

A theory of sets, by Anthony P. Morse. Academic Press, New York, 1965. xxxii + 130 pages. \$7.95.

Every so often there arises in the history of mathematics a lone thinker who shuns the public vehicle of accepted terminology and forges a private language to express his original viewpoint. Professor Morse is one of these. Alas for the poor graduate student to whom among others this book is addressed! It could have been worse; the book might have been printed in Morse code. Actually it does contain some beautiful English words such as "binarian" and "betwixt". As for the mathematical formalism, the author believes that formulas should not be read but deciphered, see page 15.

Usually formulas can be read easily because of notational redundancy. For example, in " $p \rightarrow (q \rightarrow r)$ " the second parenthesis is redundant. Some logicians prefer to reduce the redundancy and write " $p \rightarrow . q \rightarrow r$ " instead. However, " $p \rightarrow q \rightarrow r$ " becomes ambiguous, unless a special convention is made, thus forcing the reader to learn this convention. Our author does not wish to let a conceivable formula such as this go to waste. However, in place of what one might expect, he makes it stand for " $(p \rightarrow q) \wedge (p \rightarrow r)$ ".

The author is no ontological beatnik; he believes in Ockham's razor. He believes that there are only sets (meaning classes) and that propositions are special kinds of sets. One might have achieved this cheaply by saying that " $A \subset B$ " is short for " $\{x | A \subset B\}$ ", provided " $x$ " does not occur freely in " $A$ " and " $B$ ". It would then be easy to show that all mathematical concepts, including some nonsensical concepts, can be expressed as "terms", by stipulating that variables are terms and that " $\{x | A \subset B\}$ " is a term if " $A$ " and " $B$ " are. (See W.V. Quine, Logic based on inclusion and abstraction, J. Symbolic Logic, 2 (1937), pages 145-152.)

However, such conceptual economy is not achieved here. Many formulas are intentionally ambiguous and can be read either as statements about sets or as statements about propositions. It is when we are forced to do both at the same time that the going gets rough.

This reader got only until page 42, where the singleton  $\{x\}$  is defined as the intersection of all  $y$  such that  $y \rightarrow (x \in y)$ . Elsewhere we are told that  $y \leftrightarrow (0 \in y)$ , where  $0$  is the empty set. It would seem to follow that  $\{x\} = 0$ , surely not an intended consequence.

Beyond this stumbling block there come the axioms of set theory, which are asserted in the preface to be a little stronger than Kelley's, see J. L. Kelley, General Topology, Van Nostrand, 1955, Appendix. Then follow ordered pairs, relations, functions, ordinals, induction, Hausdorff's maximal principle, well-ordering, and natural numbers. More surprising are a fixpoint theorem, the Schroeder Bernstein theorem, and cardinal arithmetic, all expressed in Morse notation. There is a foreword by T. J. McMinn to help the less expert reader "see better both the forest and the trees".

Undoubtedly a willing and perseverant reader can learn a lot from this book, but the reviewer would fail in his duty if he were to recommend it for general bedtime reading.

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Proceedings of Symposia in Pure Mathematics. Vol. IX:  
Algebraic groups and discontinuous subgroups, edited by Armand Borel and George D. Mostow. (Proceedings of the Symposium in Pure Mathematics of the American Mathematical Society held at the University of Colorado, Boulder, Colorado, July 5 - August 5, 1965). American Mathematical Society, Providence, R.I., 1966. vii + 426 pages. \$10.20.

Extracts from the Foreword: "The present volume consists of the Institute lecture notes, in part slightly revised, and of a few papers written somewhat later. From the beginning, it was understood that a comprehensive exposition of the arithmetic aspects of algebraic groups should be the central aim of the Institute . . . the papers in this book are intended to serve various purposes: to supply background material, to present the current status of the topic, to describe some basic methods, to give an exposition of more or less known material for which there is no convenient reference, and to present new results."

This book gives an exciting picture of a subject in action and bears striking witness to the fruitful interaction between the theory of algebraic groups and the disciplines of algebra, arithmetic and geometry. It will be indispensable for all who are interested in the modern developments. Perhaps one third of the articles are expository or semi-expository (and usually fairly condensed). Many unsolved problems are posed in the valuable reports on special topics. The five main subject headings, and the contributors to each are as follows.

I. Algebraic Groups, Arithmetic Groups. (A. Borel, T.A. Springer, J. Tits, F. Bruhat, N. Iwahori, T. Tamagawa, B. Kostant, H. Matsumoto, N.D. Allan). II. Arithmetic properties of algebraic groups, Adèle Groups. (T. Tamagawa, T. Ono, J.G.M. Mars, R.P. Langlands, T.A. Springer, M. Kneser, P. Cartier). III. Automorphic functions and decompositions of  $L^2(G/\Gamma)$ . (A. Borel, R. Godement, R.P. Langlands, I. Satake, Y. Ihara). IV. Bounded symmetric domains, Automorphic Forms, Moduli. (W.L. Baily Jr., J. Igusa, G. Shimura, M. Kuga, D. Mumford, I. Satake, W.F. Hammond, P. Cartier). V. Quotients of symmetric spaces. Deformations. (S. Murakami, H. Garland, G.D. Mostow).

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