# I. ELECTRODYNAMICS OF THE PULSAR MAGNETOSPHERE AND WAVE ZONE

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# STRUCTURE OF THE PULSAR MAGNETOSPHERE

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## 1. INTRODUCTION

Motivation for the study of the pulsar magnetosphere is multifold. First of all, energy-wise the magnetosphere is the dominant problem, for the energy emitted in the radio pulses is at most one per-cent of the energy loss inferred from the secular increase in the rotation period of the central neutron star. With the possible exception of gamma-ray emission in a few cases, the pulsing observed in all wavelengths can be regarded as a diagnostic of a more basic phenomenon. A successful model should in fact tell us the total rate of energy loss from the star and its distribution between a low-frequency wave, a possible wind, and emission in the radio, optical, X-ray and gamma-ray bands. As a necessary preliminary, the model should predict the strength of the currents leaving and returning to the star, the location of relativistic particles, and the conditions for transition to a quantum magnetosphere via pair production (Sturrock 1971). But even if magnetospheric theory did not offer such hopes of ultimate links with observation, the problem is so well-defined that it is a challenge to the theorist: it is just intolerable that we should not know how a rotating magnetized neutron star comes to terms with its environment.

Most of the discussion below concentrates on the aligned or nearly aligned cases, with the magnetic and rotation axes nearly parallel rather than anti-parallel, so that any effects due to a finite work function for ion emission are eschewed (cf. Ruderman, this volume). Likewise the electrodynamics is treated classically, in the hope that understanding of the global classical problem will at least indicate when quantum effects should become important.

## 2. THE VACUUM MAGNETOSPHERE AND THE GOLDREICH-JULIAN CRITIQUE

We adopt the simplest model of a perfectly conducting neutron star, rotating rigidly with angular velocity $\Omega\underset{\sim}{k}$, and with a dipolar surface flux distribution for the frozen-in magnetic field. The dipole axis is



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in general inclined at angle $\chi$ to the unit vector $\mathbf{k}$. Such oblique systems are assumed to be "quasi-steady", with time-variations due just to the rotation of the essentially non-axisymmetric structure: i.e. we impose the operator equivalence (applicable to scalars and to cylindrical or spherical polar components)

$$\partial/\partial t = -\alpha\partial/\partial\phi \tag{1}$$

where $\phi$ is the azimuthal angle in cylindrical polar coordinates $(\tilde{\omega},\phi,z)$ based on $\mathbf{k}$. When $\chi = 0$ the system is axisymmetric and so (1) implies time-independence. Applied to Faraday's law, (1) yields

$$\mathbf{E} = -(\alpha\mathbf{k}\times\mathbf{r})\times\mathbf{B}/c - \nabla\psi, \tag{2}$$

a natural division of the electric field into "corotational" and "non-corotational" parts. Inside the perfectly conducting star $\nabla\psi$ vanishes, and $\psi$ may be conveniently normalised to zero within the star. The essence of the magnetosphere problem depends on a reliable determination of $\nabla\psi$ in the different domains surrounding the star. The simplest model appeals to the very small scale-height of a thermally supported atmosphere with a reasonable temperature, and so fixes $\psi$ by assuming the star to be surrounded by a strict vacuum, with no external sources for $\mathbf{E}$ and $\mathbf{B}$. In a prophetic paper (written before the discovery of pulsars), Pacini (1967) applied the vacuum solution already constructed by Deutsch (1955) to a rotating neutron star. Far enough away the field reduces to that of a point dipole, which emits classical magnetic dipole radiation of frequency $\alpha$, at the rate

$$B_s^2\ R^6\alpha^4\ \sin^2\chi/6c^3 \tag{3}$$

where $B_s$ is the surface polar field-strength. Equating this to the observed energy loss $-I\alpha\dot{\alpha}$ ($I$ being the moment of inertia) yields the canonical value $B_s \simeq 10^{12}$ gauss (Gold 1968).

If $\chi = 0$, expression (3) vanishes - the aligned vacuum model is axisymmetric and so is "dead". The Goldreich-Julian ("GJ") critique (1969) was applied to this axisymmetric model, but is extensible to the non-aligned case (Cohen and Toton 1971; Mestel 1971; §7 below). A vacuum exterior solution in general has a non-vanishing component $E_{\parallel}$ along $\mathbf{B}$, whereas inside the star $\mathbf{E}$ is the corotation field and so is normal to $\mathbf{B}$. The associated discontinuity in the normal component $E_r$ at the stellar surface implies a surface charge-density which is subject to Maxwell stresses that greatly exceed the restraining gravitational forces. GJ proposed that as a far better approximation the vacuum conditions should be replaced by the simple "plasma condition"

$$0 = -\mathbf{E}.\mathbf{B} = \mathbf{B}.\nabla\psi. \tag{4}$$

The zero value of $\psi$ within the star would thus be propagated into the magnetosphere, so that $\mathbf{E}$ approximates to the corotation field outside

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as well as inside the star. The required charge density field is given by

$$\rho_e = \nabla.\underset{\sim}{E}/4\pi = -(\alpha/2\pi c)\underset{\sim}{k}.\{\underset{\sim}{B} - \tfrac{1}{2}\underset{\sim}{r}\times(\nabla\times\underset{\sim}{B})\}. \tag{5}$$

In order of magnitude $|\rho_e| \simeq \alpha B/2\pi c$ (unless $|\nabla\times B|$ is locally abnormally large). The build-up of the magnetosphere by the action of electrical forces implies that it should be charge-separated: $\rho_e$ is not the small difference between a large density of ionic and electronic charge, as in a normal plasma, but is now given by either $-n_e e$ or $n_i Ze$, depending on whether (5) is locally negative or positive. Alternatively, one can argue that in a magnetosphere consisting initially of a mixture of electrons and ions, the finite non-electromagnetic forces on the charges require small $\underset{\sim}{E}_{\parallel}$ components which will rapidly drain away the "wrong" charge species. Then, e.g. in an electron domain,

$$B^2/8\pi\rho c^2 \simeq (eB/mc)/\alpha \equiv \omega_g/\alpha \ , \tag{6}$$

and this ratio of a microscopic to a macroscopic frequency is so large, that it justifies a description of the GJ magnetosphere as "relativistically force-free" , or as an "infinitely conducting vacuum".

Anticipating that the GJ charges will have speeds comparable to corotation, one sees that both the displacement and the particle currents contribute $\sim (\alpha\tilde{\omega}/c)^2 B/\tilde{\omega}$ to $|\nabla\times\underset{\sim}{B}|$. Thus well within the light-cylinder "l-c", defined by $\tilde{\omega}_c = c/\alpha$, the field stays nearly curl-free. Most workers assume that near the star the field is unchanged from the effective vacuum dipole, as in the Deutsch-Pacini solution. The Erlangen group (Schmalz et al. 1979, 1980; Schmalz, this volume) are an exception. They have implicitly terms like $rP_1$ as well as $P_1/r^2$ in the magnetic scalar potential near the star - i.e. they postulate that strong currents near and beyond the l-c are causing marked modification to the field structure near the star. The question is important, since by (5) when $|\nabla\times\underset{\sim}{B}|$ is small, a charge-separated GJ region consists of electrons when $B_z \simeq \underset{\sim}{B}.\underset{\sim}{k} > 0$ and of ions when $\underset{\sim}{B}.\underset{\sim}{k} < 0$ (cf. §4).

The GJ condition $\underset{\sim}{E}.\underset{\sim}{B} = 0$ must break down somewhere: if extrapolated to infinity it leads to unacceptable paradoxes in both aligned and non-aligned cases (Michel 1975; Mestel et al 1976). It is perhaps helpful to think of the plasma as insisting that however small the particle rest-mass m, the component $\underset{\sim}{E}_{\parallel}$ cannot vanish everywhere but instead enforces a relativistic mass $\gamma m$ large enough to ensure that inertial forces and possibly also radiation damping are locally important. The consequent break-away of particles from field-lines may be essential for the existence of a return current to the star, and so to the macro-structure of the magnetosphere. Theories of pulsar radiation depend on knowing in which magnetospheric region relativistic acceleration occurs in different geometries, and on the associated particle densities.

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## 3. DEAD MODELS: MAGNETOSPHERIC GAPS

The aligned, axisymmetric pulsar does not present to a distant observer a fluctuating macroscopic charge-current field, so there is no electrodynamic requirement that it emit energy; as noted, a vacuum aligned model would be dead. One can still accept the GJ argument that $\mathbf{E}.\mathbf{B} \simeq 0$ at the stellar surface, and yet ask whether the charges in the magnetosphere can adjust themselves to mechanical equilibrium, but still without any energy-angular momentum loss from the star (see Michel and Pellat, this volume). That this is a non-trivial problem emerges when one studies the simplest possible case, with the whole of the region within the l-c filled with mass-less GJ charges that corotate with the star (as enforced by the $c(\mathbf{E}\times\mathbf{B})/B^2$ drift), and with a vacuum beyond. The electromagnetic problem is then completely determined. Within the l-c $\psi = 0$, and the purely poloidal magnetic field (defined everywhere in terms of a stream function P and the unit toroidal vector $\mathbf{t}$ by $\mathbf{B} = -\nabla P\times\mathbf{t}/\tilde{\omega}$) satisfies

$$(1 - \alpha^2\tilde{\omega}^2/c^2)(\nabla\times\mathbf{B}.\mathbf{t}) = - 2(\alpha\tilde{\omega}/c)^2 \, B_z/\tilde{\omega} \tag{7}$$

(Pryce (private communication); Michel 1973; Mestel and Wang 1979). With no singularities in $\nabla\times\mathbf{B}$ within the l-c, $B_z = 0$ on the l-c; one can then find the solution of (7) which reduces near the star to a point dipole. Beyond the l-c

$$\nabla\times\mathbf{B} = 0, \quad \nabla^2\psi = -2\alpha B_z/c; \tag{8}$$

these have unique solutions, subject to continuity of $B_{\tilde{\omega}}$ and of $\partial\psi/\partial z$ on the l-c and finiteness at infinity. However, it is at once apparent what is wrong with this model: the discontinuities in $E_{\tilde{\omega}}$ and $B_z$ require charge-current sheets $\sigma$, $J_\phi$ on the l-c, and the associated net Maxwell stresses have local $\tilde{\omega}$- and z-components

$$2\pi(\sigma^2 - J_\phi^2/c^2); \quad B_{\tilde{\omega}}(\sigma - J_\phi/c). \tag{9}$$

If the solution of Maxwell's equations so constructed were such that $J_\phi = \sigma c$ - as charge-separation and corotation at the l-c would require - then both expressions (9) would vanish, and one could then hope to make a small modification - e.g. a thin gap separating the GJ magnetosphere from the l-c - to allow for centrifugal forces due to finite rest mass. In fact, $J_\phi \neq \sigma c$, so this simplest of all non-vacuum models has merely transferred the problem of unbalanced stresses from the star to the l-c (cf. also Pilipp 1974).

The example just discussed is an object lesson for the whole magnetosphere problem. It is very tempting to study separately contiguous domains with different dynamical properties - e.g. a flow domain and a neighbouring corotating domain - and then to link them up by imposing continuity of the tangential component of $\mathbf{E}$, and allowing surface charges to support normal discontinuities. In a mixed non-relativistic plasma, the corresponding stresses are easily balanced by the magnetic

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stresses maintained by a surface current due to a minute drift of electrons relative to ions. In a charge-separated plasma, however, all currents are convection currents and this freedom is missing: the introduction of surface charges may very well yield unbalanced stresses and so cast doubt on the validity of the model studied.

A dead magnetosphere will certainly have "gaps" within the l-c. This possibility was first studied by Holloway (1973), who concentrated on the zone defined by field-lines that close within the l-c, for which there is no immediate objection to the corotation of the GJ charges with the star. The dipolar field-lines near the star have points with $B_z = 0$, along the radial line with polar angle $\bar{\theta} = \cos^{-1}(1/\sqrt{3})$ which separates electron and ion regions. Holloway noted that the completely filled GJ magnetosphere could be modified by the widening of the line $\bar{\theta}$ into a vacuum gap of finite width. The electron domain is linked with the star and so still satisfies $\psi = 0$; within the gap $\underset{\sim}{B}$ and $\psi$ again satisfy (8); while in the ion domain $\underset{\sim}{E}.\underset{\sim}{B} = 0$ again, so that $\psi$ is a function of the magnetic stream function P but is non-zero. Correspondingly the ion zone will satisfy isorotation, with the local $\Omega = \alpha(P)$, but will not corotate with the star. In the simplest model (valid only well within the l-c), with the gap a thin wedge defined by polar angles $\theta_1 < \bar{\theta} < \theta_2$, the solution for $\psi$ that is continuous in $\nabla\psi$ at both $\theta_1$ and $\theta_2$ requires that $\theta_2 - \bar{\theta} \simeq \bar{\theta} - \theta_1$, and $\alpha(P)/\alpha = \{1 - (\theta_2 - \theta_1)^2/\sqrt{2}\}$ : the ion zone rotates uniformly but more slowly than the star. This is certainly a hint that a complete magnetosphere model (whether dead or alive) may have a sub-rotating ion domain that can extend beyond the l-c.

More recently, Michel (1979) has suggested that the GJ condition $\underset{\sim}{E}\cdot\underset{\sim}{B} = 0$ can be satisfied both at the star and also over a surface S that both intersects the star and separates charged and vacuum domains. This is achieved by having terms in both $P_2/r^3$ and $r^2P_2$ in the electric potential in the vacuum domain. This is again clearly a local calculation - it remains to be shown how such a gap can be fitted into global solutions which satisfy appropriate boundary conditions at infinity. But both examples suggest how one may get more flexibility into model building: e.g. if a model requires a net positive charge within the l-c, it is encouraging to have the possibility that a large section of the electron domain can be removed without necessarily violating the GJ stellar boundary condition $\underset{\sim}{E}\cdot\underset{\sim}{B} = 0$.

## 4. WIND MODELS

The electrically-driven wind was proposed by GJ as appropriate for the domain defined by field-lines that cross the l-c. They were however immediately faced with a contradiction between their initial assumption of charge separation and their plasma condition (4). In a steady state the electron flow from the polar regions must be balanced by an ion flow in a surrounding collar; but the ions would find themselves flowing through a domain that is negative according to the GJ expression (5)

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(see also Okamoto 1974). The suggested resolution with ions flowing through a sea of corotating electrons is unlikely to survive the introduction of the small but finite non-electromagnetic forces. To retain a charge-separated wind model, one is forced to abandon the neglect of the non-corotational potential; near the star (5) must be replaced by

$$\rho_e \simeq -\alpha B_z/2\pi c - \nabla^2\psi/4\pi, \tag{10}$$

and the $\nabla\psi$ term will yield relativistic acceleration near the star of both species (cf. §6). A systematic attack on this problem is being undertaken by the Erlangen group (Schmalz et al 1979, 1980; Schmalz this volume). The essential features of their work are:

(1) The magnetic field adopted is (as already noted) markedly non-dipolar near the star, but still has a negative GJ density in the ion domain, so that the $\nabla^2\psi$ term in (10) is essential.

(2) Relativistic inertia is built-in: particles do not move strictly along magnetic field-lines, but suffer "intertial drifts" (cf. (11) and (12) below).

(3) Radiation losses are assumed everywhere negligible.

(4) In an electron domain the characteristic parameter of the problem is $\varepsilon = (\alpha/(eB_s/mc))(c/\alpha R)^2 \simeq 10^{-12}$ for a rapid pulsar; in an ion domain $\varepsilon \sim 10^{-9}$. In their asymptotic solution far from the star, the particles have $\gamma$-values $\sim 1/\varepsilon$.

The group's published work prompts the following comments and queries:

(1) The choice of field structure near the star puts a severe *a priori* constraint on the distant currents that must be present in an ultimate fully self-consistent model.

(2) The motivation for experimenting with non-dipolar curl-free fields is unclear. They avoid a change of sign in $(\rho_e)_{GJ}$ within the *closed* field-line domain, but the presence of electrons and ions in different regions in that domain causes no particular problems, as there is no wind flow.

(3) In their asymptotic solution the particles are no longer accelerated and so do not radiate. But with such enormous $\gamma$-values predicted, the neglect everywhere of radiation damping becomes questionable. The associated drag could modify radically the whole macroscopic flow.

(4) If the Erlangen wind model can be shown to be classically viable, even with radiation losses included, pair production may in fact transform the magnetosphere into a dense quasi-magnetohydrodynamic model (cf. Kennel et al 1979).

## 5. MODELS WITH CIRCULATION

As pointed out by Jackson (1976a, b), the charging up of the star through the emission of a polar electron current can be off-set by a return electron current replacing the GJ ion current. This avoids the difficulty of having positive charges in a domain with $(\rho_e)_{GJ}$ negative,

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and offers the possibility of retaining GJ conditions near the star and indeed through much of the domain within the l-c. The proposals by the Sussex group (Mestel et al 1979) and by Rylov (1977, 1979) differ in important aspects, but concur in having flow of particles across field-lines near the l-c. The spinning of the star sets up potential differences over the stellar surface. Trans-field flow of the electrons is inhibited near the star, but once they have acquired sufficiently high $\gamma$-values they will drift across field-lines because of intertial force and radiation damping. The azimuthal component of the equation to the motion of a cold, radiating electron gas can be approximated by

$$\underset{\sim}{v}.\nabla(e\tilde{P}/c) = -(\Omega\tilde{\omega}^2/c^2)\mathbb{P} = -(\text{angular momentum radiated per second}), \quad (11)$$

where $\underset{\sim}{v}$ is the circulation speed, and $\mathbb{P}$ the radiated power. The inertial drift is included in the definition

$$\tilde{P} \equiv P + (c/e)\gamma m\Omega\tilde{\omega}^2 \quad : \quad (12)$$

the radiation drag enforces deviation of the flow from the dissipation-free streamlines $\tilde{P}$ = constant and enables the electrons to return to the star with non-relativistic speeds after radiating energy and angular momentum.

In a non-dissipative domain (with gravitation neglected), the energy and angular momentum integrals yield

$$\Gamma \equiv \gamma(1 - \alpha\Omega\tilde{\omega}^2/c^2) - e\psi/mc^2 = \text{constant on streamlines}, \quad (13)$$

a result valid also in quasi-steady systems (Endean 1972; Burman and Mestel 1978). (With dissipation included, (12) is replaced by $\underset{\sim}{v}.\nabla\Gamma = -\mathbb{P}(1 - \alpha\Omega\tilde{\omega}^2/c^2)/mc^2$). In the original Sussex proposal, the outflow was pictured as driven by the centrifugal sling-shot term $\gamma\alpha\Omega\tilde{\omega}^2/c^2$ in (13), moderated by a small $\psi$-field, and the inflow by a $\psi$-field that overcomes the centrifugal term. Such models would predict a total power much below the maximum Deutsch value (3), and correspondingly very slow speeds $v^*$ near the star. Following difficulties in the attempted construction of self-consistent $\psi$ and $\underset{\sim}{B}$ fields, the proposed model has been modified to allow $v^*/c \lesssim 1$; the GJ domain extends far from the star but ends well before the l-c, when the total particle speed approaches c (cf. §6). The $\underset{\sim}{E}_{\parallel}$ in the non-GJ domain within the l-c is responsible for accelerating the outward flow to highly relativistic energies and decelerating the return flow. Sufficiently far beyond the l-c the circulating electrons feel the Coulomb field of the net positive charge within the l-c, which together with energy radiation ensures that in a steady state they return to the star.

A model in which dissipation occurs near and beyond the l-c has no difficulty in satisfying the condition that the integrated energy loss per second be $\alpha$ times the angular momentum loss, as required by the fact that the energy and angular momentum sources are respectively $\frac{1}{2}I\alpha^2$ and

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$I\alpha$ (Gold, 1978 Texas Conference; Holloway 1977). This is not an extra condition to be imposed on the problem, but will be automatically satisfied in a fully self-consistent theory. The point is that a proposed model in which all the dissipation occurs near and beyond the l-c cannot be challenged a priori on these grounds. Models with substantial energy radiation near the star must have compensating energy loss beyond the l-c in order to carry off an excess of angular momentum and so balance the books.

A rough preliminary estimate for the power $\mathcal{P}$ is $2e^2\gamma^4\alpha^2/3c$, so that the drag term in (11) is comparable with the Lorentz force if

$$\gamma^4 \simeq [(\omega_g)_{1c}/\alpha]\,[(c/\alpha)/(e^2/mc^2)]\ (v/c)\ . \tag{14}$$

Both the familiar first bracket in (14) and the second - the ratio of the l-c radius to the classical electron radius - are very large numbers; with $(v/c) \simeq 1$ the required values of $\gamma$ are $\sim 10^8$, and are insensitive to changes in v/c and $\alpha$. The electron energies are $\sim 6 \times 10^{13}$ eV, and the radiated photons would be gamma rays of energy $\sim 6 \times 10^{11}$ eV. (If other dissipative processes intervene, then the required trans-field drift would occur at lower particle energies. R. Epstein (private communication) has pointed out that the inverse Compton effect on thermal photons from a sufficiently hot neutron star surface would similarly yield gamma rays). With $(B)_{1c}$ anticipated to be $\simeq B_s(\alpha R/c)^3$, as in Michel (1973) and Mestel and Wang (1979), pair production (Erber 1966) is negligible, though it would be significant with the quasi-radial field of the Erlangen group.

In rapid or moderately rapid pulsars, the potential difference between the outflow and inflow regions in the polar cap is itself sufficient to accelerate electrons to such high energies: for with a polar cap angle $\theta_c \sim (\alpha R/c)^{\frac{1}{2}}$, one estimates $\gamma \simeq e(\alpha B_s R^2/c)\,(\alpha R/c)/mc^2 \simeq 2 \times 10^7/P^2$, where $P = 2\pi/\alpha$. Thus if $P \ll 1$ sec, as for the Crab and Vela, one can argue that it is only too easy to get the super-relativistic particles required by (14) (with $v/c \simeq 1$); rather, one needs a GJ domain with $\mathbf{E}.\mathbf{B} \simeq 0$ near the star to ensure that particles do not become too energetic and radiate too soon, so violating the Gold-Holloway condition. For slower pulsars the issue is less clear. Assuming that the proposed class of circulation models extends through arbitrary $\alpha$-values, is the estimate $\gamma = 2 \times 10^7/P^2$ always an upper limit? If so, then (14) will predict a sharp drop in the speed v of particle flow across the field as P increases, and the pulsar would effectively die. Alternatively (and perhaps more plausibly), does the system respond by building up a particle and hence a $\psi$ field which ensures that particles in the dissipation domain always have enough energy to radiate at a high rate prescribed by the gross dynamics and electrodynamics? In other words, does the actual dissipation process play an active or a passive role; does it determine the power emitted, or is the problem analogous to Riemann's classical work on non-linear sound waves, where the dissipation-free equations themselves predict the inevitable onset of shock-

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wave dissipation? A definitive answer must await successful construction of a class of models. Meanwhile, careful study (Burman 1980) of the consequences of the dissipation-free integral (13) can indicate conditions under which particles will spontaneously approach large $\gamma$-values and so necessarily begin to radiate. The behaviour of the non-corotational potential $\psi$ is crucial, and this in turn depends on the charge distribution and so on the gross dynamics. Much of the difficulty of the magnetosphere problem derives from the absence of a reliable first approximation to the $\psi$-field, which could serve as the basis of an iterative scheme.

## 6. FLOW OF ELECTRONS NEAR THE STAR: FIELD-LINE CURVATURE

None of the proposals for the aligned case has been established, so we do not yet know whether there are no currents entering and leaving the star (§ 3), or whether particles of both signs leave the star with relativistic energies (§ 4), or whether particles of one sign leave and return to the star as non-relativistic currents (§ 5). The possibilities are by no means exhausted: one can picture e.g. models with a circulation superposed on a wind. Whatever the correct model for the aligned case (and it is not clear that there is a unique solution), one can always expect some current flow in oblique geometry (cf. §7). The crucial point is that the strength of the currents at the star should emerge for all cases from a consistent global model of the magnetosphere. In advance of this, it is instructive to suppose the local current density J known, and study the behaviour of the electron energies. For both aligned and quasi-steady cases, the non-corotational potential near the star satisfies

$$\nabla^2\psi = 4\pi ne - 4\pi Ne \tag{15}$$

where $N = -\alpha B_z/2\pi ce$ is the GJ electron number density (a local constant) and n is the actual number density. Likewise we define the GJ velocity $V = -J/Ne$, the actual velocity $v = -J/ne$, and the non-dimensional parameter $\tilde{J} = -J/Nec = V/c$. The simple GJ assumptions are that $\psi = 0$, $n = N$ and (implicitly) that global conditions do not demand currents such that $\tilde{J} > 1$. (In the Erlangen model, $-4\pi Ne$ is replaced in their ion current domain by the corresponding positive quantity, so that no solution with $\psi = 0$ is possible). However, in fact the electrons leave the stellar surface where $\psi = 0$ and $\nabla\psi = 0$, but where $n >> N$, $v << V$, and $\nabla^2\psi \neq 0$; the $\psi$ field accelerates electrons to $\gamma$-values given by $(\gamma - 1) \simeq e\psi/mc^2$ (cf. (13)). In a one-dimensional approximation, with distance $\tilde{s}$ along field-streamlines measured in units of $(mc^2/4\pi Ne^2)^{\frac{1}{2}}$, the non-dimensional $\tilde{\psi} \equiv e\psi/mc^2$ satisfies

$$\nabla^2\tilde{\psi} \simeq d^2\tilde{\psi}/ d\tilde{s}^2 = -1 + \tilde{J}(\tilde{\psi} + 1)/(\tilde{\psi}(\tilde{\psi} + 2))^{\frac{1}{2}} \tag{16}$$

and

$$(\tilde{\psi}')^2 = -2\tilde{\psi} + 2\tilde{J}(\tilde{\psi}(\tilde{\psi} + 2))^{\frac{1}{2}} \tag{17}$$

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Thus if $\tilde{J} < 1$, then (16) and (17) imply that $\tilde{\psi}$ and so also n, v and $\gamma$ have a steady (or quasi-steady) oscillatory form, with wavelengths typically $\sim 1$ cm, and with $\gamma$ having maxima of $(1 + \tilde{J}^2)/(1 - \tilde{J}^2)$ (Mestel and Pryce, unpublished). This is intuitively satisfactory. Provided the prescribed currents do not demand that the GJ charge density move superluminally, there is no monotonic acceleration to relativistic energies; the GJ assumption $\psi = 0$ remains an excellent approximation in the mean, though the fitting onto the stellar surface enforces superposition of stationary micro-oscillations. But if $\tilde{J} \geqslant 1$, then the oscillatory behaviour is replaced by monotonic, with relativistic acceleration near the star (Michel 1974; Fawley et al 1977, Arons 1979).

The quantities N, V, J and $\tilde{J}$ are only locally constant. For both aligned and oblique problems we are interested in the large-scale variations of $\tilde{\psi}$ and $\gamma$ in a flow region, bounded by corotating regions with $\tilde{\psi} \simeq 0$. Continuity of flow along the field-lines requires that $J \propto B$, while the GJ density $N \propto B_z$; hence $\tilde{J} \propto B/B_z$, and its behaviour and so also that of $\tilde{\psi}$ and $\gamma$ depends on how the field-lines curve with respect to the rotation axis. The one-dimensional approximation for $\nabla^2\tilde{\psi}$ is no longer adequate. An elegant rigorous treatment is given by Scharlemann et al (1978). We are content (following Sturrock 1979) to write

$$\nabla^2\tilde{\psi} \simeq d^2\tilde{\psi}/d\tilde{s}^2 - \tilde{\psi}/\tilde{D}^2 \qquad (18)$$

where $\tilde{s}$ is again an appropriate non-dimensional length along the field, and $\tilde{D}$ is a lateral scale: the boundary condition $\tilde{\psi} = 0$ at the edges of the flow domain requires that the lateral contribution to $\nabla^2\psi$ be negative.

In the aligned or nearly aligned cases, for which all or most of the field-lines curve away from $\underset{\sim}{k}$, $\tilde{J}$ increases outwards; equations (16) and (18) then predict that $d\tilde{\psi}/d\tilde{s}$ and $\tilde{\psi}$ will certainly be monotonic increasing once $\tilde{J}$ has exceeded unity. Thus unless $\tilde{J}$ is very small near the star, a postulated GJ domain (with small-scale oscillations superposed) must end before the l-c. This is the situation envisaged in the modified Sussex proposal (with the neglected terms of order $\Omega^2\tilde{\omega}^2/c^2$ retained as the l-c is approached). In the terminology of Scharlemann et al (1978) (see also Cheng and Ruderman 1980), the field-lines in the small or zero obliquity cases are curved "unfavourably" for steady relativistic flow; however, there is no contradiction, as different questions are being asked. The Berkeley group are trying to fit a local domain of steady relativistic flow into an otherwise force-free magnetosphere, so they impose the boundary condition $\tilde{\psi}' = 0$ at the ends of their domain; whereas in the Sussex proposal a non-vanishing, positive $\tilde{\psi}'$ is an essential feature.

In highly oblique cases, all field-lines near the star curve towards the rotation axis, so that $\tilde{J}$ decreases outwards. This "favourable" curvature does enable a relativistic acceleration domain to be fitted into a force-free magnetosphere - the boundary condition $\tilde{\psi}' = 0$

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can be satisfied. Scharlemann et al show that particles can then reach $\gamma \sim (eB_s/mc\alpha)(\alpha R/c)^3$ near the star. In moderate obliquity cases there will be both "favourably" and "unfavourably" curved field-lines. However, the terminology does depend implicitly on the boundary condition imposed on $\tilde{\psi}'$; in the absence of a globally-constructed magnetospheric model for any obliquity, the concepts should perhaps be used with caution.

## 7. THE OBLIQUE ROTATOR

A non-zero obliquity angle $\chi$ implies that the charge-current system fluctuates with the rotation frequency and so should emit a low frequency wave analogous to the Deutsch-Pacini wave. However, if GJ conditions hold near the star when $\chi = 0$, they should likewise hold at least for $\chi$ small but finite. In illustrating how global considerations fix the currents near the star, and specifically how the star may come to terms with both the Sommerfeld boundary condition at infinity and GJ conditions well within the l-c, the following example is instructive (Mestel and Wang 1981; see also Burman and Mestel 1979). The domain within the l-c is idealized by supposing it filled with the GJ charge density, without any gaps, so that $\underset{\sim}{B}$ satisfies the "relativistic force-free equation" (Mestel 1973; Endean 1974):

$$\nabla \times \underset{\sim}{B}^{\star} = \Lambda \underset{\sim}{B}, \quad \underset{\sim}{B}^{\star} = \{B_{\tilde{\omega}}(1 - \alpha^2\tilde{\omega}^2/c^2),\ B_\phi,\ B_z(1 - \alpha^2\tilde{\omega}^2/c^2)\} . \tag{19}$$

The terms $\alpha^2\tilde{\omega}^2/c^2$ in $\underset{\sim}{B}^{\star}$ include both the effect of the corotation of the GJ charges and of the displacement current, subject to the GJ condition $\underset{\sim}{E}.\underset{\sim}{B} = 0$. The total material current is $\underset{\sim}{j} = (\rho_e)_{GJ}(\alpha \underset{\sim}{k} \times \underset{\sim}{r}) + c\Lambda\underset{\sim}{B}/4\pi$ - the sum of corotation current and a flow along the field-lines. Far beyond the l-c the electromagnetic field reduces to an outgoing vacuum wave; we again idealize by extending this domain all the way back to the l-c. The appropriate solution of (19) must be linked up with the vacuum wave by continuity of $B_{\tilde{\omega}}$, $E_\phi$, $E_z$.

If $\Lambda$ is put equal to zero, implying no currents linking the star and the l-c, then from equations (19) it follows that the $\tilde{\omega}$-component of the Poynting vector vanishes at the l-c, and there is no supply of energy to the wave. Thus the Sommerfeld boundary condition requires that there be current flow through the GJ domain. When $\chi = 0$, this electromagnetically-driven current vanishes, and the present model reduces to that discussed in §3, which we saw is in fact unacceptable because it predicts unbalanced stresses at the l-c. However, as we are concerned to bring out the effects of obliquity and the consequent time-dependence, let us temporarily ignore the mechanical breakdown of that attempt to construct an aligned model, and write the magnetic field within the l-c of a slightly oblique rotator as

$$\underset{\sim}{B} = \underset{\sim}{B}^{(1)} \cos\chi + \underset{\sim}{B}^{(2)} \sin\chi, \tag{20}$$

where $\nabla \times \underset{\sim}{B}^{(1)\star} = 0$, identical with the Pryce-Michel equation (7). The

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currents supplying energy to the vacuum wave field $\underset{\sim}{B}^{(2)}$ beyond the l-c are approximated as flowing along $\underset{\sim}{B}^{(1)}$:

$$\nabla \times \underset{\sim}{B}^{(2)*} \sin\chi = \Lambda \underset{\sim}{B}^{(1)} \cos\chi \tag{21}$$

with

$$\Lambda = \exp\{i(\phi - \alpha t)\} \sin\chi \, F(P^{(1)}). \tag{22}$$

The function $F(P^{(1)})$ must be chosen judiciously, so that there are no bogus local energy sources along the l-c. The models are then completely determined; typically, they predict 2-3 times more energy flow to infinity than in the Pacini model, since the basic magnetic dipole radiation is augmented by electric and magnetic multipole contributions (especially electric quadrupole) from the fluctuating GJ charges. In contrast to the equatorially symmetric current flow in the aligned models of §4 and §5, the electron flow associated with the rotating perpendicular component of the oblique dipole is anti-symmetric, leaving from one pole, flowing along the l-c from one hemisphere to the other, and returning to the star at the other pole.

This highly idealized model has the same limitations as the axi-symmetric limit of §3 - it predicts unbalanced stresses at the l-c. However, some of its qualitative features may very well persist in a future self-consistent treatment. If the Michel-Pellat dead models exist, then a slight tilt should cause emission of a low-frequency wave and an associated anti-symmetric current system. Similarly, if the aligned models are as in §5, the slightly oblique generalization will have a current system consisting of a superposition of equatorially symmetric and anti-symmetric components; and as $\chi$ increases, the fraction of the total energy loss emitted as gamma radiation will decline, and that in the low-frequency wave will increase. Definitive observational results on gamma-ray pulsars are anxiously awaited.

As in §6, self-consistency of the above perturbation treatment of the idealized model requires that $|\underset{\sim}{j}^{(2)}|/|(\rho_e)_{GJ}^{(1)}| < c$. Since $j^{(2)} \propto \sin\chi$, this is clearly valid at small $\chi$, and for large $\chi$ the perturbation treatment in any case breaks down. However, the model suggests that the high obliquity problem should be fundamentally different from the aligned or nearly aligned cases. The energy requirements of the low-frequency wave increase with obliquity, essentially like $\sin^2\chi$; but near the star the GJ charge density $\propto \underset{\sim}{B}.\underset{\sim}{k} \propto \cos\chi$, so that for large or moderate obliquity the GJ charges will be unable to yield the currents necessary in a GJ magnetosphere to supply the energy carried by the wave. Thus when $\chi$ is not small one can conjecture that while GJ conditions will continue to hold on field-lines that close within the l-c, along field-lines that cross the l-c the parameter $j/(\rho_e)_{GJ}c$ must exceed unity near the star, with consequent relativistic acceleration. It is not yet clear, however, whether the electromagnetic field along the "open" field lines will approximate to a vacuum field, with energy

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carried primarily by the displacement current, and with super-relativistic acceleration of the few charges present, or whether the system will insist within a classical framework on a current much greater than the maximum GJ value. The answer to this classical query will affect the properties of the associated quantum magnetosphere formed when pair production is introduced.

The low-frequency wave will exert a precessional as well as a braking torque on the star (Michel and Goldwire 1970; Mestel and Wang 1981), normally in a sense such as to reduce $\chi$. One can expect a similar (though possibly weaker) effect from a wind emitted by a star with a basically dipolar field (cf. Mestel and Selley 1970). However, the reaction of the star to the torque depends on the elastic properties of the crust (Goldreich 1970; Lamb, this volume), so that the time-evolution of $\chi$ is more complicated than in the analogous problem for a gaseous star.

Besides the enforced relativistic acceleration near the star, the highly oblique cases offer more obvious opportunities for the spontaneous generation of high $\gamma$-values near the l-c. As pointed out originally by Kahn (1971, and this volume; see also Burman and Mestel 1979; Burman 1980), and exploited by da Costa (Manchester Ph.D. thesis 1976), particles that are moving in the sense of the rotation along forward pointing field-lines will inevitably become highly relativistic as the l-c is approached, and so are likely to radiate. And even for particles that are on backward-pointing field-lines and so have no difficulty in passing through the l-c, the dissipation-free equations can often predict that $\gamma \to \infty$ on critical surfaces beyond the l-c, implying in fact departure from flow along the field and likely radiation.

## 8. CONCLUSIONS

The discussion suggests the following (non-exhaustive) set of possibilities.

(1) The aligned rotator is dead, with no sensible current flow out of or into the star.

The slightly oblique rotator emits a generalized Deutsch-Pacini low-frequency wave, with electron currents flowing between the star and the neighbourhood of the l-c, and probably also some high-frequency radiation from near the l-c.

Moderately or highly oblique rotators will have relativistic particle acceleration near the star, and also at singular regions near and beyond the l-c. Transition to a mixed plasma via pair production is expected, with probable wind emission.

(2) The aligned model is a substantial radiator (probably of gamma-

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rays) from near and beyond the l-c. Current flow near the star is non-relativistic or mildly relativistic.

The slightly oblique rotator emits both a low-frequency wave and l-c gamma radiation. The highly oblique rotator is as above.

(3) The classically treated aligned model emits a wind that is relativistic near the star as well as further out. Pair production and transition to a mixed plasma is expected for all obliquities.

## REFERENCES

Arons, J.: 1979, Space Sci. Rev. 24, p. 437.
Burman, R.R.: 1980, Australian J. Phys. (in press).
Burman, R.R. and Mestel, L.: 1978, Australian J. Phys. 31, p. 455.
Burman, R.R. and Mestel, L.: 1979, Australian J. Phys. 32, p. 681.
Cheng, A.F. and Ruderman, M.A.: 1980, Astrophys. J. 235, p. 576.
Cohen, J.M. and Toton, E.T.: 1971, Astrophys. Letters 7, p. 213.
Deutsch, A.J.: 1955, Ann. d'Astrophys. 18, p. 1.
Endean, V.G.: 1972, Nature Phys. Sci. 237, p. 72.
Endean, V.G.: 1974, Astrophys. J. 187, p. 359.
Erber, T.: 1966, Rev. Mod. Phys. 38, p. 626.
Fawley, W.M., Arons, J., and Scharlemann, E.T.: 1977, Astrophys. J. 271, p. 227.
Gold, T.: 1968, Nature 218, p. 73.
Goldreich, P. and Julian, W.H.: 1969, Astrophys. J. 157, p. 869.
Goldreich, P.: 1970, Astrophys. J. Letters 160, p. L11.
Holloway, N.J.: 1973, Nature Phys. Sci. 246, p. 6.
Holloway, N.J.: 1977, Mon. Not. R. Astr. Soc. 181, p. 9P.
Jackson, E.A.: 1976a, Nature 259, p. 25
Jackson, E.A.: 1976b, Astrophys. J. 206, p. 831.
Kahn, F.D.: 1971, Paper read at Royal Astronomical Society.
Kennel, C.F., Fujimura, F.S., and Pellat, R.: 1979, Space Sci. Rev. 24, p. 407.
Mestel, L.: 1971, Nature Phys. Sci. 233, p. 149.
Mestel, L.: 1973, Astrophys. Space Sci. 24, p. 289.
Mestel, L., Phillips, P., and Wang, Y.-M.: 1979, Mon. Not. R. Astr. Soc. 188, p. 385.
Mestel, L. and Selley, C.S.: 1970, Mon. Not. R. Astr. Soc. 149, p. 197.
Mestel, L. and Wang, Y.-M.: 1979, Mon. Not. R. Astr. Soc. 188, p. 799.
Mestel, L. and Wang, Y.-M.: 1981, Mon. Not. R. Astr. Soc. (in press).
Mestel, L., Wright, G.A.E., and Westfold, K.C.: 1976, Mon. Not. R. Astr. Soc. 175, p. 257.
Michel, F.C.: 1973, Astrophys. J. 180, p. 207.
Michel, F.C.: 1974, Astrophys. J. 192, p. 713.
Michel, F.C.: 1975, Astrophys. J. 196, p. 579.
Michel, F.C.: 1979, Astrophys. J. 227, p. 579.
Michel, F.C. and Goldwire, H.C.: 1970, Astrophys. Letters 5, p. 21.
Okamoto, I.: 1974, Mon. Not. R. Astr. Soc. 167, p. 457.
Pacini, F.: 1967, Nature 216, p. 567.

https://doi.org/10.1017/S0074180900092597 Published online by Cambridge University Press

Pilipp, W.G.: 1974, Astrophys. J. 190, p. 391.
Rylov, Yu.A.: 1977, Astrophys. Space Sci. 51, p. 59.
Rylov, Yu.A.: 1979, Astrophys. Space Sci. 66, p. 401.
Scharlemann, E.T., Arons, J., and Fawley, W.M.: 1978, Astrophys. J. 222, p. 297.
Schmalz, R., Ruder, H., and Herold, H.: 1979, Mon. Not. R. Astr. Soc. 189, p. 709.
Schmalz, R., Ruder, H., Herold, H., and Rossmanith, C.: 1980, Mon. Not. R. Astr. Soc. 192, p. 409.
Sturrock, P.A.: 1971, Astrophys. J. 164, p. 529.
Sturrock, P.A.: 1979, Paper read at IAU, Montreal.

## DISCUSSION

MANCHESTER: In discussions of period evolution, alignment of the magnetic axis is often considered as an alternative to magnetic decay. Can you comment on the rate of energy loss from a nearly aligned system relative to that from an oblique system?

MESTEL: That is still unresolved. Our original proposal, with essentially centrifugal sling-shot driving, implied a much lower energy loss than for a highly oblique system, by several orders of magnitude. Our newer model (if viable) will certainly predict a greater power, but I cannot yet say what fraction of the classical Deutsch-Pacini estimate.

HEINTZMANN: The angular momentum must be transported from the neutron star surface to infinity. That implies that one has to change the currents near the star also.

MESTEL: The Gold-Holloway condition is an integral constraint. It implies that radiation within the light-cylinder must be balanced by radiation beyond. In a self-consistently constructed model, this will be automatically satisfied. Also, radiation damping included in the dynamics, whether near to the star or further out, will automatically modify the currents so as to yield the correct torque on the star.

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