Introduction to Calculus, by K. Kuratowski. Pergamon Press, 1961. 315 pages. 35/.

Although the number of calculus texts at present on the market is only just not denumerable, very rarely is one written by as eminent a mathematician as Kuratowski, and this book, which is a translation (and a slight revision) of the Polish edition of 1946 will arouse much interest.

In spite of the title, this book does not compare with the usual texts entitled "Calculus" (or Introduction thereto) used in Canadian (and American) universities. It is about a quarter the weight of, say, Taylor's or Johnson and Kiokemeister's book, and presupposes greater maturity on the part of the reader. Indeed, I would classify it under "Analysis" rather than "Calculus". The most nearly comparable well-known books are Hardy's "Pure Mathematics" and volume I of Courant's "Differential and Integral Calculus", it being nearer to Hardy than to Courant, though it lacks Hardy's plethora of exercises (a supplementary book of problems is, however, promised). Kuratowski keeps strictly to functions of one variable, leaving functions of several variables to a second volume. However, by including series of functions, he manages to cover some of the fundamental ideas of the theory of binary functions, notably the idea of uniform convergence.

The first section covers induction, bounds, and continuity. Real numbers are treated in three sub-sections; an informal explanation called "various kinds of numbers" right at the start; a very brief sketch of the axioms for a complete ordered field; and an equally brief description of the use of Dedekind's Section to construct the reals from the rationals. The remaining sections of Chapter I cover sequences and series, where Kuratowski goes a little deeper than Hardy and Courant into convergence-tests giving Kummer's and Raabe's criteria; but the subject starts very brusquely. The definition and explanation of series consists of the following two sentences:-

"If a real number corresponds to each positive integer, then we say that an infinite sequence is defined. For instance, positive even numbers constitute an infinite sequence $2, 4, 6, \ldots, 2n, \ldots$; namely, to the number 1 there corresponds the number 2, to the number 2, the number 4, to the number 3, the number 6, and generally, to the positive integer n there corresponds the number 2n."

A student meeting sequences here for the first time may well need help in seeing what this is all about.

Chapter II deals with functions and their limits. Again, the opening definition seems to me to be weak. It runs:-

"If to any x belonging to a certain set there corresponds a number $y = f(x)$, then a function is defined over this set." The substitution of "each" for "any" would improve this a little. Limits are defined in terms of sequences: ℓ is the limit of f at a if $f(x_n)$ tends to ℓ whenever $\{x_n\}$ tends to a. This conveniently exploits the fact that sequences were treated before functions, and enables several later proofs to be shorter than in the more familiar treatment. Uniform continuity is treated from early on, and the chapter ends with a section on uniform convergence and power series and an optional subsection on mathematical logic which makes the point that the difference between uniform and point-wise convergence corresponds to a permutation of quantifiers.

Chapter III covers differentiation and contains no surprises. Chapter IV, on integration, contains one very interesting novelty. The definite integral is defined as follows. If f is continuous on the closed interval $[a, b]$ and if F is any primitive of f on this interval then $\int_a^b f(x)dx$ is defined to be $F(b) - F(a)$. That the integral is a limit of a sum follows by uniform continuity, and the use of integrals to approximate areas is justified by polygonal approximations. The Riemann integral is treated later and regarded as "a generalization of the notion of a definite integral to a certain class of discontinuous functions".

The statements of the theorem on integration by substitution is one of the best I have seen: the conditions are given in terms of piece-wise continuity, a very slight generalization theoretically but very useful in practice.

This chapter also contains a good justification of the use of integration to calculate centres of mass. This justification lies somewhere between calculus and analysis and is rarely included in books on either subject.

To sum up: we have here a compact book, with some interesting novelties, less prolix than Hardy, less expensive than Courant, less taciturn than Landau, and capable of bearing them honourable company.

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https://doi.org/10.1017/S0008439500026345 Published online by Cambridge University Press