PART II

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DEPENDENCE OF THE LUNISOLAR PERTURBATIONS IN THE EARTH ROTATION ON THE ADOPTED EARTH MODEL

Nicole CAPITAINE
Observatoire de Paris and Bureau International de l'Heure, France

INTRODUCTION

If no perturbation exists, the motion of the Earth around its center of mass would be a rigid rotation around a fixed axis in space with constant angular velocity.

In fact, many perturbations disturb this ideal motion and produce variations in both the celestial orientation of the rotation axis and the Earth's angular velocity.

The mechanisms responsible for these perturbations are the changes in the total angular momentum due to external torques and also the changes in the inertia tensor of the Earth (due to deformations or motions of matter) or in the relative angular momentum in the terrestrial frame (due for instance to winds or to turbulent flow inside the core).

The combined gravitational force of the Sun and Moon or, more exactly the luni-solar tidal force (which is the component of this force function of the position of the points on the Earth) is probably the best modeled mechanism which affects the rotation of the Earth.

This tidal force exerts a torque on the Earth because of the Earth dissymetry with respect to it, which varies with the positions of the Moon and Sun relative to the Earth. It also raises oceanic and bodily tides because of the non-rigidity of the Earth, and thus, produces periodic terrestrial deformations.

The resulting perturbations in the rotation of the Earth around its center of mass are a secular deceleration and periodic variations of the Earth's angular rate, the precession and nutation of the Earth's axis of rotation, which is its main spatial motion, and the corresponding motion of the Earth's axis of rotation in the Earth's fixed frame defined as "diurnal polar motion" or "diurnal nutation" which is a minor, but non negligible component of the polar motion.

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 155–169.

In the case of a rigid Earth, the lunisolar potential does not deform it and, for a model with axial symmetry around the principal axis of inertia, the Earth's angular velocity is constant. The lunisolar perturbations in the Earth's angular rate are specifically due to the non-rigidity of the Earth.

The secular retardation, which is of the order of 2 ms/century for the length of the day, is due to the tidal energy dissipation in the Earth producing a phase lag between the lunar attraction and the radial tidal deformation. This is a very complex problem in itself which has been considered for instance by Melchior (1979) and Lambeck (1980). It will no longer be considered here.

The periodic variations in the rate of rotation with amplitudes of the order of 0.1 ms, are due to zonal tides inducing an axisymmetric deformation of the Earth which changes its polar moment of inertia. Their amplitudes depend on the elasticity of the Earth as computed by Woolard (1959) and on the existence of the oceans and of the fluid core inside the Earth as computed by Merriam (1980), Yoder et al.(1981) and Wahr et al.(1981).

The other lunisolar perturbations in the rotation of the Earth have the same expression for all the Earth's models, but their amplitudes depend on the deformability of the Earth and on the existence of the fluid core. This is not the case for the most important motion in space, the precession of the rotation axis in space around the normal to ecliptic in 26 000 years, which does not depend on the internal structure of the Earth. But it is the case for the most important nutations of this axis in space. The differences between the amplitudes corresponding to a rigid Earth and to an elastic Earth with fluid core reach 0.02.

The purpose of this paper is to review the theories and results concerning the periodic lunar and solar fluctuations in the rotational motion of the Earth for some adopted Earth models.

The motivation for this purpose is to get a better understanding of the dynamical motion of a realistic Earth model which has to be considered in the reductions of very precise observations such as lunar laser ranging, satellite laser ranging or satellite Doppler tracking.

Such an understanding is useful for using these very precise observations in order to solve for some parameters of the Earth's model and to obtain values which can improve our knowledge of the Earth's structure and dynamical behaviour.

1. SUCCESSIVE IMPROVEMENTS OF THE THEORIES CONCERNING THE LUNISOLAR EFFECTS ON THE EARTH'S ROTATION

Hipparchus (during the second century BC) was the first to observe the motion of precession, Copernic (in 1543) was the first to describe correctly this motion and Newton, during the 17th century, the first to explain it and approximatly compute it both with the solar nutation. Bradley, during the 18th century, discovered by observations the existence of the lunar nutation, theoretically explained just afterwards by D'Alembert. Euler (during the 18th century) presented the results with more elegant equations and was the first to recognize the possibility of a free wobble of the rotation axis within the Earth.

During the 19th century, Poinsot, Lagrange and Liouville, gave contributions to the resolution and formulation of these equations ; Laplace considered the effects of tides on the rotation of the Earth and concluded that the effect of oceans on the precession-nutation is the same as if they became solid. Hopkins (1839) was the first to consider the dynamical effects of a fluid core inside a rigid shell.

Oppolzer (1880) pointed out the existence of the "diurnal variation of latitude" due to the "diurnal polar motion". Tisserand (1891) gave a resolution of the equations for a rigid Earth and introduced the concept of "the mean axes" in the case of a deformable Earth.

The discovery of the Chandler wobble in 1891 gave rise to theoretical investigations on the effect of the Earth elasticity and the fluidity of the core in the rotation of the Earth. This led to the works of Newcomb (1892) about the effect of elasticity and Hough (1895), Sloudskii (1896) about the effect of the fluid core in a rigid shell. Poincaré (1910) confirmed the existence of a new free mode in the motion of the rotation axis within the Earth due to the fluid core and considered its consequence, because of a resonance effect, on the amplitudes of nutation.

Jeffreys (1928) was the first to point out that zonal tides must induce changes in the length of the day.

Woolard (1953) gave a numerical solution for his theory of rotation in the case of a rigid Earth which was taken as a reference by IAU from 1964 and he used the same expansions in 1959 to derive a series for the tidal variations in UT1.

Jeffreys (1948, 1949) extended the Poincaré's studies by including inertial effects in the core and elasticity of the mantle.

Jeffreys and Vicente (1957 a,b) and Molodensky (1961) considered more realistic theoretical models for the core and respectively the Tacheuki's (1950) and Altermann et al.'s (1959) solutions for the elastic parameters of the mantle.

Their analytical resolutions for the elastic displacements in the two media neglect ellipticity and rotation in both mantle and fluid core.

Shen and Mansinha (1976) extended these theories to include in their equations more complete models of the fluid core and solve them by numerical techniques considering elliptical and rotational effects in the fluid core but not in the shell nor at the boundaries.

In order to improve the dynamical theory of the rotation of the Earth, Kinoshita (1977) developed the most rigorous analytical solution for a rigid Earth.

In the same time, the elastic theories were improved in order to obtain more reliable coefficients. Nearly all these elastic theories solve the problem of harmonic oscillations of the Earth from the linearized infinetinimal equations of displacements, truncated for numerical reasons, and used the equation of angular momentum as an additional equation. Sasao et al. (1977) used an analytical approach of the problem by considering the equations of angular momentum for the whole Earth and for the core, with dissipative core mantle coupling, completed by the most important aspects of elastic deformation in the mantle and hydrodynamical behaviour in the core.

Wahr (1979) gave a further extension of these elastic theories which accounts more completely for the Earth's ellipticity and rotation, using a normal mode expansion together with linearized equations developed by Smith (1974).

His results for the Earth's nutational motion are based on one of the most reliable Earth models actually available, which is the model 1066 A of Gilbert and Dziewonski (1975) constructed from a large volume of recent seismological data.

These nutation coefficients will be very probably adopted by IAU instead of the ones of the Molodensky's second model provisionally adopted in 1979.

2. DYNAMICAL APPROACH OF THE ROTATIONAL MOTION

The Earth is considered here as an ellipsoïd of revolution around its polar axis.

The two considered frames are centered at the Earth's center of mass G.

The (r) = (Gxyz) frame is attached to the Earth, its axes being defined as the axes of inertia for a rigid Earth and as the Tisserand's principal axes of the mantle for a non rigid Earth, which are such that the relative angular momentum in the mantle be equal to zero (Tisserand 1891, Munk & Mac Donald 1960).

The (R) = (GXYZ) frame is attached to the ecliptic and equinox of a given epoch of reference t_o.

$\vec{\omega}$ is the rotation vector of (r) relative to (R) with components $\omega_1 = \Omega\, m_1$, $\omega_2 = \Omega\, m_2$, $\omega_3 = \Omega\,(1 + m_3)$ along the terrestrial axis, Ω being the mean rotation rate.

m_1, m_2 are of the order of 10^{-6} and m_3 of the order of 10^{-8}.

Ψ, φ, θ are the Euler's angles between the two frames.

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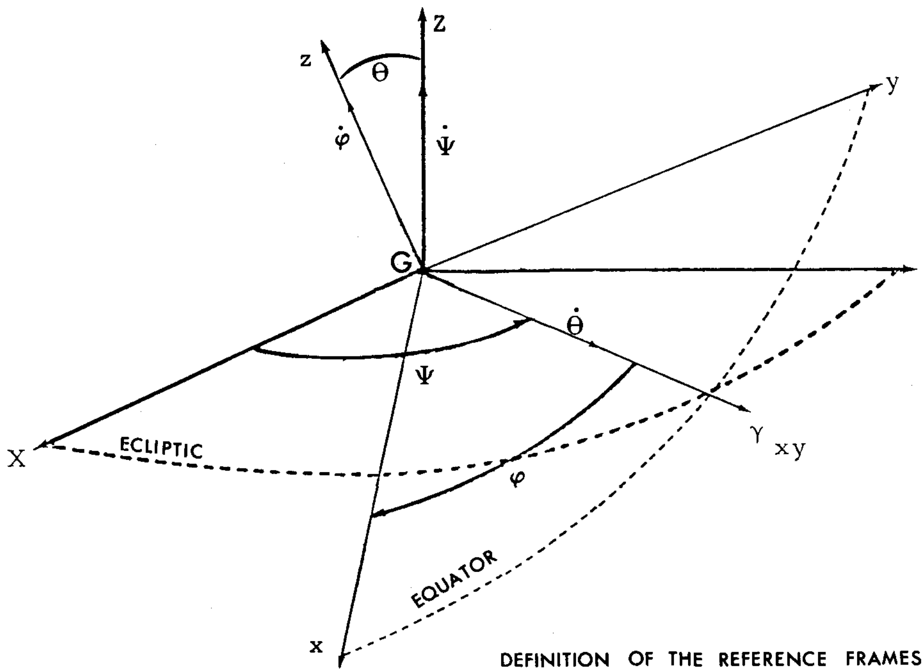

DEFINITION OF THE REFERENCE FRAMES

$\vec{L}$ is the resulting lunisolar torque acting on the Earth with components L_1, L_2, L_3 along the terrestrial axes.

There are two equivalent dynamical methods to obtain the motion of rotation of the (r) frame relative to (R).

One is to write the Lagrange's equations of the conservative system constituted of the Earth, including the atmosphere or the equation of the angular momentum referred to the rotating frame (r). This mathematical approach provides the classical Euler's dynamical equations which can be written, at the first order in m, m_3, if A and C are respectively the equatorial and polar momentum of inertia and m, L the complex notations :

$$m = m_1 + im_2 \quad , \quad L = L_1 + iL_2 :$$

$$\begin{cases} A\Omega\dot{m} + (C-A)\,\Omega m = L \\ C\,\Omega\,\dot{m}_3 = L_3 \end{cases} \qquad (1)$$

The lunisolar torque L can be expressed in function of time,using Woolard's or Doodson's expansion into the fundamental arguments of the Sun and Moon (Woolard 1953, Doodson 1922).
The differential equations (1) of the first order in m and m_3 can be easily solved in the case of a rigid Earth and easily extended to the case of simple Earth models. These equations, classicaly used for the free polar motion, have been solved by McClure (1973) for the "diurnal polar motion" in the cases of rigid and deformable Earth.

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The corresponding solution in space, $\Delta\theta, \Delta\Psi$ can be obtained through the Euler's kinematical relations :

$$\begin{cases} \dot{\theta} + i\,\dot{\Psi}\sin\theta = -\,\Omega\, m e^{i\varphi} \\ \dot{\varphi} = \omega_3 - \dot{\Psi}\cos\theta \end{cases} \qquad (2)$$

followed by a simple integration with respect to time.

The method used by Woolard (1953) for computing the nutational motion of a rigid Earth was to transform, by the use of the relation (2), the Euler's equations into differential equations of the second order in θ, Ψ, φ and to solve them by successive approximations in order to obtain the motion in space of the rotation axis and the Gz axis. His theory and values for the coefficients of nutation, based on the observed value of the constant of nutation (which is the coefficient of the principal nutation in obliquity) have been adopted by the IAU from 1964 until 1979.

The other dynamical method used to obtain the rotation of a planet around its center of mass is to consider the canonical equations verified by the Andoyer's variables.

This was the method used by Tisserand (1891) and in a more rigorous way, with a more sophisticated mathematical technique, by Kinoshita (1977). This method gives, in the same resolution, the motions of the angular momentum axis in space and within the Earth and then the corresponding motions of the rotation axis and figure axis.

Also, Kinoshita adopted the ecliptic of date as the unique reference plane which avoids the laborious transformations of the Woolard's expansions for the exterior torque. He also used the improved Eckert's expressions for the lunar orbital motion and new values for the fundamental considered constants as the constant of precession, the ratio of the masses of the Moon and the Earth. It is why the nutation coefficients given by Kinoshita (1977), slightly modified for account of the final values of the IAU - 1976 constants, are taken as a reference for a rigid Earth.

Sasao et al. (1977) extended the first dynamical approach to an elastic Earth with a fluid core plus inertial and dissipative torques at the core-mantle interface whereas Kubo (1979) extended the second one to a rigid Earth with a rigid core, plus a frictional and a kind of inertial coupling between the two parts.

However, other authors consider the problem of the elastic mantle and fluid core through elastic and hydrodynamical equations that we shall rapidly consider now.

3. ELASTIC APPROACH OF THE ROTATIONAL MOTION

The Earth is, in fact, a very complex stratified body with various

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parts, the main ones being the elastic mantle, the fluid outer core and the solid inner core.

The deviation of the rotational motion of the Earth from its equilibrium state of uniform rotation around a fixed axis in space can be considered as an infinitesimal displacement s(x, t) of each particule in its solid, elastic or fluid medium.

This displacement s, subject to an applied body force, has to satisfy the linearized elastic or hydrodynamical equations of motion and the boundary conditions at the interface between two different parts.

The equation of motion in a continuum medium is :

$$\rho \frac{Ds_i^2}{Dt^2} = \rho \frac{\partial U}{\partial x_i} + \sum_j \frac{\partial p_{ij}}{\partial x_j} \qquad (3)$$

where U is the potential of the body force, including the additional part due to the deformation, p_{ij} the elements of the stress tensor and ρ the density.

The rheological behaviour of the material and some hydrodynamical proprieties in the fluid core have also to be considered.

The first member of the equation (3) has to be written in the non-rotating frame (R) and its second member has to be expressed, using the strain-stress relationships and observational values for the parameters ρ, λ, μ . The analytical development of the solution becomes, then, exceedingly complex, even with crude approximations of the Earth.

It can be shown that such a solution, corresponding to an harmonic perturbing potential, can be expressed as an infinite sum of harmonic free oscillations (toroïdal and spheroïdal ones).

When neglecting rotation and ellipticity, this sum is reduced to two terms and the system is equivalent to the Altermann et al.'s one (1959) of six equations (in the mantle) or five ones (in the core) of the first order, which can be solved by numerical integration, using the values of density and Lame's parameters inside the Earth based on seismic data.

Wahr was the first to consider rotation and ellipticity in the equations for the two parts, which couple together the toroïdal and spheroïdal modes of the same degree and to obtain a tenth degree system in the mantle and a seventh degree one in the core.

He solved it by a very efficient technique, using values of density, Lame's parameters and ellipticity of surfaces inside the Earth deduced from five recent Earth's models, one of which is the model 1066A mentioned in the part 1.

4. DEPENDENCE OF THE DYNAMICAL EQUATIONS ON THE EARTH MODEL

The results obtained through the most complex and precise elastic approach of the Earth's rotational motion (Wahr 1981), especially for the amplitudes of nutation, are the most precise ones. But it appears that this very complex treatment gives only slight modifications of these amplitudes as compared to the ones corresponding to less realistic modelizations of the Earth.

As the purpose of this paper is to show how the greatest features of the Earth model affect its nutational motion, we shall consider the very simpler dynamical approach based on the angular momentum equation applied to a rigid Earth, to an elastic Earth and to an elastic Earth with a fluid idealized core.

4.1. Rigid Earth model

The Euler's dynamical equations (1) can be written, if $\sigma_r = (C-A)\Omega/A$ is the frequency of the Euler's free wobble :

$$\begin{cases} \dot{m} - i\sigma_r m = \dfrac{L}{A\Omega} \\ \\ m_3 = 0 \end{cases} \qquad (4)$$

The lunisolar torque L can be expressed, following Melchior and Georis (1968) or Mc Clure (1973), by using the Doodson's expansion of the tesseral part of the tide generating potential (Doodson 1922), as :

$$L = -(C-A)\ \Omega^2\ K_☾ \sum_j A_j\ e^{-i(\omega_j t+\beta_j)} \qquad (5)$$

where $K_☾ = 3Gm_☾ / c_☾^3\,\Omega^2$, A_j are real coefficients, partly due to the Moon, partly to the Sun and the arguments $\omega_j t + \beta_j$ are the sum of the sidereal time and of linear combinations of the mean elements of the Moon and the Sun.

The Euler's equations are then written :

$$\dot{m} - i\sigma_r m = -\left[(C-A)/A\right]\ \Omega K_☾ \sum_j A_j\ e^{-i(\omega_j t + \beta_j)} \qquad (6)$$

4.2. Elastic Earth model

The terrestrial frame being the Tisserand's one, the relative angular momentum in (r) is equal to zero.

We denote by $c = c_{13} + ic_{23}$ and c_{33} the variable parts of the third column elements of the inertia tensor relative to the (r) frame.

In this case, the Liouville's equations corresponding to the projected angular momentum equations in the terrestrial frame, neglecting products of quantities m and c/C (of the order of 10^{-6}), can be written :

$$\begin{cases} \dot{m} - i\sigma_r m = \dfrac{L}{A\Omega} - \dfrac{ic\Omega}{A} - \dfrac{\dot{c}}{A} \\ m_3 = \dfrac{1}{C\Omega}\displaystyle\int_0^t L_3 \, dt - \dfrac{c_{33}}{C} \end{cases} \tag{7}$$

As we do not consider here the displacements of matter on the Earth's surface, the c and c_{33} terms are only due to the Earth's deformations produced by the tidal and centrifugal potentials.

Following the Love's representation, the Earth's deformation, as produced by these perturbing potentials, with energy concentrated in the second degree harmonic, increases the second degree harmonic of the Earth's gravitational potential and then, its coefficients J_2, C_{21}, S_{21}. The resulting variations of the elements of the Earth's inertia tensor are such that :

$$\begin{cases} c = \kappa(C-A)m + i\,\kappa(C-A)\,K_{☾} \displaystyle\sum_j A_j \; e^{-i(\omega_j t + \beta_j)} \\ c_{33} = \dfrac{4}{3}\,\kappa\,(C-A)\,m_3 + \dfrac{2}{3}\,\kappa\,K_{☾}\,(C-A) \displaystyle\sum_j A'_j \cos(\omega_j t + \beta_j) \end{cases} \tag{8}$$

where A'_j are similar coefficients to A'_j, but corresponding to the zonal part of the Doodson's expansion, and $\kappa = k_2/k_s$, k_2 and k_s being respectively the Love number of degree 2 and the secular Love number as considered by Munk and Mac Donald (1960) in order to represent the secular deformation of the Earth under its mean rotation.

The Liouville's equations can then be written, if $\sigma_0 = \sigma_r(1-\kappa)$ is the frequency of the Chandlerian free wobble and $n_j = \Omega - \omega_j$:

$$\begin{cases} \dot{m} - i\sigma_0 m = -\dfrac{C-A}{A}\Omega K_{☾} \displaystyle\sum_j A_j \left(1 - \kappa \dfrac{n_j}{\Omega}\right) e^{-i(\omega_j t + \beta_j)} \\ m_3 = -\dfrac{2}{3}\,\kappa\;K_{☾}\dfrac{(C-A)}{C} \displaystyle\sum A'_j \cos(\omega_j t + \beta_j) \end{cases} \tag{9}$$

4.3. Elastic Earth with fluid core

If the core mantle interface were perfectly spheric and if no dissipative effect occurs in the boundary layer, the motion of the core would be completely independent of the one of the mantle.

In the case of the spin rate ω_3, Wahr (1979) showed a discontinuity in the tidal variations of rotation, across the core-mantle interface and deduced a decoupling between these two parts for such variations. This can be explained by the non efficient effect of the third component of the inertial, viscous, topographic and electromagnetic torques at such periods, as evaluated by Yoder et al. (1981) and Wahr et al. (1981).

In the case of nutational motions, the more important coupling between the mantle and the core is the inertial torque due to the dynamical ellipticity $e_f = \frac{C_f - A_f}{A_f}$ of the core, which is an axisymmetric ellipsoïd with respective inertia momentums A_f and C_f.

This torque, denoted by $\vec{N} = \int_S (p - \rho_f U)\, \vec{r} \wedge d\vec{S}$, p being the fluid pressure, ρ_f the density of the fluid assumed to be homogeneous and incompressible and U the Earth's gravitational potential, is produced by the non-hydrostatic part π of the fluid pressure due to its asymmetric distribution with respect to the core-mantle interface, denoted S.

Non negligible damping effects are also produced by the dissipative torques (viscous and electromagnetic ones) acting on the boundary layer and which can be written in complex notation for the two first components (Sasao et al 1977) :

$\Gamma = - K(1 + i\eta)\,\Omega m_f$, η being respectively equal to 0.1 and 1 for the viscous and electromagnetic couplings and $m_f = \omega_f/\Omega = m_{f_1} + i m_{f_2}$, if $\vec{\omega}_f$ is the relative core flow vorticity.

The Liouville's equations, corresponding to the angular momentum equations for the whole Earth and the fluid core, written in complex notation for the two first components are then :

$$\begin{cases} \dot{m} - i\sigma_r m = \dfrac{L}{A\Omega} - \dfrac{i\, c\Omega}{A} - \dfrac{\dot{c}}{A} - i\dfrac{h_f}{A} - \dfrac{\dot{h}_f}{A\Omega} \\ \dot{m} - i\, e_f \Omega\, m = \dfrac{L_f + N + \Gamma}{A_f \Omega} - \dfrac{i\Omega c_f}{A_f} - \dfrac{\dot{c}_f}{A_f} - \dfrac{i\, h_f}{A_f} - \dfrac{\dot{h}_f}{A_f \Omega} \end{cases} \qquad (10)$$

where $c_f = c_{13f} + i c_{23f}$ represents the variable part of the inertia tensor of the core induced by the elastic yielding in the mantle, $h_f = A_f\, \Omega\, m_f$ the relative angular momentum in the core and

$L_f = (C_f - A_f)\, L/(C-A)$ the exterior torque acting on the core.

Taking into account the tidal and rotational potentials as well as the potential generated by the core flow, N can be written :

$$N = i\,(C_f - A_f)\,\Omega^2(\pi - m) - L_f + i\,\Omega^2 c_f$$

And then, considering the dynamical effect of the fluid core on the Love number k, through two dimensionless coefficients β and γ of the order of 10^{-3}, as given by Sasao et al. (1977), and the expressions of L_f, N, Γ, c, c_f, h_f, the equations (10) can be written :

$$\begin{cases} A\dot{m} - iA\sigma_o m + A_f(\dot{m}_f + i\Omega m_f) = -(C-A)\Omega K_{\ast} \sum_j (1 - \frac{\kappa n_j}{\Omega}) A_j \quad e^{-i(\omega_j t + \beta_j)} \\ A_f \dot{m} + A_f \; \dot{m}_f + \{A_f \; i(1+e_f)\,\Omega \quad + K(1+i\eta)\}\, m_f = -A_f \quad K_{\ast} \sum_j A_j \omega_j e^{-i(\omega_j \, t + \beta_j)} \end{cases} \tag{11}$$

This is a differential system in m and m_f giving rise to two free modes with respective frequencies σ_1 , nearly diurnal, and σ_2 Chandlerian, such that :

$\sigma_1 = -\Omega\left[1 + \frac{A}{A-A_f}(e_f - \beta)\right] + i\delta$, δ being a damping factor due to the dissipativetorques, and :

$$\sigma_2 = \frac{C-A}{A-A_f}(1-\kappa)\Omega = \frac{A}{A-A_f}\,\sigma_o \quad .$$

5. DEPENDENCE OF THE SOLUTIONS ON THE ADOPTED EARTH MODEL

5.1. Solutions for the diurnal polar motion and celestial nutations

Each forced "diurnal nutation" of the rotation axis within the Earth can be written : $m_j = \alpha_j \; e^{-i(\omega_j t + \beta_j)}$ (12)

The corresponding circular nutation of the Gz axis in space, derived from the Euler's kinematical relation (2) and a simple integration with respect to time, is :

$$\Delta\theta_j + i\,\Delta\Psi_j \sin\theta = \xi_j \; e^{i(n_j t - \nu_j)} \tag{13}$$

with $\xi_j = -\Omega\alpha_j / n_j$ (14)

The resolution of (6) for a rigid Earth, gives :

$$\alpha_{R_j} = -\, iK_{\ast} \frac{C-A}{C}\left[A_j / (1 - \frac{A\,n_j}{C\,\Omega})\right] \tag{15}$$

The resolution of (9) for an elastic Earth gives :

$$\alpha_{E_j}/\alpha_{R_j} = \xi_{E_j}/\xi_{R_j} = 1 - \frac{\kappa\, n_j}{\Omega} \tag{16}$$

which shows that the Earth's elasticity modifies the amplitude of each circular nutation (within the Earth or in space) in a ratio included between 0.98 and 1.02 and equal to 1 for $n_j = 0$ (corresponding to precession).

The resolution of (11) for an elastic Earth with fluid core and dissipative torques at the boundary gives (Sasao et al. 1977) :

$$\alpha_{Ef\,j}/\alpha_{R_j} = \alpha_{E_j}/\alpha_{R_j} - \frac{A_f}{A-A_f}\left(\frac{n_j}{\omega_j + \sigma_1}\right)\left(1 - \frac{\gamma}{e} + [\frac{\gamma}{e} - \kappa]\frac{n_j}{\Omega}\right) \tag{17}$$

and a similar expression, obtained by replacing α by ξ, for the amplitude of the corresponding spatial circular nutation.

The elasticity of the Earth and the existence of an ellipsoïdal fluid core are both responsible for the modification of the amplitudes of nutation from the ones corresponding to a rigid Earth; a resonance effect occurs for $-\omega_j = \sigma_1$ in the part of the ratio due to the fluid core.

The total factor is included between 0.996 and 1.250 and is equal to 1 for the precession ($n_j=0$).

The amplitude of the elliptical nutation with frequency n_j is obtained by combination of the modified amplitudes of the two circular nutations with respective frequencies n_j and $n_{-j} = -n_j$.

5.2. Solution for the angular rate

In the case of a rigid Earth, the Earth's angular rate is constant.

In the case of an elastic Earth, the tidal variations of the angular rate, as given by the second line of (9) in 4.2. have amplitudes of the order of 10^{-13} rad/s corresponding to 0.1 ms in the observed Universal Time UT1. The principal terms are of respective periods 13.66d, 27.55 d, 9.3 y, 18.6 y. for those of lunar origin and are annual, semi-annual ones for those of solar origin.

The theoretical expressions for these variations, can be obtained from the Woolard's (1953) developments, as by Woolard (1959), or from the Kinoshita (1977) ones, as by Yoder et al.(1981), and from the Doodson's or Cartwright & Edden's (1973) expansions of the tidal potential, as by Wahr et al.(1981).

In the case of an elastic Earth with fluid core, Wahr et al.(1981) have shown that the two parts of the Earth are decoupled for the tidal variations. So, as each part conserves angular momentum separately, only the changes in the mantle moment of inertia influence the changes in rotation rate. This lack of core-mantle coupling lowers the Love number k by 0.032, so reduces the amplitudes of the tidal variations in UT1 by about 11% from the case of an elastic Earth as appears for the 1066A-oceanless Earth model (Wahr et al. 1981).

Similar evaluations have been made by Merriam (1980) and Yoder et al.(1981).

As also recalled by these authors, Agnew & Farell (1978) have shown that the equilibrium ocean tides increase the Love number k by 0.038, so increase the amplitudes of the tidal variations in UT1 by about 13%.

So the effects of the equilibrium ocean and the fluid core almost cancel and the amplitudes of the tidal variations in UT1 corresponding to an Earth model with equilibrium ocean and fluid core are nearly the same as for a purely elastic Earth (Yoder et al. 1981).

6. COMPARISON OF THE LUNISOLAR PERTURBATIONS IN THE EARTH ROTATION OBTAINED FOR A FEW EARTH MODELS.

The results for the largest elliptic spatial nutations and for the largest tidal variations in UT1, as obtained for a few Earth models, are given in the following two Tables.

Table 1. Amplitudes of the elliptical spatial nutations (epoch J2000.0) corresponding to a few Earth models. These amplitudes are referred to the celestial ephemeris pole, except for the ones marked with * , which are referred to the rotation axis.

Earth model		Rigid Earth		Deformable Earth	Elastic oceanless Earth with fluid core		
Period	Author	Woolard (1953)	Kinoshita (1977)	Capitaine (1980)	Molodensky (1961)	Sasao et al. (1977)	Wahr (1979)
18.6y	Obliquity	9".2109* 9".2099	9".2278	9".2281	9".2044	9".2018	9".2025
	Longitude	-6".8617* -6".8603	-6".8743	-6".8750	-6".8441	-6".8407	-6".8416
182.6d	Obliquity	0".5519* 0".5547	0".5534	0".5526	0".5719	0".5739	0".5736
	Longitude	-0".5064* -0".5094	-0".5082	-0".5073	-0".5232	-0".5249	-0".5245
13.7d	Obliquity	0".0884 0".0948	0".0949	0".0930	0".0972	0".0977	0".0977
	Longitude	-0".0810* -0".0879	-0".0881	-0".0861	-0".0899	-0".0904	-0".0905

It shows that the deformability of the Earth has a minor effect on the modification of nutation as compared to the one of the fluid core, whereas the phenomenom is inverse for the modification of the frequency of the free wobble. The amplitudes obtained by Wahr (1979) are the most precise ones though neither the frictional torques nor the oceans are taken into account.

The amplitudes of the diurnal nutations of the rotation axis within the Earth are nearly unchanged by the Earth model, due to their order of size. The largest modification occurs for the term of argument T - 2☾ (T being the sidereal time and ☾ the mean longitude of the Moon. Its amplitude is 0."0067 for a rigid Earth, 0".0065 for an elastic Earth and 0".0069 for an elastic Earth with fluid core.

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Table 2. Tidal variations in Universal Time UT1 (in 10^{-4} s) corresponding to a few Earth models.

Earth model / Period \ Author	Deformable Earth with k = 0.299	Deformable Earth with fluid core with equilibrium ocean	Deformable Earth with fluid core oceanless
	Woolard (1959)	Yoder et al. (1981)	Wahr et al.(1981)
18.6 y	-1539.9	-1617.3	-1380.0
9.3 y	8.1	7.9	6.9
1 y	14.7	15.4	13.1
182.6 d	45.8	48.3	41.2
27.6 d	7.9	8.3	7.1
13.66d	7.4	7.7	6.6
13.63d	3.1	3.2	2.7

It confirms the conclusion given in 5.3. and the most precise amplitudes are given by Yoder et al. (1981).

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