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ARTICLE

# Acyclic population ethics and menu-dependent relations

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**Abstract**
It has been shown that the Mere Addition Paradox occurs in a choice-functional approach with Path Independence (Stewart R.T., 2021, Path independence and a persistent paradox of population ethics, Journal of Philosophy, forthcoming). The present study is a three-part response to this finding. First, I show that Path Independence is not an essential property leading to this paradox and that logically weaker properties can get the same result. Second, I present a rationalizable choice function that does not yield the paradox. And third, I argue that menu-dependent relations are nicely examined if Path Independence is relaxed to Property $\alpha$ (or equivalently, Contraction Consistency).

**Keywords:** path independence; acyclicity; menu-dependent relations; population ethics; mere addition paradox

## 1. The Choice-functional Approach to Population Ethics

In 'Path independence and a persistent paradox of population ethics', Rush Stewart (2021) develops a choice-functional approach to population ethics.[1] The fundamental element of his argument is a choice function $C$ that assigns permissible alternatives (populations in the context of population ethics), $C(S)$, to each menu (i.e. set of available options). If a population $A$ is an element of $C(S)$, then $A$ is said to be permissible (or chosen) under menu $S$. For each menu $S$, $C(S) \subseteq S$ and $C(S)$ is nonempty. An advantage of Stewart's approach is that a wider range of concerns can be incorporated than in the standard approach with a better-than relation/preference. For example, menu-dependent relations can be captured using a choice-functional framework.[2]

[1] Blackorby *et al.* (2005) is a pioneering study on the choice-functional approach.
[2] See Levi (1986) and Sen (1997) for analyses of individual choices that cannot be captured by the standard choice function or maximization.

The paradox mentioned in the title of Stewart's paper is an extension of the Mere Addition Paradox, which he obtains by using a choice function. This paradox is understood as a logical relation between the following three conditions.

**Mere Addition.** If $A$ and $B$ are populations that contain only positive utility levels, then $A \cup B \in C(\{A \cup B, A\})$.

**Non-Anti-Egalitarianism.** Let $A$ and $B$ be two populations of the same population size. If all individuals have the same utility level in $B$ and the total sum of utility in $B$ is higher than that in $A$, then $A \notin C(\{A, B\})$.

**Repugnant Conclusion.** For any large population $A$ with blissful utility levels, there is another much larger population $B$ such that all individuals are at a positive, but drab utility level, and $B \in C(\{A, B\})$.

Stewart (2021) shows that Mere Addition and Non-Anti-Egalitarianism imply the Repugnant Conclusion if the following choice axiom is satisfied:

$$C(S \cup T) = C(C(S) \cup C(T)) \text{ for all menus } S, T.$$

According to this axiom, any large choice problem can be decomposed into a collection of smaller choice problems, which is called *Path Independence* (Plott 1973). Stewart's novel contribution is the introduction of this axiom into the formal analysis of population ethics. By using Path Independence as a necessary condition of plausible population ethics, Stewart establishes a paradox, which is a natural extension of Ng's (1989) axiological impossibility result. Ng (1989) uses a transitive relation, but Stewart replaces transitivity with Path Independence, which implies that transitivity might be unnecessary. Notably, Stewart motivates his result by arguing that Path Independence is a minimally acceptable choice constraint on axiology.

In this paper, I will show that Path Independence can be replaced with logically weaker properties. Let us define

$$AR^*B \Leftrightarrow A \in C(\{A, B\}),$$

wherein $R^*$ represents what is called the *base relation* in the literature (Sen 1971; Suzumura 1983). $A$ is at least as good as $B$ with regard to the base relation if and only if $A$ is permissible in the choice between $A$ and $B$. It can be proved that $R^*$ is quasi-transitive (and complete) if Path Independence is satisfied (Suzumura 1983: 95–96). Quasi-transitivity is defined as the transitivity of the asymmetric part (strict preference) $P^*$ of $R^*$ (i.e. $AP^*B$ if and only if $\{A\} = C(\{A, B\})$). Thus, if $AP^*B$ and $BP^*C$, then $AP^*C$ for all $A, B$, and $C$.[3]

To see that the quasi-transitivity of the base relation is crucial to Stewart's main observations, imagine some population $A$ with blissful utility levels and add a large number of individuals with positive but negligible utility levels resulting in a much larger population $A^+$. Then, consider an egalitarian population $E$ that has the same population size as $A^+$, but with a higher total utility level, while the utility of each

[3]See Cato (2016) for a summary of well-known properties of binary relations.

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individual is at a drab level.[4] By Mere Addition, $A^+ \in C(\{A^+, A\})$, which implies that $A^+R^*A$. Non-Anti-Egalitarianism requires that $A^+ \notin C(\{A^+, E\})$, implying that $\{E\} = C(\{A^+, E\})$. Therefore, $EP^*A^+$. If $AP^*E$, then quasi-transitivity implies that $AP^*A^+$, which contradicts $A^+R^*A$. It follows that $ER^*A$, and thus that the Repugnant Conclusion holds true. In short, the quasi-transitivity of the base relation leads to the Mere Addition Paradox.

Notice that this proof does not depend on Path Independence. However, the quasi-transitivity of the base relation logically follows from Path Independence (Suzumura 1983: 95), which means that my proof entails Stewart's observation as a corollary. For this reason, it can be argued that the quasi-transitivity of the base relation is more essential for the Mere Addition Paradox in the choice-functional framework than Path Independence, which suggests that Path Independence is not an indispensable property in Stewart's observation.

To see this from another perspective, consider the following two alternative choice axioms:

$$[\text{Weak } \alpha]: \quad \text{if } A \in C(\{A, B, D\}) \cap C(\{A, B\}), \ \text{ then } A \in C(\{A, D\}),$$

and

$$\left[\text{Property } \delta\right]: \quad \text{if } A,\ B \in C(S) \subseteq S \subseteq T \text{ and } A \neq B,\ \text{ then } \{A\} \neq C(T).$$

According to Weak $\alpha$, if $A$ is permissible under $\{A, B, D\}$ and $\{A, B\}$, then it is also permissible under $\{A, D\}$. Property $\delta$ requires that if both $A$ and $B$ are permissible under $S$, which is a subset of $T$, then $A$ cannot be the only permissible population under $T$. Both of these axioms are implied by Path Independence, and therefore, *if* Path Independence is considered to be plausible, then these axioms must be considered plausible as well because of this logical relation; see Appendix A. Together, Weak $\alpha$ and Property $\delta$ imply that the base relation is quasi-transitive, and these two axioms are, therefore, sufficient to obtain the Mere Addition Paradox, which means that Path Independence can be replaced by weaker properties. The unfortunate fallout hereof is that the impossibility of finding an appropriate theory in population ethics is even more fundamental.

Given the observation that Weak $\alpha$ and Property $\delta$ are logically implied by Path Independence, that the conjunction of Weak $\alpha$ and Property $\delta$ is indeed weaker than Path Independence can be seen in the following example. Consider the following choice function:

$$\{A_1\} = C(\{A_1, A_2\}), \{A_1\} = C(\{A_1, A_3\}), \{A_1\} = C(\{A_1, A_4\}),$$

$$\{A_2\} = C(\{A_2, A_3\}), \{A_2\} = C(\{A_2, A_4\}), \{A_3\} = C(\{A_3, A_4\}),$$

$$\{A_1\} = C(\{A_1, A_2, A_3\}), \{A_1\} = C(\{A_1, A_2, A_4\}),$$

$$\{A_2\} = C(\{A_2, A_3, A_4\}), \{A_1, A_2\} = C(\{A_1, A_2, A_3, A_4\}).$$

This satisfies Weak $\alpha$ and Property $\delta$ but violates Path Independence. Notice that the violation of Path Independence occurs when the choice of $\{A_1, A_2, A_3, A_4\}$ is considered. This suggests that the requirement of Path Independence is imposed

[4]These are the three populations shown in figure 2 in Stewart (2021).

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even when many options are included, and that when it comes to very complex paths with many options, Path Independence may be too demanding. For example, Path Independence requires that

$$C\left(\bigcup_{m=1}^{M} S_m\right) = C(C(C(C(S_1) \cup S_2) \cup S_3) \ldots S_M)$$

where each $S_m$ contains over 1,000,000 populations. (See Suzumura 1983: Theorem 2.4.) In some issues in population ethics, such as Parfit's Continuum Argument, a very large number of populations may be considered as options, but in the present context, it would be natural to restrict the requirement for consistency to cases with a limited number of options such as Weak $\alpha$.[5] Our result shows that the impossibility (of finding an appropriate theory in population ethics) holds true even with this restriction.

Before proceeding to the next topic and the main point of this paper, let me summarize the implications of the brief review of Stewart's results described above. First, the choice-functional approach is useful for extending existing work in population ethics. Second, Path Independence is not the logically weakest requirement for obtaining the Mere Addition Paradox because the quasi-transitivity of the base relation is mathematically more essential than Path Independence, as shown above. Third, given this latter point, Stewart's result can be substantially generalized.

## 2. Acyclic Population Ethics

In this section I will examine the Mere Addition Paradox in a choice-functional approach that uses an acyclic better-than relation. Since it is possible to infer a choice function by using the better-than relation, the relational approach can be connected with the choice-functional approach. Key to this connection is the acyclicity of the better-than relation, because a choice function inferred from the better-than relation is well-defined only when the relation is acyclic. Related to this, Stewart (2021) notes that the 'choice pattern' that Frick (2022) advocates is consistent with maximizing an acyclic binary relation and even offers an example of the relation in footnote 27. However, his binary relation is limited to the case of three options, and neither its applicability to the case with $m$ options for $m \geq 4$, nor the general idea behind it is entirely clear. Hence, by offering a general principle that can apply to any number of options, in some sense, I complement Stewart's point.

Let us define a population principle called 'Theory $\alpha$'. I will show that Theory $\alpha$ yields a choice function that satisfies Non-Anti-Egalitarianism, Mere Addition, Weak $\gamma$ (explained below) and Property $\alpha$, and that, moreover, avoids the Repugnant Conclusion. As an auxiliary step, I introduce the concept of an *extension*. Given two populations $F$ and $G$, population $F$ is an extension of population $G$ if and

[5]Notably, such a restriction is considered in the literature of the revealed preference theory. The Strong Axiom can be recovered even if the requirement is weakened by applying the Weak Axiom of Revealed Preference only when two or three options are included. See Sen (1971).

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only if there is a nonempty sub-population $F'$ of $F$ such that $F'$ is distinct from $G$ and $F = F' \cup G$.[6] That is, $F$ is obtained by adding $F'$ to $G$. Notably, $F$ is an extension of $G$ when $G$ is a proper sub-population of $F$ (i.e. $F \supsetneq G$); $F \not\supsetneq G$ means that $F$ is not an extension of $G$.

Theory $\alpha$ is defined as follows: For all two populations $A$ and $B$, population $A$ is strictly better than population $B$ (formalized as '$AP_\alpha B$') if and only if either of the following two conditions holds: (i) the population size of $A$ is the same as that of $B$, and the total sum of utility in $A$ is greater than that in $B$, or (ii) the population size of $A$ is smaller than that of $B$, $B$ is not an extension of $A$, and the average utility in $A$ is greater than that in $B$.[7] Formally:

$$AP_\alpha B \Longleftrightarrow$$

$$((N_A = N_B) \wedge (TU_A > TU_B)) \vee ((N_A < N_B) \wedge (B \not\supsetneq A) \wedge (AU_A > AU_B)),$$

where $N_A$ is the population size of $A$, $TU_A$ is the total sum of the utilities in $A$ and $AU_A$ is the average utility level in $A$ (where $N_B$, $TU_B$ and $AU_B$ are defined in parallel).[8] Notably, $A$ is strictly better than $B$ only when the population size of $A$ is less than or equal to that of $B$.

It is worth mentioning that Theory $\alpha$ is a hybrid of total utilitarian and average utilitarian methods of assessment. It conditionally uses total utilitarianism and average utilitarianism, switching between the two. It may be interesting to compare the proposed principle with the class of variable population principles (Hurka 1883; Ng 1986). For a variable population principle, there is a non-decreasing function $f$ such that $A$ is better than $B$ if and only if

$$f(N_A)AU_A > f(N_B)AU_B.$$

When $f(n) = n$ for all natural numbers $n$, this principle corresponds to total utilitarianism, and when $f(n) = 1$ for all natural numbers $n$, it corresponds to average utilitarianism. Hence, variable population principles take an intermediate position between the two. Theory $\alpha$ is similar in nature to variable value principles in its reconciliation of the two extreme types of utilitarianism, but there is a significant difference. While in the traditional approach to variable principles a continuous function $f$ is used to construct an intermediate principle that is neither total utilitarianism nor average utilitarianism, in Theory $\alpha$ a radical switch is made by combining the two directly. And more importantly, while a variable value principle cannot avoid the mere addition paradox (Bossert *et al.* 2023), Theory $\alpha$ can. Hence, Theory $\alpha$ can be seen as a new arrangement that avoids the mere

[6]For example, $F = (1, 3, 5)$ is an extension of $G = (1, 3)$, while $F = (1, 3, 5)$ is not an extension of $G' = (1, 4)$.

[7]Instead of the average utility level, we can use the worst-off or the best-off. That is, (ii) can be replaced with (ii′) or (ii″) formulated as follows: (ii′) the population size of $A$ is smaller than that of $B$, $B$ is not an extension of $A$, and the worst-off in $A$ is better than that in $B$; (ii″) the population size of $A$ is smaller than that of $B$, $B$ is not an extension of $A$, and the best-off in $A$ is better than that in $B$.

[8]I do not identify indifference relations $I_\alpha$ or weak preference relations $R_\alpha$ in this definition. However, $R_\alpha$ is easily obtained by letting $AR_\alpha B \Longleftrightarrow \neg(BP_\alpha A)$, and $I_\alpha$ is obtained by letting $AI_\alpha B \Longleftrightarrow \neg(BP_\alpha A) \wedge \neg(AP_\alpha B)$.

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addition paradox while embracing the spirit of traditional variable value principles, at least to some extent.

Significantly, theory $\alpha$ is acyclic.[9] Acyclicity is a necessary and sufficient condition for the non-emptiness of the set of maximal elements. In a sense, acyclicity guarantees that the decision-making over the populations is decisive (Sen 1970: Lemma 1*1). The claim that Theory $\alpha$ is acyclic can be proved as follows. If this theory is cyclic, there is a chain of populations, $A^1, A^2, \ldots, A^K$, such that $A^1 P_\alpha A^2$, $A^2 P_\alpha A^3, \ldots, A^{K-1} P_\alpha A^K$, and $A^K P_\alpha A^1$. As mentioned above, $A^1$ is strictly better than $A^2$ only when $N_{A^1} \leq N_{A^2}$. Similarly, $A^2$ is strictly better than $A^3$ only when $N_{A^2} \leq N_{A^3}$. This holds for all adjacent pairs in $A^1, A^2, \ldots, A^K$. Therefore, it follows that

$$N_{A^1} \leq N_{A^2} \leq N_{A^3} \cdots N_{A^{K-1}} \leq N_{A^K} \leq N_{A^1},$$

which immediately implies that

$$N_{A^1} = N_{A^2} = N_{A^3} \cdots N_{A^{K-1}} = N_{A^K} = N_{A^1}.$$

Therefore, all of $A^1, A^2, \ldots, A^K$ have the same population size. This being the case, total utilitarianism applies, and because of the preference cycle, it follows that

$$TU_{A^1} > TU_{A^2} > TU_{A^3} \cdots TU_{A^{K-1}} > TU_{A^K} > TU_{A^1},$$

which is a contradiction, implying that Theory $\alpha$ is not cyclic and, thus, proving our claim.

Next, consider the following choice function that is inferred from Theory $\alpha$: $A$ is permissible under menu $S$ if and only if there is no population $A^*$ in menu $S$ such that $A^*$ is strictly better than $A$ with regard to Theory $\alpha$. It is easy to see that this choice function coincides with the set of maximal elements under Theory $\alpha$.[10] I will refer to this as the 'Choice-Functional Theory $\alpha$' hereafter. Because it is defined as the set of maximal elements, we can take advantage of the large body of literature on choice theory that has been developed since Arrow (1959). It is well known that if a better-than relation is acyclic, the set of maximal elements inferred from it satisfies several noteworthy properties (Suzumura 1983: Ch. 2):

(i) Non-Emptiness: $C$ is well-defined (i.e. $C(S)$ is non-empty for all menus $S$);
(ii) Property $\alpha$: $C(T) \cap S \subseteq C(S)$ for all menus $S, T$ such that $S \subseteq T$ if $C(T) \cap S$ is nonempty;
(iii) Weak Path-Independence: $C(S \cup T) \subseteq C(C(S) \cup C(T))$ for all menus $S, T$;
(iv) Generalized Condorcet Property:[11] $\{A \in S | AR^*B \text{ for all } B \in S\} \subseteq C(S)$.

[9] In general, a relation $R$ is said to be *acyclic* if and only if there is no chain of alternatives, $A^1, A^2, \ldots, A^K$, such that $A^1PA^2, \ldots, A^{K-1}PA^KPA^1$, where $P$ is the asymmetric relation associated with $R$. Equivalently, acyclicity requires that if $A^kPA^{k+1}$ for all $k \in \{1, 2, \ldots, K-1\}$, then $\neg x^K P x^1$. See Suzumura (1983).

[10] In general, given menu $S$ and a relation $R$, $A$ is a maximal element in the menu if and only if there is no $A^*$ in this menu such that $A^*$ is strictly better than $A$ with regard to $R$.

[11] Generalized Condorcet Property is not logically implied by Path Independence. These properties are independent of one another. Generalized Condorcet Property requires that if, for all $B$ in menu $S$, $A$ is permissible in the choice between $A$ and $B$, then $A$ is permissible under $S$.

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According to Property $\alpha$, if $A$ is permissible under $T$ that includes all options in $S$, then $A$ is permissible under $S$. Property $\alpha$ is logically equivalent to Weak Path-Independence (Ferejohn and Grether 1977: 24).

The choice axiom Weak $\gamma$, which was introduced by Stewart in an earlier paper (Stewart 2020), follows from Generalized Condorcet Property and Property $\alpha$. This choice axiom states that if $A$ is the only permissible population under $S$ and is permissible under $T$, then it is also permissible under the union of the two menus, $S \cup T$. Formally:

$$[\text{Weak } \gamma]: \textit{If } \{A\} = C(S) \text{ and } A \in C(T), \text{ then } A \in C(S \cup T).$$

To see that Weak $\gamma$ follows from Generalized Condorcet Property and Property $\alpha$, assume that Generalized Condorcet Property and Property $\alpha$ are satisfied and that the antecedent of Weak $\gamma$ is true (i.e. $A$ is uniquely permissible in menu $S$ and permissible in menu $T$). That is, we have $\{A\} = C(S)$ and $A \in C(T)$. Then, by Property $\alpha$, $A$ is at least as good as all populations in $S$ with regard to the base relation,[12] and $A$ is also at least as good as all populations in $T$ with regard to the base relation.[13] This implies that $A$ is at least as good as all populations in $S \cup T$ with regard to the base relation. Again, by Generalized Condorcet Property, $A$ is permissible in the menu $S \cup T$. Hence, Weak $\gamma$ is satisfied.

The proof that Choice-Functional Theory $\alpha$ avoids the Mere Addition Paradox consists of three steps. First, it is shown that Mere Addition is satisfied. Take two populations, $A$ and $B$, that contain only positive utility levels, and for which it is the case that $N_{A \cup B} > N_A$ and that $A \cup B$ is an extension of $A$. Notice that neither (i) nor (ii) in Theory $\alpha$ applies to these populations, and therefore, that $A$ is not preferred to $A \cup B$ according to Theory $\alpha$. This means that $A \cup B$ is permissible (in the choice between $A \cup B$ and $A$) under Choice-Functional Theory $\alpha$.

Second, to show that Non-Anti-Egalitarianism is satisfied, assume that two populations $A$ and $B$ have the same population size, that all individuals in $B$ have the same utility level, and that the sum total utility in $B$ is greater than that in $A$. Since total utilitarianism applies to these populations, $B$ is strictly better than $A$ according to Theory $\alpha$. This implies that $A \notin C(\{A, B\})$ under Choice-Functional Theory $\alpha$, which satisfies Non-Anti-Egalitarianism.

Third (and final), take any large population $A$ with blissful utility levels and another much larger population $B$ such that all individuals are at a positive but drab utility level. It holds that $N_A < N_B$ and $AU_A > AU_B$. Moreover, $B$ is not an extension of $A$ because the individuals in $A$ enjoy blissful utility levels while those in $B$ do not. Theory $\alpha$ suggests that $A$ is strictly better than $B$, and therefore, $B$ is not permissible (i.e. $B \notin C(\{A, B\})$) implying that the Repugnant Conclusion is avoided.

Before proceeding, let me recapitulate the findings of this section. There is at least one choice-functional population theory that can avoid the Mere Addition Paradox.[14] The theory proposed in this section satisfies Property $\alpha$ and Generalized

[12]For all $B \in S$, $\{A, B\} \cap C(S) = \{A\}$, and thus, Property $\alpha$ implies that $A \in C(\{A, B\})$.

[13]For all $B \in T$, $A \in \{A, B\} \cap C(T)$, and thus, Property $\alpha$ implies that $A \in C(\{A, B\})$.

[14]One may wonder if Theory $\alpha$ can avoid the Sadistic Conclusion proposed by Arrhenius (2000). The answer is that it avoids the Strong (or Very) Sadistic Conclusion, but entails the Sadistic Conclusion. However, if the average utility level is replaced with the worst-off in Theory $\alpha$, the Sadistic Conclusion is avoided too. See Footnote 7 and Appendix B. This implies that not only the Mere Addition Paradox but also

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Condorcet Property, which means that Weak $\gamma$ is satisfied. Moreover, a further implication is that the theory is rationalizable. That is, there is a relation under which the set of maximal elements coincides with the actual choice for each menu (Suzumura 1983: 36). I showed this point by providing an example of such a relation in this section. While I mentioned only one possibility, there may be many other rationalizable theories that can avoid the paradox, and some of them might be considered plausible.[15] Our example is based on a single binary relation, but it is worth noting that the general choice-functional setting would allow a wide range of non-binary theories, or theories with menu-dependent preferences. In the next section, I will examine how the possibility pointed out in this section (i.e. Theory $\alpha$) is related to menu-dependent relations.

## 3. The Possibility of Menu Dependent Relations

After obtaining 'the persistent paradox', Stewart (2021) discusses the possibility of relaxing Path Independence by means of an extension of the menu-dependent relations proposed by Frick (2022).[16] In relation to this idea, Stewart mentions his choice axiom Weak $\gamma$. Recall that this choice axiom states that if $A$ is the only permissible population under $S$ and is permissible under $T$, then it is also permissible under the union of the two menus, $S \cup T$. (A formal definition was given in the previous section.) This is indeed a natural way of relaxing Path Independence.

Although Stewart (2021) only mentions Weak $\gamma$, the menu-dependent-relation approach has been previously established in the revealed preference theory. Indeed, Tyson (2008) and Cato (2014) provide a set of results on rationalizability of menu-dependent relations. Consider a menu-dependent relation, $R_S$, on menu $S$. A *system* $\langle R_S \rangle$ of menu-dependent relations is a collection of binary relations such that for each menu $S$, $R_S$ is a transitive and complete relation over $S$.[17] As pointed out correctly by Stewart (2021: sec. 5), representing menu-dependent relations becomes trivial if there is no restriction.[18] He also suggests that 'at the very least, there is a constructive challenge here to complete a seriously incomplete case', and asks: 'What principles relate the various menu-relative relations such that rational choice theory is not entirely trivialized?' (Stewart 2021: sec. 5).

In response to this 'constructive challenge', the present section considers the following restriction: a system $\langle R_S \rangle$ is said to be *nested* if and only if for all menus $S, T$ such that $S \subseteq T$,

Arrhenius's impossibility result can be overcome in acyclic population ethics. This observation strengthens the relevance of the series of theories proposed in the present study.

[15]See two alternative theories in Footnote 7.

[16]Stewart (2021) mentions an alternative interpretation of Frick's argument in terms of *option individuation* of choice alternatives, which is indeed another approach to introduce context-dependency. However, about this interpretation Stewart (2021) writes that '[t]here are … a couple of reasons to think that this second interpretation may not be what Frick intends in the end'. I may agree with Stewart's assessment, and therefore, do not examine this interpretation here.

[17]The concept of a system of menu-dependent relations was introduced by Tyson (2008) to examine the satisficing criterion.

[18]Stewart (2021: sec. 5) writes: 'If we allow menu-relative relations to rationalize choice … we can rationalize *any* pattern of choices with some set of transitive menu-relative relations.'

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$$(AR_TB \text{ and } A, B \in S) \Rightarrow AR_SB.$$

According to nestedness, if $S$ is a subset of $T$, then $A$ is at least as good as $B$ in menu $S$ whenever $A$ is at least as good as $B$ in menu $T$ and both $A$ and $B$ are elements of $S$.[19] Nestedness is a well-behaving restriction because a choice function satisfies Property $\alpha$ if and only if there exists a nested system of menu-dependent transitive relations that rationalizes the choice function (Cato 2014: Theorem 3).

Three points are worth discussing here. First, Frick's (2022) proposal can be meaningfully understood in my theory, which suggests that representing menu-dependent relations is worthwhile. Stewart (2021: sec. 5) writes:

> Certain egalitarians, for example, may find considerations of equality compelling grounds to regard $A^+$ as impermissible in the presence of $E$ in the three-option menu $\{A, A^+, E\}$, yet think that such considerations are insufficient to exclude $A^+$ from the choice set for the menu $\{A, A^+\}$. In the latter menu, invoking egalitarian considerations to exclude $A^+$ has the drastic effect of reducing the number of lives worth living ... we have $C(\{A, A^+, E\}) = \{A\}$ since $E$ effectively blocks $A^+$ even though $E$ is not itself permissible. Johann Frick advocates precisely this choice pattern in his paper (2022).

It is easy to confirm that $C(\{A, A^+, E\}) = \{A\}$ and $C(\{A, A^+\}) = \{A, A^+\}$ under Choice-Functional Theory $\alpha$, which was proposed in the previous section, and therefore, the proposed theory is compatible with what Johann Frick advocates. One may wonder, however, *why* the acyclic population theory can capture Frick's attempt to consider menu-dependent relations. The answer is that any acyclic population theory is compatible with Property $\alpha$, which is (necessary and) sufficient for generating a nested system of menu-dependent transitive relations over the populations. Examining a choice function with Property $\alpha$ is not a trivial issue, and therefore, Frick's proposal is meaningful in the context of choice theory, regardless of whether it is based on sufficient strength of rationality.

Second and related to the previous point, it seems important to examine population ethics with Property $\alpha$ because a system of transitive menu-dependent relations is associated with the choice function, as proved by Cato (2014: Theorem 4). As alluded to above, this is considered a plausible trial for the 'constructive challenge' suggested by Stewart (2021: sec. 5). Of course, a potential problem is the acceptability of nestedness. However, even if nestedness is considered objectionable, this would open up the possibility of exploring a plausible restriction on the system of menu-dependent relations.[20] For such explorations, Theory $\alpha$ is a natural starting point because it satisfies various well-known properties of the choice theory, as proved in the previous section.

[19] This is referred to as nestedness* by Cato (2014). A similar, but not identical condition was proposed by Tyson (2008). In Tyson's definition, a system $\langle R_S \rangle$ of menu-dependent relations is said to be *nested* if and only if for all menus $S, T$ such that $S \subseteq T$, $AR_SB \Rightarrow AR_TB$. That is, if $S$ is a subset of $T$, then $A$ is at least as good as $B$ in menu $T$ whenever $A$ is at least as good as $B$ in menu $S$.

[20] Another choice axiom provides a different type of restriction over menu-dependent relations (Tyson 2008).

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Third, any path-independent choice function is rationalized by a system of menu-dependent relations because Path Independence implies Property $\alpha$. Nevertheless, this observation does not mean that a path-independent choice function is necessarily rationalized by menu-dependent relations. Indeed, Moulin (1985) established the equivalence between path-independence and pseudo-rationalizability. A choice function $C$ is said to be pseudo-rationalizable by $\langle R_i \rangle_{i \in I}$ if, for all menus $A$,

$$C(A) = \bigcup_{i \in I} \{x \in A | xR_i y \text{ for all } y \in A\}.$$

Here, $R_i$, which is a relation on $X$, applies across all menus. Moulin's result states that any path-independent choice function can be pseudo-rationalizable by a set of transitive and complete relations (Moulin 1985: Theorem 5 and Lemma 6). This suggests that Property $\alpha$ is also equivalent to a similar type of rationalizability somehow. That is, even Property $\alpha$ might admit rationalization by means of a set of menu-independent relations with certain properties; see Duggan (2019) who proposes proto-rationalizability, which is equivalent to Property $\alpha$.[21] Indeed, related to this, Stewart (2020: Theorem 4) shows that the conjunction of Property $\alpha$ and Weak $\gamma$ is equivalent to what is called weak pseudo-rationalizability, which is rationalizability by a set of acyclic relations.

## 4. Concluding Remarks

In closing I want to offer some remarks on the role of acyclicity or quasi-transitivity of betterness relations. As shown above, switching from quasi-transitivity to acyclicity gives a possibility result. A missing piece here is Suzumura consistency, which has been substantially developed by Bossert and Suzumura (2010).[22]

I also think that Stewart's (2021) choice-functional approach is quite significant for the development of population ethics, especially when taking account of the recent call for new directions for population ethics beyond the standardized Repugnant Conclusions by Zuber *et al.* (2021). The current deadlock in population ethics is related to the well-trodden approach based on better-than binary relations. Stewart's observations come down to the properties of the base relation for the same reason. That is, his three conditions are formulated over binary menus. There can be various ethical issues that cannot be captured by binary choices, however, and to explore such issues, it seems that the choice-functional framework is a plausible approach for theorists of population ethics.

[21]As another possibility, one might consider the existence of *external norms*. If there are such factors, menu-independent preferences can rationalize path-independence choice functions or other choice patterns. This type of argument is related to the argument of Sen (1993). See also Bossert and Suzumura (2009), who introduce the concept of norm-conditional rationalizability.

[22]See Suzumura (1976, 1983) for early works on this property. See also Bossert *et al.* (2005). Suzumura consistency is a property of coherence requiring that any preference cycle can only include indifferences. It logically implies acyclicity, while it is independent of quasi-transitivity. Despite its importance in social decision-making problems, Suzumura consistency has not yet been fully explored. I believe that examining this property in the context of population ethics can be a fruitful direction for future research.

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## Appendix A. Auxiliary Results

**Proposition A.1.** Path Independence implies Weak $\alpha$.

**Proof.** Path Independence implies Property $\alpha$. It is obvious that Weak $\alpha$ follows from Property $\alpha$. Thus, the claim holds. ∎

**Proposition A.2.** Path Independence implies Property $\delta$.

**Proof.** By way of contradiction, assume that $A, B \in C(S) \subseteq S \subseteq T$, whereas $\{A\} = C(T)$. By Path Independence, $C(T) = C(C(S) \cup C(T))$. Therefore, $\{A\} = C(C(S) \cup \{A\})$. Because $A \in C(S)$, it follows that $\{A\} = C(C(S))$. From Path Independence, $C(S) = C(C(S))$ by considering the case in which $S = T$ in its definition. We obtain $\{A\} = C(S)$, which is a contradiction because $B \in C(S)$. The proposition is thus proved. ∎

**Proposition A.3.** If Property $\delta$ and Weak $\alpha$ are satisfied, then the base relation is quasi-transitive.

**Proof.** To the best of my knowledge, this has not been explicitly proved, and thus, I provide the proof here. By way of contradiction, assume that there exist $A', B', D'$ such that $A'P^*B'$, $B'P^*D'$, and $D'R^*A'$. If $D' \in C(\{A', B', D'\})$, Weak $\alpha$ implies that $D' \in C(\{B', D'\})$, which contradicts $B'P^*D'$. If $B' \in C(\{A', B', D'\})$, Weak $\alpha$ again implies $B' \in C(\{A', B'\})$, which contradicts $A'P^*B'$. Thus, $\{A'\} = C(\{A', B', D'\})$. By Weak $\alpha$, $A' \in C(\{A', D'\})$, and thus, $\{A', D'\} = C(\{A', D'\})$. This contradicts Property $\delta$. This proof follows the same strategy as that provided by Sen (1986: 1099), who uses a similar but different definition of Weak $\alpha$. ∎

## Appendix B. Sadistic Conclusions

Considering that Arrhenius (2000) proved that if the Repugnant Conclusion is avoided, it is likely that the Sadistic Conclusion holds, an obvious question is whether Theory $\alpha$ (and/or the variants suggested in footnote 7) can avoid the Sadistic Conclusion. This appendix aims to answer that question.

The Sadistic Conclusion is an impossibility result that is considered just as important as the Mere Addition Paradox in population ethics. If one accepts the view that a population with positive utilities must always be better than a population with negative utilities, then this conclusion is undesirable. Following Arrhenius (2000), I define two variants of the Sadistic Conclusion:

**Very Sadistic Conclusion.** There are populations $A$ and $B$ such that $A$ contains only positive utility levels, $B$ contains only negative utility levels, and $A \notin C(\{A, B\})$.

**Sadistic Conclusion.** There are three populations $A$, $B$ and $G$ such that $A$ contains only positive utility levels, $B$ contains only negative utility levels, and $A \cup G \notin C(\{A \cup G, B \cup G\})$, where $G$ may be an empty population.

Although both variants have similar undesirable implications, avoiding the second variant tends to be more demanding than avoiding the first.

It is easy to see that Choice-Functional Theory $\alpha$ (or Theory $\alpha$) can avoid the Very Sadistic Conclusion. If it would not, then there would be $A$ and $B$ such that $A$ contains only positive utility levels while $B$ contains only negative utility levels and $BP_\alpha A$. This would imply that $TU_B > TU_A$ or $AU_B > AU_A$ by the definition of Theory $\alpha$, but this is impossible because both $TU_A$ and $AU_A$ are positive, while both $TU_B$ and $AU_B$ are negative. Consequently, Choice-Functional Theory $\alpha$ (or Theory $\alpha$) avoids the Very Sadistic Conclusion. On the other hand, Choice-Functional Theory $\alpha$ entails the Sadistic Conclusion. To see this, consider the following three populations:

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$$G = (97), A = (1, 1, 1), B = (-1)$$

Notice that both $G$ and $B$ have only one individual, and that $AU_{G\cup A} = 100/4 = 25$ and $AU_{G\cup B} = 96/2 = 48$. Furthermore, $2 = N_{G\cup B} < N_{G\cup A} = 4$, and $G \cup A$ is not an extension of $G \cup B$. Consequently, by the definition of Theory $\alpha$, $G \cup B$ is strictly better than $G \cup A$, and therefore, Choice-Functional Theory $\alpha$ implies that $A \cup G \notin C(\{A \cup G, B \cup G\})$, which means that the Sadistic Conclusion holds.

However, a variant of Theory $\alpha$ proposed in footnote 7 can avoid the Sadistic Conclusion as well. In this variant, the average utility level is replaced by the worst-off as follows:

$$AP_\alpha B \iff$$
$$((N_A = N_B) \wedge (TU_A > TU_B)) \vee ((N_A < N_B) \wedge (B \not\supseteq A) \wedge (\min U_A > \min U_B)),$$

where $\min U_A$ (resp. $\min U_B$) represents the worst-off in $A$ (resp. B). According to this theory, population $A$ is strictly better than population $B$ if and only if either of the following two holds: (i) the population size of $A$ is the same as that of $B$, and the total sum of utility in $A$ is greater than that in $B$, or (ii′) the population size of $A$ is smaller than that of $B$, $B$ is not an extension of $A$, and the worst-off in $A$ is better than that in $B$.

To show that this principle can avoid the Very Sadistic Conclusion as well as the Sadistic Conclusion, I will focus on the Sadistic Conclusion because the argument for the Very Sadistic Conclusion is similar. Take three populations $G$, $A$ and $B$ such that $A$ contains only positive utility levels while $B$ contains only negative utility levels. Then, it is the case that $TU_{G\cup A} > TU_{G\cup B}$ and $\min U_{G\cup A} \geq \min U_{G\cup B}$. Note that the Sadistic Conclusion holds only when $TU_{G\cup B} > TU_{G\cup A}$ or $\min U_{G\cup B} > \min U_{G\cup A}$. Hence, the Sadistic Conclusion is avoided.

As in the case of the original Theory $\alpha$, this worst-off-based variant is an acyclic population principle that can avoid the Mere Addition Paradox. Consequently, the corresponding choice function satisfies Non-Anti-Egalitarianism, Mere Addition, Generalized Condorcet Property, and Property $\alpha$, and avoids the Repugnant Conclusion. Moreover the above argument shows that the Sadistic Conclusion can also be avoided. This suggests that in addition to the Mere Addition Paradox, the paradox associated with the Sadistic Conclusion can also be overcome if transitivity is relaxed to acyclicity or a rationalizable choice function.

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