Published online by Cambridge University Press

of how it varies with forward speed. The analysis of take-off performance in the ground cushion, and in particular of the effect of wind on the performance, has in consequence to be based on rather arbitrary assumptions.

The performance out of the ground cushion is, however, in some ways the more important aspect of take-off because in certain circumstances (mountainous country for example) it may not be possible to rely on full assistance from the ground cushion. For this reason it is customary to measure a helicopter take-off to a height well clear of the cushion; the height commonly used at present is 100 ft. compared with the 50 ft. normally used with fixed wing aircraft.

The take-off of a helicopter may be made in different ways. It may be possible to make a vertical take-off, ascending at zero forward speed (to cover the no-wind case) to a height of 100 ft. Most often helicopters make what can be described as a cushioned take-off in which the aircraft ascends to a height within the ground cushion, then accelerates forward to gain translational lift before climbing away; in this case the acceleration should be made without loss of height. For helicopters in an overloaded condition it may be necessary to make a running take-off in which the speed needed to develop lift to become airborne is obtained in a ground run.

Similar division can be made for the normal power-on landing of a helicopter; the engine-off landing case is considered later.

Measurement of Take-off and Landing Performance.

The take-off and landing performance of a helicopter can be measured using the standard F.47 take-off camera. Because of the low speeds involved, however, it is very important to make the tests in calm conditions with little wind. It is possible to allow for the effect of a slight wind on the take-off distance by starting the take-off with backward motion to follow a marker moving with the wind ; distances during the take-off are measured relative to this marker. The time to height performance in a zero forward speed take-off can be fairly easily made with the helicopter moving backward with the wind. It is necessary for accuracy in this method to take into account the presence of a wind gradient ; the rough measurements we have so far been able to make in connection with our tests did not reveal any appreciable gradient up to the greatest height of take-off measurement.

It has been found in practice that the performance at low speed is more reliably determined by finding the curve of rate of climb against speed; the range of speed can be taken up to the best climbing speed, or further, to fix more firmly the shape of the curve. The curve then shows the rate of climb vertically to the ground in winds of different strengths.

Theoretical Analysis of Low Speed Performance.

In connection with the investigation of low speed performance, estimates have also been made of the variation of rate of climb with speed. In vertical flight the momentum theory is known to be optimistic, but it is possible to make reasonable estimates of the vertical performance using a curve of rotor operating coefficients based on empirical data ; in the form of curve now commonly used V_i/v_0 (= x) is plotted against u_i/v_0 (= s) in place of the 1/F against 1/f relationship originally used by GLAUERT.

For low forward speed the simple momentum theory will evidently not produce results consistent with those for vertical flight. An alternative basis has been proposed by HAFNER (Ref. 7); he points out that writing the momentum thrust formula in the form

$$x^{2} \cos^{2} i + (x \sin i + y)^{2} = \frac{1}{y^{2}}$$

shows that the plot of x cos i against x sin i for a constant value of y is a circle; the centre and radius of this circle can be determined by the established data for vertical flight. For performance estimation it appears more convenient to present the data in the form shown for part of the field in Fig. 7a with the curves plotted for constant values of s.

Performance estimates from this chart show the rate of climb to increase with speed more rapidly then observed in flight tests. It seems possible, however, to modify and ultimately fully determine the set of curves by analysis of reliable flight data, in the same way as has been done in the vertical flight case. It is clear that the analysis is more involved than for vertical flight and that a considerable body of performance data is needed to outline the curves. We are analyzing the performance

figures using engine power from makers' curves, to determine in the first place a curve of u against V_H; it is then possible to determine, for a selected value of u, the disc incidence i, from the energy equation with the observed value of V_c. From some results already obtained it appears that in the regions so far explored the curves may take the modified form shown by the dotted line curves in Fig. 7a. In Fig. 7b the data from Fig. 7a has been transformed to the original form showing the deviation of the v curves from the theoretical circular form.



Fig. 7a. Section of chart of axial flow through rotor.



Fg. 7b. Section of chart of induced flow at rotor.

PERFORMANCE AFTER POWER FAILURE.

General.

The take-off and landing tests defined in the previous section are intended to determine the distances required when particular operating techniques are used; the technique may be the one giving the best performance (that is the shortest distance), or it may be that in which a safe emergency landing is possible following power failure at any stage. More is said of the latter type in the sections dealing with operating techniques. At the present stage it is convenient to consider the performance of a helicopter after power failure; it is necessary for this performance to be satisfactory from the airworthiness angle, but information on it is also necessary for the analysis of operating techniques.

Only a single-engined and single-rotor helicopter is considered here, but the performance of this type of aircraft is best suited for illustrating the principles involved in the change-over from powered to engine-off operation, and in an autorotative glide and landing.

The Journal of the Helicopter

The Transition to Steady Autorotation.

Not a great deal has yet been written about the transition from powered operation to steady autorotation, although it is a performance aspect of obvious practical importance; it is essential, for example, for safe operation to know the height required to reach steady autorotation, and it is also important to know how the rotor speed varies and in particular whether it tends to fall to a dangerously low value early in the transition.

The general nature of the performance in the transition in forward flight of a single rotor helicopter is illustrated in Fig. 8; the records given were obtained on the Hoverfly I. Following the simulated (or real) engine failure the rotor speed starts to decrease and it is essential to reduce the blade pitch very quickly; in the example given the pitch was reduced exceedingly quickly but even so the rotor speed decreased by a considerable amount, tending to fall for a short period even with the pitch at its lowest value.

The variation of rotor speed depends on the way in which the change of flow through the rotor disc takes place. In powered level flight the flow through the rotor is downwards, while in steady power-off flight the resultant flow is upwards. Until the upwards flow is sufficiently established after engine cut the rotor speed tends to decrease.

The combination of pitch and rotor speed reduction results in a large loss of lift and in a high initial downward acceleration. The height lost in reaching steady autorotation also depends on the way in which the new rotor flow (and therefore the thrust) builds up. In the example in Fig. 8 the height lost is about 100 ft.



The height required to make the transition appears to depend in general on the relationships between the initial and final values of the flight and rotor speeds. From a relatively high speed, for example, it is possible by tilting the disc back (subject to aircraft control) and thereby reducing the flight speed, to build up the flow quickly. From low speed on the other hand it may be necessary to tilt the disc forward to gain speed and to await the increase in the disc incidence resulting from the steepening of the glide path. From the point of view of rotor speed it is evidently advantageous to have a relatively high rotor speed because of the larger reserve of energy. Further, at lower rotor speeds the blade pitch in powered operation is higher and a greater change is involved in going in to autorotation ; there may be a relatively greater initial loss of rotor speed before the reduction of pitch is completed because of higher blade drag at high pitch. Serious consequences could follow engine failure with the rotor speed at its normal minimum operating value.

Flight tests are being made to investigate the variation with the initial flight speed of the height to reach a steady glide. From the results on the Hoverfly I it appears that on this aircraft the height to reach steady autorotation at a speed giving approximately the best angle of glide varies from about 160 ft. at 40 ft./sec. to 60 ft. at 100 ft./sec.

Measurement of Transitional Performance.

Owing to the varying airflow conditions during the transitional motion, the readings of the ordinary airspeed indicator and altimeter are unreliable, the position errors not being known accurately. The use of trailing instruments to avoid downwash effects is not satisfactory because of possible differences in the motions of the aircraft and instruments, and also because of lag in pressure transmitting instruments. To overcome this difficulty tests are now being made with accelero-meters; measurements are made of fore-and-aft and normal accelerations, and of attitude, to find the change in speed and height in any form of transitional motion (including landings). The initial steady conditions are found from the normal flight instruments.

Theoretical Analysis of Transitional Motion.

A study of the theory of transitional motion is being made in connection with the flight test investigation. The first analysis of this type of motion appears to have been given by HOHENEMSER (Ref. 8). His theory is developed on the assumption that the flight velocity remains constant during the motion; it appears from this analysis that the height lost during the change to steady autorotation increases slightly with speed.

For fuller investigation of the transitional motion, and in particular for an indication of the optimum manner of attaining steady autorotation it is necessary to permit variation of the flight velocity during the manoeuvre. An extended form of theoretical analysis in which this is done has recently been developed at A.F.E.E. As in Hohenemser's work, it is assumed that the helicopter pitching attitude and the disc attitude to the horizontal are constant. The effects of unsteady aerodynamic conditions are not considered; it is assumed that the rotor thrust is given by the normal formulae for steady conditions.

The motion of the helicopter is considered relative to axes fixed in space, 0x horizontal forwards, and 0z vertical downwards. With variables μ , ν (= v₀/V), i (the disc incidence) and τ (= gt/v₀) the equations of motion reduce to the form :

$$V \frac{di}{d\tau} = -T' \cos i + v^2 \cos (i + a) + \frac{h' \sin i \cos i}{\mu}$$
$$\frac{d\nu}{d\tau} = T' \sin i - v^2 \sin (i - a) + d' + \frac{h' \cos i}{\mu}$$
$$v \frac{d\mu}{d\tau} = -T' \mu (\sin i - \lambda \mu^2 u') + v^2 \mu \sin (i - a) + h'(\frac{1}{2}\lambda - \cos i)$$

where $h' = C_D k/8$ and $\lambda = WR^2/J$. $T' = T/2\pi\rho R^2 V^2$, and $u' = u/2\pi\rho R^2 V^2$, can be shown to be functions of i and μ , and these, in conjunction with the above equations determine the transitional motion for selected values of the rotor disc attitude, α , and of the blade pitch; the motion can also be determined when these parameters are varied in a step-by-step manner.

The numerical work involved in applying this analysis is fairly heavy. It does not seem possible while including variation of flight velocity to reduce the equations to a form giving a general analytical solution, but certain approximations, possible without much loss of accuracy, result in simplification of the analysis; in the initial stages of the transition, for example, when there is a great loss of lift, the rotor and fuselage drag forces are comparatively unimportant and may be neglected. Considerably more simplification is possible at the later stage when the rate of descent approximates to the steady value (the rotor speed being then possibly little more than its minimum value). It may then be assumed that the thrust is approximately equal to the weight and only two equations are required to determine the motion.

Estimates have been made for a modern type helicopter for different rotor disc attitudes, to show the effect of increasing or decreasing the flight speed in the early stages of the transitional motion (Fig. 9). The initial speed is the same (112.6 ft./sec.) in all cases and it is assumed that the initial rotor speed has been reduced to allow for the loss in speed occurring while the pitch is being reduced.

The Journal of the Helicopter



Association of Gt. Britain.

It will be seen that with the disc tilted forward (a = -5 deg.) the flight speed first decreases slightly and then increases steadily; for a = 0 deg. the speed decreases slightly and for a = 7 deg. it decreases more quickly. The fall-off in rotor speed is in no case large, varying from 7% with a = -5 deg. to 2% with a = 7 deg. In the first two cases the rate of descent approaches a constant value while the rotor speed is passing its minimum; with a = 7 deg., however, the rate of descent appears to be approaching a constant value but then starts to increase again. The change appears to occur in the neighbourhood of the best gliding speed and although the connection has not been determined analytically it appears reasonable to find that the relative gain in utilizing the translational energy should decrease as the speed varies away from a speed corresponding to the best gliding speed. It may not, of course, be possible in practice to make full use of the translational

It may not, of course, be possible in practice to make full use of the translational energy because of the effect of disc position on helicopter control; nor do the different rotor positions necessarily lead to reasonable steady flight conditions. The performance from the above analysis should in fact be taken only as an indication of the trend in the performance for different control positions.

Transitional Motion in Vertical Flight.

The transition from powered vertical flight to steady autorotation in forward flight is similar to the forward flight case. The transition to vertical autorotation will involve greater loss of height and of rotor speed if the rotor descends through its own downwash. No flight tests have yet been made to investigate this type of transition; it is not likely to occur much in practice because the flight path can normally deviate sufficiently from the vertical to avoid the downwash stream.

A theoretical analysis has also been made of the vertical transitional motion. An estimate of the performance in still air for the helicopter is given in Fig. 10; the height lost in reaching steady descent is about 300 ft. A rough estimate indicates that at least 450 ft. will be required to attain steady conditions if descent is through the powered state downwash.

Engine-off Landings.

If engine failure occurs very near the ground it is possible by increasing the blade pitch to maintain the lift for a sufficiently long period to make a safe landing. We intend making an investigation of this type of landing but so far the only information available is theoretical (Fig. 11). Using the transitional motion analysis with a constant rate of descent of 12 ft./sec. it is estimated that the descent can be controlled



Fig. 10. Estimated performance for the transition from hovering flight to vertical autorotation of a modern type helicopter : the effect of the powered condition rotor downwash is not included.

16

in vertical flight from a height of about 45 ft.; from forward flight at 60 ft./sec. descent could be controlled from 95 ft. while the forward speed was reduced to 30 ft./sec.

If failure occurs sufficiently far from the ground steady autorotation can be attained. A normal autogyro type landing can then be made by tilting the disc back for a pull out, slowing down the forward speed, and in addition increasing the blade pitch to reduce the rate of sink. The use of rotor energy in this way is limited either by blade stalling or by excessive upward coning of the blades.

It remains to be considered whether a safe landing can be made before steady autorotation has been reached. It has already been noted that during the transition following pitch reduction the lift builds up quickly to the steady value. If the rotor speed is above the minimum flying value it appears possible to use some of the remaining available rotor energy to reduce the rate of descent by raising the pitch and so increasing the thrust.



Theoretical estimates of the landings possible in this way can be made by the transitional motion analysis; it will not be possible to check fully the limiting cases in flight tests but a partial check may be attempted to determine whether the tendencies in the motion are as predicted theoretically.

STABILITY AND CONTROL.

General Handling Tests.

The final assessment of the handling qualities of an aircraft, on the ground and in the air, and of the convenience and comfort of the cockpit arrangements, must be based on pilots' impressions. For the guidance of pilots a comprehensive list has been drawn up of points to be covered and tests to be made, so that the aircraft can be judged from all aspects. Particular attention is given to the characteristics of the helicopter with the C.G. at the extreme fore and aft positions.

It will be noted that the current types of helicopter have the normal stick and rudder controls with the collective pitch lever as an additional flying control; the characteristics of this control are related to the stability of the rotor in its operating states.

Longitudinal Stability.

In addition to the general handling tests measurements are made of stick position and stick force trimmer positions against speed. The slopes of these lines are a simple indication of the stick fixed and stick free stability with respect to speed.

It would be helpful if the stick position and stick force data could be analyzed to give a parameter related to the stability of the helicopter in the way that the static margin is for fixed wing aircraft. The effect of C.G. shift on a single rotor helicopter, however, is practically only to change the fuselage attitude by an amount independent of speed.

A simple theoretical analysis of the forces on the helicopter in steady level flight. shows where the complication in the static stability analysis arises.

Assuming the rotor thrust equal to the weight, the equation of translational motion is

$$W (\theta - \beta) = H + D$$

= When + WdeV²

where
$$\theta$$
 is the angle of the rotor shaft to the vertical and β is the flapping angle measured positive upwards.

Taking moments about the C.G. (assumed to be on the rotor axis)

 $O = M = W\beta h + Hh + MR + Mf$

where

 M_R = pitching moment due to rotor = W C_F $\Omega^2 \beta$.

 M_f = fuselage pitching moment = $W(l_b + l_{\theta}\theta)V^2$.

In addition the combined cyclic pitch and flapping angles are determined by the condition for zero rolling moment, giving $a_c + (\beta) = 2a_{\alpha\mu}$ approximately (Ref. 9). Using these relations we can eliminate β and θ from the pitching moment equation

and so determine a_c . The form of the final equation is much simplified if we neglect $l\theta$; then

 $O = M = -\alpha_c(h + C_F\Omega^2) + 2\alpha_0\mu(h + C_F\Omega^2) + h_0h_\mu + V^2l_b.$

The stick fixed static stability with respect to speed is given by $(dM/dV)_{ac}$;

$$7 \qquad \left(\frac{dM}{dV}\right)_{\alpha_{c}} = \frac{2\alpha_{0}\mu}{V} \left(h + C_{F}\Omega^{2}\right) + \frac{h_{0}h\mu}{V} + 2Vl_{b}$$
$$= \left(h + C_{F}\Omega^{2}\right) \left(\frac{d\alpha_{c}}{dV}\right)_{M} = 0$$

The form of this relationship does not make it possible to eliminate the unknown helicopter characteristics by varying, for example, the C.G. position as on fixed wing aircraft nor to obtain a convenient parameter of the static stability by analysis of the trim curves.

There are in any case, however, grounds for arguing that static stability with respect to speed is not a satisfactory criterion of the steady flying qualities of the helicopter. The different aspects of this problem have been well presented in recent papers by GUSTAFSON & REEDER (Refs. 10 and 11). They show that the rotor is stable with respect to speed but not with respect to angle of attack; in other words the rotor disc tends to tilt in the direction of an initial change of attitude and an unstable moment results. In addition it is possible for the fuselage to be unstable with respect to angle of attack also.

It may also be noted that it is generally agreed that stick-free stability is nearer to pilots' impressions than stick-fixed stability; a similar form of analysis would be developed in the stick-free case with the condition $a_c = constant$ replaced by one determined by the fact that the stick force is zero.

Longitudinal Manoeuvrability.

Considerably more experience will be required in control and manoeuvrability tests before it is possible to determine the most suitable form for the tests to take. The interpretation to be placed on results obtained in these tests is also the subject for further study.

Tests in which the stick is displaced quickly from a trimmed position, released for stick-free, and held for stick-fixed, are normally made. So far we have relied on pilots' impressions only in these tests but intend recording the subsequent motion as the Americans have done in the work already referred to (Refs. 10 and 11). The American tests showed that it is possible for the normal acceleration to build up at an increasing rate, and not reach a maximum until 3 or 4 seconds have elapsed. In some cases as the manoeuvre developed a form of stick force reversal took place. It is, of course, desirable to have stick forces in the stable direction, so that, for example, a pull force is required to maintain positive normal acceleration in a pull-out at constant speed.

This leads one to consider how far one can apply the manoeuvre theory approach of GATES & LYON (Ref. 12) to helicopters. It appears possible for helicopters, as

18

The Journal of the Helicopter

annrovimately

for fixed winged aircraft, by comparison of the equations for a steady glide with the equation for a circular path with normal acceleration at the same speed, to find the stick movement required to produce the acceleration. A preliminary investigation has shown, however, that the analysis becomes very involved and some simplifying assumption will be necessary to make it of possible practical value.

Dynamic Stability.

Measurements are also made of the dynamic longitudinal stability of the helicopter. In these tests the aircraft is trimmed in steady flight and the speed is then increased or decreased by a selected amount (5 or 10 m.p.h.). The control is held to maintain this speed for a short time and then returned quickly to the original position ; it is important to ensure that the return to position is accurate and a stick position indicator must be used. The subsequent motion of the helicopter is recorded and indicates different degrees of stability or instability. Owing to the lag and uncertain position error of the pitot system, a record is also taken of fuselage attitude and this may be a more reliable indication of the motion. A thorough analysis of the different types of motion occurring has been made at R.A.E. and the work done on the Hoverfly I was well described in STEWART's Lecture in 1948 to the Royal Aeronautical Society (Ref. 13).

Additional Tests.

Vibration Tests.

Considerable attention is now being given to the vibration characteristics of helicopters. Vibration is one of the major problems on helicopters and records will have to be taken at all positions affecting the comfort of pilots and passengers. An apparent increase in vibration with altitude is also requiring investigation. The measurement of helicopter vibrations is complicated by the presence of very low frequencies and shortage of suitable instruments is limiting the work that can at present be done. The Miller acceleration pick-up vibration recorder seems the most suitable ; at present only a 16-channel set up is available, but a two or three channel system of the same kind would be very useful.

Rotor Blade Motions.

The rotor system has a certain amount of freedom of motion relative to the fuselage and it is desirable to investigate whether there is any possibility of interference between rotor and fuselage in any flight condition; the general motion of the rotor in fact relative to the flapping and drag stops is of interest. This is another aspect, however, for which instrument development is required, and at present only general impressions can be used to decide whether the rotor layout is satisfactory in operation or not.

OPERATING TECHNIQUES.

General.

The performance obtainable from an aircraft depends on the way in which it is operated. We may consider, for example, the "best" and the "safe" performances, the difference being probably most easily illustrated in the take-off or landing cases. Whatever the type of performance the aircraft should be safe from the point of view of airworthiness and the distinction lies in the different consequences following power failure. The technique used to obtain the best take-off performance does not necessarily permit a safe emergency landing after power failure; the technique for the safe performance, on the other hand, does.

The best performance is assumed to be that in which, for example, the take-off distance is a minimum and is required for some operational roles; it is also in some circumstances the most suitable basis for the comparative assessment of performance. The safe performance is required for normal civil operation; the techniques for safe operation, particularly in take-off and landing, still require investigation and these are discussed in the next section. This is followed by a section on blind flying on helicopters. The general use of helicopters depends on it being possible to operate in blind conditions and the instruments required and the appropriate flying techniques are discussed.

Take-off Techniques.

We are concerned with the technique for take-off in which it is possible to make a safe landing in the event of power failure at any stage. The performance after power cut on a single-rotor and single-engined helicopter has been discussed in a previous section ; it appears that the greatest height from which it is possible to make a safe landing by increasing pitch and so utilizing immediately the rotor energy, increases with speed. Similarly the least height from which it is possible to go into autorotation (without necessarily attaining steady autorotation) and make an engine-off landing, decreases with speed.

In assessing the performance after an actual power failure it is necessary to include some allowance for the delay in the pilot's reaction. It appears possible for 1.5 to 2.5 secs. to elapse before pitch adjustment is started and in this period the rotor speed can fall off by a considerable amount. An illustration of the variation of rotor speed before and after the pitch is altered is given for the vertical flight case in Fig. 12.



Fig. 12. Illustrative diagram of performance of single rotor helicopter following power failure in hovering flight.

Engine cut occurs at (a) and pitch adjustment is made at (b); curve A shows the variation of rotor speed when the pitch is reduced to the autorotative value, and B the rotor speed when the pitch is increased. The corresponding rates of descent are also given. For completeness an indication is given of the type of motion resulting when the pitch is increased at (c), while the rotor speed is building up by autorotation. A safe landing is possible in this case if the rate of descent is not greater than the maximum landing value when the rotor speed falls to the minimum operating value as shown in curve C.

Referring back now to the height limits for a safe landing following power failure for speeds above that at which the upper and lower boundaries intersect it is theoretically possible to make an emergency landing from any height. Information on these boundaries, which is essential for the determination of safe take-off techniques, is not easy to obtain, and so far only rough figures for earlier type helicopters are available. For the Hoverfly I the band of height appears to extend in vertical flight from 30 to 400 ft., and narrows to zero at a forward speed of about 70 m.p.h. The

aim in some modern types of helicopter is to reduce this band to much smaller proportions. It is doubtful, however, whether it is possible to eliminate it completely on single engined machines without requiring exceptionally skilled pitch manipulation by the pilot. It appears more likely for safety to be obtained through a multi-engine arrangement, but even in this case it is probable that at low speeds some height loss will be involved in the transition from twin to single-engine operation.

On a single-engined helicopter a safe take-off can, however, be made by keeping below the lower height band until sufficient speed is attained. When the distance for an emergency landing forward is added to that for take-off the overall ground requirement may be prohibitively great. Consideration must also, therefore, be given to take-off in a curved path; this would be acceptable providing a technique can be used in which it is possible to avoid a cross wind emergency landing. This means delaying turning flight until a height is reached from which it is possible to turn back into wind or to turn to the wind before landing.

A backwards take-off has also been suggested; this would have the advantage of making the take-off area visible to the pilot if an emergency landing must be made and would result in some saving in forward distance, limited, however, by the fact that it would be possible to rise safely only to the height from which a landing could be made following power cut at zero airspeed. In addition it seems undesirable for the helicopter to travel backwards unless the pilot has a satisfactory rearwards view.

It may be that even with the most favourable techniques the distance or space required for a safe take-off is greater than can normally be accepted. The only procedure then open appears to be to find out how to keep the unsafe period to a minimum in a take-off in the maximum acceptable space.

Blind Flying.

The problems of instrument flying on helicopter are now fairly well known. It has been found possible, using the standard blind flying instruments, to fly helicopters in blind conditions, and in fact regular night and blind flying operations have now been carried out by B.E.A. Notwithstanding, however, it still remains true that considerable concentration is required and it becomes very tiring in disturbed air conditions. It is doubtful whether control could be maintained for a prolonged period in rough air conditions by other than a pilot well practised in helicopter instrument flying.

Difficulty arises mainly in connection with the longitudinal control; the current type of helicopter is unstable longitudinally and in addition the fuselage attitude gives a less satisfactory indication of the flight state than on a fixed wing aircraft. The attitude varies only with speed (apart from random disturbances) and gives no indication of the flight path angle; the random variations in attitude can be comparatively large and tend to mask the steady attitude variation. The only definite indication of speed is obtained in consequence from the A.S.I., which is not satisfactory as the main flying instrument because it indicates only changes which have occurred and not a tendency to change, which is of greater assistance to the pilot.

It is clear that instrument flying on helicopters would be much simplified by an improvement in longitudinal stability. At the same time it can be argued that the fault lies partly in the blind flying instruments for, theoretically speaking, the instruments should enable the aircraft to be flown in much the same way as in contact conditions. A possible reason for the difference found so far in practice is evident from an analysis of the flying technique in contact conditions; near the ground pilots do not rely only on the horizon for attitude assessment but also judge their flight path relative to known objects on the ground. Away from the ground it is necessary to assess the flight state relative to the horizon and, indirectly (even in contact conditions) from instruments; pilots appear to find flying in this way less comfortable than near the ground, and it is to the away-from-the-ground condition that instrument flying corresponds. The apparent increase in rotor vibrations with height will also tend to make flying at altitude less comfortable than near the ground, but this is separate from and additional to the difference already considered.

It seems that with present types of helicopter the difficulty of blind flying cannot be easily resolved. Some improvement might be effected by arranging for the bar in the artificial horizon to vary with airspeed as well as with fuselage attitude ; in other words, by increasing the sensitivity to speed without affecting the sensitivity to disturbances. There is some ground for thinking, however, that fuselage attitude

will not ultimately be found a completely satisfactory guide to follow for speed control of a helicopter. A more direct approach is to use the rotor disc attitude to the horizontal. This is complex, however, in helicopters with hinged blades, because it involves firstly, measurement of blade flapping angles to give the disc attitude to the fuselage and secondly, combination with the fuselage attitude to give the disc attitude to to the horizontal. The results of flight tests now in progress should indicate whether it is necessary to consider undertaking development work along these lines.

CONCLUDING REMARKS.

There are many aspects of type testing and operating techniques on which I have not touched but sufficient has been said, I think, to give an impression of the methods being used and of the problems that face us. I am conscious that in parts at least there is emphasis on what might be done rather than on what is being done; this, however, is representative of the present position in helicopter flight testing, for it is very much in a state of development—and all the more interesting a field of work because of that.

Acknowledgments.

The testing methods here described have been developed in the main during the past few years at the Airborne Forces Experimental Establishment, and I am indebted to my colleagues at this Establishment for their co-operation in this work. I should like to thank in particular S/Ldr. CABLE and his fellow pilots for their ever-willingness in even the most umpromising looking tasks; and MR. OLIVER and the others who have assisted in the preparation of this paper. I wish to acknowledge also the assistance derived from discussions on operating techniques with representatives of the Ministry of Civil Aviation.

Finally I wish to thank the Chief Scientist of the Ministry of Supply for permission to present this paper.

REFERENCES.

Ref.	No. Author.	Title, etc.
1.	Glauert.	Heavy flexible cable for towing a heavy body below an aeroplane.
		R. & M. No. 1592. February, 1934.
2.	Squire.	The flight of a helicopter.
		R. & M. No. 1730. 1936.
3.	Wald.	A method for rapid estimation of helicopter perfor-
		mance.
	0.177	Journal of Aero. Sciences. April, 1943.
4.	O'Hara.	A method of performance reduction for helicopters.
_	D	A.F.E.E. Report Res. 26. October, 1947.
5.	Drees.	A theory of airflow through rotors and its application
		to some helicopter problems.
6	Data	The ground effect on lifting propallors
0.	Belz.	NACA Tech Mem No 836 1027
7	Hofner	Rotor systems and control problems in the beliconter
1.	Hamer.	Aeronautical Conference London Sentember 1947
8	Hobenemser	The theory for the control of the blade angle of the
0.	1 tone motor.	Heliconter. Wright Field Trans.
		Report No. F-TS-1004-RE. September, 1946.
9.	Hohenemser.	Performance of rotating wing aircraft.
		N.A.C.A. Tech. Mem. No. 871. 1938.
10.	Gustafson and Reeder.	Helicopter stability.
		N.A.C.A. Research Memo. No. L7K04.
11.	Reeder and Gustafson.	On the flying qualities of helicopters.
	a	N.A.C.A. Tech. Note No. 1799. November, 1948.
12.	Gates and Lyon.	A continuation of longitudinal stability and control
		analysis.
10	Story and	R. & M. No. 2027. February, 1944.
13.	Stewart.	Flight testing of helicopters.
		Journal of Royal Aero. Society. May, 1948.

22

	b	= number of blades.	T′	$=\frac{T}{2\pi c P^{3} V^{3}}$
	с	= rotor blade chord at $r = 0.7R$.		$2\pi\rho R^{*}V^{*}$
	CD	= blade profile drag coefficient at the mean effective lift coefficient.	u 11:	= 10 for more than to refer disc.
	ď	D		
	u	$=2\pi\rho R^2 V^2$	u′	$=rac{\mathrm{d}}{2\pi ho\mathrm{R}^2\mathrm{V}^2}$
	dc	$=\frac{D_0}{2\pi\rho_0 \mathrm{e}^2\mathrm{R}^2\mathrm{100^2}}.$	v	= induced velocity at rotor.
	do	$=rac{D}{WV^2}$	vo	$= \left(\frac{\mathrm{W}}{2\pi\rho_{\mathrm{o}}\cdot\mathrm{R}^{2}\mathrm{e}^{2}}\right)^{\frac{1}{2}}$
	D	= fuselage drag.	v	= flight speed.
	\mathbf{D}_{0}	= fuselage drag at 100 fp s.	Vc	= rate of climb.
	e	= tip loss factor.	$V_{\rm H}$	= horizontal component of speed.
	Ε	= ratio of effective power at rotor to total power.	V′	= resultant air flow velocity at rotor.
	f, F	= rotor operating coefficients.	w	$=\frac{\Omega_{i}R}{1}$
	h′	$=\frac{C_{Dk}}{2}$	W	Vo — sircraft weight
		8	w	V
	$\mathbf{h_{o}}$	$=\frac{H}{W}$	x	$=\frac{v_1}{v_0}$
	н	= transverse rotor force.	Хн	$= \frac{V_i \cos \chi}{\chi}$
	i	= disc incidence.		Vo
	J	= rotor moment of inertia.	у	$=\frac{\mathbf{v}_{i}}{\mathbf{v}_{o}}$
	k	$=\frac{bc}{\pi R}$	z	$=\frac{V_c\sqrt{\sigma}}{T}$
	Μ	= pitching moment.	a	v_0 = rotor disc attitude to horizontal.
	$M_{\mathbf{F}}$	= fuselage pitching moment.	u	- blade evolie nitch emplitude
	M_R	= pitching moment due to rotor.	ac	
-	р	$=\frac{8Eq_c}{Cr}$ - 1.	a _o	= blade collective pitch.
	p	= engine power	β	= blade happing angle.
	PR	= power required for rotor torque	μ	$=\frac{\mathbf{v}}{\Omega \mathbf{R}}$
		due to profile drag.	•	
	qc	$=\frac{P}{bcR_{\rho}\Omega^{3}R^{3}}$	v	= V = air density.
		$C_D k \Omega_i R$	p T	- relative air density
	rc	= 8e ² v _o	U	gt
	R	= rotor radius.	τ	$=\frac{\mathbf{v}}{\mathbf{v}_0}$
	s	$=\frac{\mathbf{u}_{\mathbf{i}}}{\mathbf{v}_{\mathbf{o}}}$	x	= angle of flight path to horizontal.
	t	= time.	Ω	= rotor speed.
	Т	= rotor thrust.	$\Omega_{\mathbf{i}}$	$=\Omega\sqrt{\sigma}$
	Assor	siation of Gt. Britain.		. 23
		······································		