Canad. Math. Bull. Vol. **18** (3), 1975

ON COUNTABLY, σ-, AND SEQUENTIALLY BARRELLED SPACES

BY
T. HUSAIN* AND S. M. KHALEELULLA

The first author [2] calls a Hausdorff locally convex (abbreviated to l.c.) space (E, u) countably barrelled if each $\sigma(E', E)$-bounded subset of E', which is the countable union of equicontinuous subsets of E', is itself equicontinuous.

De Wilde and Houet [1] call a l.c. space (E, u) σ-barralled (which is the same as ω-barrelled of [5]) if each $\sigma(E', E)$-bounded sequence in E' is equicontinuous.

Webb [7] calls a l.c. space (E, u) sequentially barrelled if each $\sigma(E', E)$-convergent sequence in E' is equicontinuous.

Every barrelled space is countably barrelled; every countably barrelled space is σ-barrelled; and every σ-barrelled space is sequentially barrelled. Iyahen [3] and Morris and Wulbert [6] have given examples of countably barrelled spaces which are not barrelled. In this note we give two examples of sequentially barrelled spaces which are not σ-barrelled.

EXAMPLE 1. It is known that the space l^∞ of all bounded sequences (real or complex) is a perfect sequence space and l^1 is its Köthe dual ([4], page 406). Consider $(l^\infty, \tau(l^\infty, l^1))$, where $\tau(l^\infty, l^1)$ is the Mackey topology on l^∞. Then, $(l^\infty, \tau(l^\infty, l^1))$ is sequentially barrelled ([7], page 354) and we show, that it is not σ-barrelled. Suppose it is σ-barrelled. Then, being separable ([7], page 357), it is barrelled ([1], page 260). But it is known that it is not barrelled ([7], page 357), which is a contradiction.

REMARK. That $(l^\infty, \tau(l^\infty, l^1))$ is not σ-barrelled also follows from [5], pages 100 and 102.

EXAMPLE 2. The space $(l^1, \tau(l^1, c_0))$, where c_0 is the space of null sequences, is a sequentially barrelled space ([7], page 357). That it is not σ-barrelled follows from ([5], page 102).

It is not yet known whether or not every σ-barrelled space is countably barrelled.

REFERENCES

1. M. De Wilde and C. Houet, *On increasing sequences of absolutely convex sets in locally convex spaces*, Math. Ann. **192** (1972), 257–261.

2. T. Husain, *Two new classes of locally convex spaces*, Math. Ann. **166** (1966), 289–299.

* This work was supported by an N.R.C. grant.

https://doi.org/10.4153/CMB-1975-080-x Published online by Cambridge University Press

3. S. O. Iyahen, *Some remarks on countably barrelled and countably quasi-barrelled spaces*, Proc. Edinburgh Math. Soc. (2) **15** (1966/67), 295–296.

4. G. Köthe, *Topological vector spaces* I, (English Edition) Springer-Verlag, New York, Inc. 1969.

5. M. Levin and S. Saxon, *A note on the inheritance of properties of locally convex spaces by subspaces of countable codimension*, Proc. Amer. Math. Soc. **29**, no. 1 (1971), 97–102.

6. P. D. Morris and D. E. Wulbert, *Functional representation of topological algebras*, Pacific J. Math. **22**, no. 2 (1967), 323–337.

7. J. H. Webb, *Sequential convergence in locally convex spaces*, Proc. Camb. Phil. Soc. **64** (1968), 341–364.

McMaster University,
Hamilton, Ontario,
Canada

https://doi.org/10.4153/CMB-1975-080-x Published online by Cambridge University Press