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Jecklin's paper on year-of-entry grouping for valuation of endowment policies

Sirs,

In his paper on year-of-entry grouping for valuation of endowment policies (*J.S.S.* x, 119) H. Jecklin utilizes the interesting approximate relationship

$$\frac{t/n}{1-t/n} \doteqdot F\,\frac{{}_tV_{x:\overline{n}|}}{1-{}_tV_{x:\overline{n}|}},$$

where F is independent of t. It is worth noting that, if instead of the proportionate paid-up policy t/n, the true paid-up policy ${}_tW_{x:\overline{n}|}$ is used, we have the exact relationship

$$\frac{{}_tW_{x:\overline{n}|}}{1-{}_tW_{x:\overline{n}|}} = F'\,\frac{{}_tV_{x:\overline{n}|}}{1-{}_tV_{x:\overline{n}|}}, \quad \text{where} \quad F' = \frac{1}{A_{x:\overline{n}|}}.$$

For sinking fund policies and endowment assurances at the younger ages it will be found that $(F'-1)$ approximates to twice $(F-1)$. The near constancy of F for variations in t (x and n fixed) suggests that in practice an F formula might well be a useful instrument for linking surrender values and paid-up policies by a suitable choice of a set of F's, whether the paid-up policy is on the proportionate basis or not or whether the surrender value is obtained from the paid-up policy or vice versa.

For this purpose, as well as for Jecklin's original purpose of expressing ${}_tV_{x:\overline{n}|}$ in terms of F and t/n in an ingenious year-of-entry approximate valuation system, it is desirable to examine the closeness of the F-approximation at the older ages—Jecklin gives figures at 3% on the A 1924–29 ultimate table for entry age 30 only. Figures for other entry ages are shown in Table 1 as well as for sinking fund policies. This table confirms that $F-1$ is nearly constant and approximates to $\frac{1}{2}(F'-1)$ for ages at maturity up to 60; thereafter, the approximation progressively breaks down.

It does not necessarily follow, however, that even in these cases

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the assumption of a constant value for F will produce an intolerable approximation for ${}_tV_{x:\overline{n}|}$ or a worse result than by assuming a common maturity age of, say, 60. An extreme example is given in Table 2.

Table 1. *Values of* (F − 1) *and* (F′ − 1) *for* 3 % *sinking fund policies and for endowment assurances on* A 1924–29 *ult.* 3 %

Term	Duration	Sinking fund	Age at entry					
			20	30	40	50	60	65
10	All F′ − 1	**·344**	**·339**	**·339**	**·335**	**·325**	**·298**	**·273**
	1 F − 1	·163	·176	·176	·181	·191	·237	·271
	5	·159	·173	·172	·182	·200	·266	·332
	9	·156	·169	·171	·182	·211	·307	·412
20	All F′ − 1	**·806**	**·777**	**·768**	**·737**	**·659**	**·501**	—
	1 F − 1	·362	·397	·381	·370	·325	·234	—
	5	·354	·386	·373	·371	·338	·277	—
	10	·344	·374	·367	·374	·371	·378	—
	19	·327	·356	·364	·399	·505	·817	—
25	All F′ − 1	**1·094**	**1·036**	**1·014**	**·944**	**·788**	—	—
	1 F − 1	·479	·519	·490	·446	·316	—	—
	5	·467	·507	·478	·445	·326	—	—
	10	·454	·490	·471	·446	·359	—	—
	20	·427	·463	·468	·488	·542	—	—
	24	·418	·456	·475	·532	·699	—	—
30	All F′ − 1	**1·427**	**1·321**	**1·272**	**1·133**	**·872**	—	—
	1 F − 1	·608	·656	·584	·491	·243	—	—
	5	·592	·637	·576	·482	·244	—	—
	10	·575	·613	·565	·479	·264	—	—
	20	·541	·577	·563	·522	·420	—	—
	29	·510	·565	·591	·681	·933	—	—
40	All F′ − 1	**2·262**	**1·943**	**1·756**	**1·380**	—	—	—
	1 F − 1	·902	·932	·719	·407	—	—	—
	5	·886	·898	·693	·385	—	—	—
	20	·806	·799	·655	·356	—	—	—
	39	·710	·799	·868	1·061	—	—	—

From a practical point of view, Jecklin's approximate valuation method is subject to the serious drawback that, as presented so far, it does not enable us to obtain any valuation figures other than the total of the policy values. In British practice we also need the value of the sums assured, the value of the net premiums and the amount of the net premiums. For with-profits policies we also need the value of the accrued bonuses; the value of the sums assured is also required to obtain the value of the new bonus. Most of these drawbacks would appear also to apply to the retrospective t-method studied recently in Switzerland and referred to in Jecklin's paper.

Table 2. *Specimen values of* $(F-1)$ *and* ${}_tV_{x:\overline{n}|}$ *and approximate values of* ${}_tV_{x:\overline{n}|}$ *based on a constant value of* $(F-1)$ *for an endowment assurance at entry age* 40, *term* 35 *years on* A 1924–29 *ult.* 3%. *Policy values for maturity age* 60 *are also shown.*

t	$F-1$	${}_tV_{x:\overline{n}|}$	Approx.* ${}_tV_{x:\overline{n}|}$	Percentage error	${}_tV_{25:\overline{35}|}$
1	·471	1·96	1·95	− ·5	1·66
5	·463	10·23	10·12	−1·1	8·82
10	·452	21·60	21·28	−1·5	19·09
15	·454	34·03	33·63	−1·2	30·89
20	·480	47·40	47·39	—	44·41
25	·545	61·80	62·81	+1·6	60·01
30	·675	78·18	80·21	+2·6	78·19
34	·857	94·82	95·83	+1·1	95·26

* *Note.* The approximate values of ${}_tV_{x:\overline{n}|}$ are obtained from the formula ${}_tV_{x:\overline{n}|}=t/[t+F(n-t)]$ with $F=1·48$.

It is perhaps not inappropriate, therefore, to refer to the fact that in my first paper on the n-point method (*J.I.A.* LXIV, 264) I gave a method of valuing endowment assurances in year-of-entry groupings which yields all the desired figures. This method requires the assumption of a fixed maturity age. In my second paper on the n-point method (*J.I.A.* LXXII, 377) I gave methods which could readily be adapted to year-of-entry groupings in which both variables x and n could be allowed for together, if for some reason a fixed maturity age could not be adopted. The n-point method also has the great advantage that it frees the classification from any particular valuation basis.

Yours faithfully,

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