To the Editor of the *Mathematical Gazette*

DEAR SIR,—May I clarify some matters arising from Prof. John Satterly's letter (Math. Gaz. 44 (350) 1960, December page 296)? I open with a quotation from a Bill now before Parliament:

" 9 Eliz. 2 WEIGHTS AND MEASURES

A Bill intituled an Act to make amended provision with respect to weights and measures, and for commercial purposes. A.D. 1960 "
Part I, Clause 1, (1):

" The yard or the metre shall be the unit of measurement of length and the pound or the kilogram shall be the unit of measurement of mass by reference to which any measurement involving a measurement of length or mass shall be made in the United Kingdom: and
(*a*) the yard shall be 0·9144 metre exactly;
(*b*) the pound shall be 0·453 592 37 kilogram exactly."

It is made clear elsewhere in the Bill that metre and kilogram are exactly equal to the internationally defined *mètre* and *kilogramme* respectively and that pound and kilogram are lawfully written lb and kg. The same definitions came into use throughout the Commonwealth for purposes of science and technology on 1st July 1959 and the Federal Register of that date made them effective in the U.S.A. for virtually all purposes. There can be no doubt therefore that the unified pound and kilogram now to be used are, like the *kilogramme*, units of mass. The " weights and measures " in the title of the Bill must be presumed to relate merely to the material objects used to ascertain masses and lengths.

In the case of units of force there is no Statutory document settling the matter. It has, however, been well covered by the British Standards (BS 350: Part 1: 1959 and BS 1991: Part 1: 1954 when read with Amendment No. 3 of 4th October 1960) and by the International Organisation for Standardisation (covering Anglo-American and metric units and having physicists, mathematicians, engineers, chemists and metrologists from 20 countries at the plenary sessions of the relevant committee) as well as by the *Conférence Générale des Poids et Mesures*, the supreme arbiter for metric units. The two bodies first mentioned agree that in addition to the units of force called dyne (dyn), newton (N) and poundal (pdl)—which correspond to g cm $s^{-2}$, kg m $s^{-2}$ and lb ft $s^{-2}$ respectively—there exist " technical " units of force called kilogram-force (kgf), pound-force (lbf), etc. which are the forces on bodies of mass 1 kg, 1 lb, etc. when these have the standard acceleration, defined exactly to be $g_n = 980{\cdot}665$ cm $s^{-2}$. The BSI defines analogously the ozf and the tonf. Indeed we may say that if XXXXX be the name and X the unit-symbol of any unit of mass then the unit of force called XXXXX-force, and assigned the unit-symbol Xf, is such that 1 Xf/1 X corresponds to $g_n$, exactly. There also exists a unit of mass called the slug defined as (980·665/30·48) lb; 1 lbf thus corresponds to slug ft $s^{-2}$.

It will be noted that the kilogram-force, pound-force etc. are defined by means of dynamical principles. If, for introductory teaching, a simpler definition is needed it must be that these forces are the attrac-

https://doi.org/10.1017/S0025557200215660 Published online by Cambridge University Press

tions on bodies at mass 1 kg, 1 lb, etc. in a standard gravitational environment which, may, if required, be defined as " a place where a freely falling body descends 490·3325 cm in the first second from rest ". So-called gravitational " units "—such as the lbwt defined as the *local* attraction on a body of mass 1 lb are eschewed since they depend on the *local* initial acceleration in free fall, $g$, and are thus not constant quantities. It may soon become significant that if the tensile strength of a steel is 30 tonwt/in² at London it will be 40 tonwt/in² on Mars! It will, of course, be about 30·02 tonf/in² everywhere.

Reference is made in Prof. John Satterly's letter to the different customs of engineers and physicists. Most technologists and scientists now use symbols for quantities (which are regarded a product of numerical measure and unit quantity) rather than for the mere measures of these quantities. If a body have mass $M$ and velocity $u$ then its kinetic energy is $Mu^2/2$ and this is the most convenient formula for engineers and physicists alike. Anyone who cares may write this

$$Wu^2/2g \quad \text{or} \quad W_n u^2/2g_n, \quad (\text{where } W = mg \quad \text{and} \quad W_n = mg_n)$$

but this sort of thing is dying out. Whichever of the 3 formulae mentioned be used, it is not dependent on the units to be used, since the symbols stand for quantities which will carry their units with them thereby bringing in any conversion factors required.

British and American engineers (except for a few aeronautical engineers) have always regarded the pound (lb) as a unit of mass but—most deplorably—they generally used also to write the same name and unit-symbol for the unit of force now called pound-force (lbf). By adopting distinct names and unit-symbols for the units of force they have removed ambiguity. (A few authors have also written kgm and lbm instead of kg and lb for units of mass—a deplorable practice contrary to national and international law and standards and conflicting with a convenient device mentioned later.)

It is greatly to be desired that we should abandon non-metric units and that engineers of all lands will abandon the technical units and use only the newton and dyne and their decimal multiples and submultiples. In the meantime the Standards provide units understood by technologists while at the same time avoiding ambiguity and inconsistency.

Those who find it unfortunate that the name of a quantity, " force " is now built-in to the names of the units kilogram-force, pound-force etc. may care to adopt a device advocated by the present writer. This is to write, instead, kilogram fors (kg f), pound fors (lb f), etc. and to regard the fors (with unit-symbol f) as a unit of specific force (i.e. force per mass) defined as exactly 980·665 dyn/g and therefore corresponding to the standard acceleration $g_n$. This device has, at present, no official or formal backing. It is evident, however, that it does not conflict with law, and Standards.

The use of kgf and lbf is growing both in industry and in university engineering. They now appear in the Examination papers for B.Sc(Eng.) of the University of London. Unfortunately the schools and the school-examining bodies lag behind. If the students reached us, the University teachers, having heard only of the dyne, the newton and the poundal we could well impart information about the other units of force. Better still they should already have appreciated the use of the kgf and lbf. Unfortunately however we generally have to start by trying to erase the belief that engineers are still misguided enough to use pound (lb) both for mass and force or to use a variable quantity (lbwt) as a unit. I hope Sir, that through your valued publication I may thus not only add to the historical matter provided by Professor Satterly but also encourage applied mathematicians and physicists in schools to teach in a manner not discordant with the reforms that have been made.

Yours faithfully, E. J. Le Fevre

*Queen Mary College,*
*London, E.*1.