

CORRIGENDUM

Exponential stability for small perturbations of steep integrable Hamiltonian systems – CORRIGENDUM

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Two results are claimed in [4].

On the one hand, a new proof is given of Nekhoroshev estimates of stability for small perturbations of generic steep integrable Hamiltonian systems based on simultaneous Diophantine approximation, which yields significant simplifications with respect to Nekhoroshev's original reasoning.

On the other hand, new values of the exponents of stability are provided, which generalize the sharp exponents obtained in the quasi-convex case by Lochak and Neishtadt [3] and Pöschel [6].

The first claim is valid, but the second one is not.

Actually, the value of the exponents is not justified due to an incorrect application of the Dirichlet theorem on Diophantine approximation in Lemma 4.4 on page 603 and in the alternative given at the end of page 605. More specifically, we need a refined arithmetical property, which cannot be ensured by the Dirichlet theorem. On the other hand, at the price of lower exponents in the statements of exponential stability, the complete scheme of the new proof of generic Nekhoroshev estimates introduced in [4] can be made rigorous. This is the case in [5], where Nekhoroshev estimates are proved for a class of integrable Hamiltonians called Diophantine steep, which strictly include the steep ones. In [5], the construction of [4] is performed with successive basic applications of the Dirichlet theorem. This procedure fills the gap in [4] but with lower exponents in the theorems. Actually, the first issue pointed out above [4, Lemma 4.4, p. 603] corresponds to [5, Lemma A.5.1, p. 923] and the second one [4, p. 605] does not appear at the similar step in [5] for the proof of Lemma A.6.1 on page 925.

In the papers [1, 2], written in great detail, exponential estimates of stability have been proved in the same way for a generic class of integrable Hamiltonians called Diophantine

Morse, which are reduced to the steep case with steepness indices equal to two along rational planes. In this setting, there are no simplifications with respect to the general steep case.

Given the issue in the article [4], to the best of my knowledge, the derivation of sharp exponents in the exponential estimates of stability for small perturbations of steep integrable Hamiltonians remains an interesting open problem.

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REFERENCES

- [1] A. Bounemoura. Effective stability for Gevrey and finitely differentiable prevalent Hamiltonians. *Comm. Math. Phys.* **307**(1) (2011), 157–183.
- [2] A. Bounemoura and L. Niederman. Generic Nekhoroshev theory without small divisors. *Ann. Inst. Fourier* **62**(1) (2012), 277–324.
- [3] P. Lochak and A. I. Neishtadt. Estimates of stability time for nearly integrable systems with a quasiconvex Hamiltonian. *Chaos* **2**(4) (1992), 495–499.
- [4] L. Niederman. Exponential stability for small perturbations of steep integrable Hamiltonian systems. *Ergod. Th. & Dynam. Syst.* **24**(2) (2004), 593–608.
- [5] L. Niederman. Prevalence of exponential stability among nearly integrable Hamiltonian systems. *Ergod. Th. & Dynam. Syst.* **27**(3) (2007), 905–928.
- [6] J. Pöschel. Nekhoroshev estimates for quasi-convex Hamiltonian systems. *Math. Z.* **213** (1993), 187–217.