

TOPICS IN BALANCED TERNARY DESIGNS

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In this thesis, we investigate the properties of balanced ternary designs.

We begin with a survey of the literature on balanced n -ary designs, of which balanced ternary designs are a special case. This provides a setting for our research, as well as the necessary background information.

Our main objective is to establish new existence and enumeration results. We develop methods for constructing certain balanced ternary designs. In some cases we construct entire classes of balanced ternary designs with block size four; hence obtaining necessary and sufficient conditions for their existence. We also enumerate the number of non-isomorphic balanced ternary designs with certain sets of parameters.

A number of the papers surveyed state necessary conditions for the existence of certain balanced ternary designs. We find additional conditions which must be satisfied by the parameter sets of some balanced ternary designs. We prove the existence/non-existence of two balanced ternary designs, the existence of which was left open by Billington and Robinson [2].

In the survey we document papers which give methods for constructing balanced n -ary designs from balanced incomplete block designs, affine α -resolvable balanced incomplete block designs, partially balanced incomplete block designs, group divisible designs, finite geometries, difference sets and cyclotomic classes, and Latin squares.

We give new results which construct balanced ternary designs from the incidence matrices of balanced incomplete block designs and from the blocks of 1-designs. We show that balanced ternary designs, with block size a prime or prime power, can be embedded into balanced ternary designs with the same block size. We give a recursive construction which combines the blocks of a group divisible design with the blocks of s frames and an existing balanced ternary design, to form the set of blocks of a 'new' balanced ternary design. This construction is analogous to that used by Hanani [3] to construct balanced incomplete block designs.

We survey papers which give necessary and sufficient conditions for the existence of balanced ternary designs with block size three and some balanced ternary designs with

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block size four. We extend this work and go on to give a detailed study of balanced ternary designs with block size four. We construct a number of individual designs, as well as an infinite family of cyclic balanced ternary designs, with block size four. These designs are then used, in conjunction with the above recursive construction, to construct entire families of balanced ternary designs, with block size four, index 2, and $\rho_2 = 1, \dots, 6$ (ρ_2 denotes the number of blocks in which an element occurs twice). In the case of $\rho_2 = 2$, we complete recent work by Assaf, Hartman and Mendelsohn [1]. We state necessary and sufficient conditions for the existence of such designs. Balanced ternary designs with block size four, any index and any ρ_2 are discussed.

Finally, we consider the enumeration of some balanced ternary designs. We consider certain parameter sets and state the exact number of non-isomorphic balanced ternary designs arising from these parameters. Quasi-T-multiples of balanced ternary designs are discussed, and their reducibility or irreducibility is considered.

REFERENCES

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