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THE BSL 25TH ANNIVERSARY PRIZE

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The Bulletin of Symbolic Logic was established in 1993 by vote of the ASL Council as a journal both accessible and of interest to as wide an audience as possible with the stated purpose of keeping the community abreast of important developments in all parts of logic. The first issue appeared in March 1995. In addition to reviews, meeting reports, notices and other items of general interest, the Bulletin publishes two types of papers, articles and communications. Articles are usually of an expository or survey nature, in any area of logic including mathematical or philosophical logic, logic in computer science or linguistics, the history or philosophy of logic, logic education and applications of logic to other fields. They should present topics of broad interest in an accessible way, pay attention to the history of the area, and provide explanations for a general reader of its importance, concerns and achievements. Communications, on the other hand, contain important new results and ideas; they may be preliminary announcements or short complete papers and should include enough history, background and explanation to make the significance of the work apparent to a wide audience.

In order to celebrate the 25th anniversary of the Bulletin, the ASL Council has established a one-time 25th anniversary prize in order to highlight some of the best expository papers published in the BSL, and constituted an ad-hoc committee consisting of Patricia Blanchette, Alexander Kechris, Martin Otto, Michael Rathjen, Richard Shore, and myself as chair. We were tasked with deciding the feasibility of such a selection, and if so, proposing a list of appropriate papers representing different branches of logic. For a paper to be considered, it would have to stand out among papers published not only in the BSL, but also in comparable journals in general. We distributed the 250 papers published in the BSL during its 25 years of existence among the committee members by thematic affinity, and everyone selected two or three papers in their domain for further consideration. Our criteria were that the paper should survey a broad area and/or explain deep mathematics in a manner that could enhance their impact and draw fresh interest from a widened audience, and display an excellent expository style. We decided to exclude from consideration the two papers published in the BSL that have already been awarded the Shoenfield Prize, namely

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- B. Balcar and T. Jech, *Weak distributivity, a problem of Von Neumann and the mystery of measurability*, vol. 12 (2006), no. 2, pp. 241–266.
- R. Downey, D. Hirschfeldt, A. Nies and S. Terwijn, *Calibrating randomness*, vol. 12 (2006), no. 3, pp. 411–49.

This resulted in an initial list of 15 papers, which were then discussed among the whole prize committee. The precise number of awards having been left to our discretion, we finally settled on a list of seven papers, which we felt are each outstanding, but also represent different areas and different perspectives in logic.

J. Avigad, ***Forcing in proof theory*, vol. 10 (2004), no. 3, pp. 305–333.** The standard view of forcing whether in set theory, recursion theory or nonclassical logics is semantic. This paper provides an eye-opening, purely syntactic approach in line with the goals and methods of traditional proof theory. So its concerns are with the analysis of theories much weaker than ZFC and nonclassical logics including constructive, computational, intuitionistic and all varieties of modal logics. The approach analyzes a (stronger) theory T_1 in terms of a (weaker) theory T_2 by a syntactic process of interpretation. Looking first at some specific examples, it presents a general pattern of analysis. The paper then develops applications to a wide variety of topics including subsystems of second order arithmetic, reverse mathematics and reductions of classical logic to constructive and intuitionistic logics. Remarkably, it also shows how the syntactic approach to forcing can mediate between typical semantic arguments in various areas and the detailed syntactic ones of traditional proof theory. One converts model theoretic approaches to provide constructive cut elimination theorems with explicit algorithms but without much of the direct, detailed syntactic analysis of classical proof theory. Another application is to the apparently highly nonconstructive subject of nonstandard analysis. Here forcing can provide what can be seen as a formal justification of the widespread intuition that nonstandard proofs can be translated into standard ones by replacing "nonstandard" by "large enough."

A. Kanamori, ***The mathematical development of set theory from Cantor to Cohen*, vol. 2 (1996), no. 1, pp. 1–71.** This paper gives a comprehensive overview of the development of set theory from its beginnings in Cantor's work until the period surrounding the discovery of forcing by Cohen and its application in proofs of independence results. This excellent survey covers all the main mathematical developments in set theory during this period, including the foundational work of Cantor; the axiomatization of set theory and the role of the Axiom of Choice (AC); the beginnings and development of descriptive set theory; the emergence of combinatorial set theory and the theory of large cardinals; Gödel's work on the constructible universe and the consistency of AC and the Continuum Hypothesis (CH); the role of model theoretic methods in set theory; and finally the invention of forcing by Cohen and its first applications to independence results, including the independence of AC and CH.

C. McLarty, ***What does it take to prove Fermat's last theorem*, vol. 16 (2010), no. 3, pp. 359–377.** Several famous results in cohomological number theory, such as Wiles' (Fermat's last theorem) or Deligne's (Weil conjectures), or Faltings' (Mordell conjecture), have proofs drawing on vast requisites. A contentious issue from a foundational point of view has been whether they rest on universes, often called Grothendieck universes, and thereby go beyond Zermelo-Fraenkel set theory. MacLarty's paper explains that existing proofs indeed use universes and how and why two facts coexist:

1. Universes provide a framework for organizing explicit calculational arithmetic into a geometric conceptual order. Grothendieck found ways to do this in cohomology and used them to produce calculations which had eluded a decade of top mathematicians pursuing the Weil conjectures.
2. Weaker foundations also suffice. Universes can be eliminated in favor of ZFC by known devices though at the loss of some of the desired conceptual order.

The paper also discusses prospects for proving FLT in PA or an even weaker arithmetic: "No one who has looked at Wiles' proof seriously doubts that it could be unwound into a rather high order non-conservative extension of PA, say 8th order, by perfectly routine means which however would do tremendous damage to the theoretical organization."

The message of the paper can perhaps be described as a Hilbertian pact. In an adventurous mode, mathematicians prefer to preserve what Grothendieck called the "childish ...incorrigible naïveté," allowing for many stages of infinity to accomodate their needs, whereas in a more pensive mode (like Hegel's owl of Minerva), they also appreciate that afterwards there is usually a way to retreat to safer grounds.

Y. N. Moschovakis, ***Kleene's amazing second recursion theorem*, vol. 16 (2010), no. 2, pp. 189–239.** The Kleene Second Recursion Theorem is a fundamental result in computability theory. It has a short proof of a few lines but when appropriately used it is an extremely powerful tool that has played a crucial role in the proofs of many important results in computability theory and its applications. This paper is a masterful exposition of the Second Recursion Theorem and several of its connections and applications to many areas of logic. These include incompleteness and undecidability results; Solovay's theorem in provability logic; the development of hyperarithmetic theory, including the basic method of effective transfinite recursion and its use in the proofs of such well-known results as the Spector Uniqueness Theorem and the Suslin–Kleene Theorem; and the use of the Second Recursion Theorem in the proof of the author's Coding Lemma, a

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basic tool with numerous applications in the theory of the Axiom of Determinacy.

C. Rosendal, ***Automatic continuity of group homomorphisms*, vol. 15 (2009), no. 2, pp. 184–214.** The article of Rosendal is an excellent, comprehensive survey of the phenomenon of automatic continuity for topological groups, a very timely subject in the interface of logic (especially model theory and descriptive set theory), measure theory, topology and dynamics. The basic question is under what conditions a homomorphism $\pi : G \to H$ between Polish groups has to be continuous. After some non-continuous examples (necessarily involving the axiom of choice), Rosendal surveys results by Banach-Pitts, Steinhaus-Weil, Christensen and Solecki for measurable π. He then considers conditions on the target group H (notably Dudley's Theorem for discrete normed H) and on the base group G, introducing and analyzing the notion of *ample generics* for a group action, namely that there are comeagre orbits for the diagonal action on any Cartesian power.

T. Scanlon, ***Diophantine geometry from model theory*, vol. 7 (2001), no. 1, pp. 37–57.** Hrushosvki's proof of the function field Mordell–Lang conjecture, together with his later proof of the Manin–Mumford conjecture, is one of the deepest applications of mathematical logic to other areas of mathematics. It uses sophisticated tools of model theory—geometric stability theory—in order to prove a well-known conjecture in diophantine geometry. In this excellent survey, Scanlon explains both the geometric and model-theoretic notions underlying the conjecture and its proof in a way comprehensible to a non-specialist, and traces the principal steps of the argument, indicating both generalizations and limits of the methods. It should be noted that since this survey was written, the other main stream of model theory, *o*-minimality, has also turned out to yield results in diophantine geometry, starting with Pila's proof of the André–Oort conjecture and ongoing work on the Zilber–Pink conjecture.

R. Zach, ***Completeness before Post*, vol. 5 (1999), no. 3, pp. 331–366.** This essay traces the rich early history of the formulation, within the Hilbert school, of precise metatheoretical questions about formal systems of logic. Focusing on unpublished lecture notes and manuscripts of Hilbert and Bernays from the period 1917–1923, and on Bernays' 1918 Habilitationsschrift, the essay traces the conceptual and technical path that results in the syntactic and semantic completeness results for propositional logic, along with the establishment of the soundness, decidability, consistency, and independence of propositional systems. Along the way, it is revealed that the role of Bernays in formulating and establishing these early results is considerably greater than has been generally recognized. The essay is essential reading for those who would like to understand the path from Russell and Whitehead's Principia Mathematica to logic and metalogic as we now understand them, and is particularly important for understanding the route to our now-familiar notions of the completeness and incompleteness of formal systems.

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