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Who is this book for? The use of integration and partial derivatives in places means that in the UK you would need to be an enthusiastic sixth form maths student or a science or maths undergraduate to enjoy it all. There are things such as differentiation from first principles that UK students will have met earlier, but they may enjoy rediscovering the magic now that they have a deeper knowledge of calculus. And students in other education systems will have different states of knowledge. The author himself locates the background needed as "what a science or engineering major learns in the first year of undergraduate calculus and physics".

So in summary, I would recommend this book for a school library or as a gift to a student, or as background reading for teachers and lecturers. It will doubtless greatly enrich their knowledge and appreciation of maths and its creators.

10.1017/mag.2023.40 © The Authors, 2023 Published by Cambridge University Press on behalf of The Mathematical Association FRANCIS HUNT
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**Trigonometric delights by** Eli Maor, pp. 236, £17.95 (paper), ISBN 978-0-691-20219-8, Princeton University Press (2020).

Like algebra and number theory, trigonometry is a central pillar of mathematics that can be traced all the way back to the ancient Babylonians. Any keen mathematician would struggle to get far without a firm understanding of trigonometry, and while it is a key component of school curricula across the world it has gained a reputation for being a dry and cumbersome subject, bogged down by fiddly calculations and repetitive questions. Eli Maor's book aims to address this issue, cutting through the needless formalism and monotonous exercises to try to endear trigonometry to the school and undergraduate students that are his intended audience. A number of topics have been selected for aesthetic reasons or for their applications to other sciences, and they are all examined through a historical lens in an attempt to make the journey more engaging for readers and thus to allow them to share in the author's "love affair" with the subject.

The first nine chapters are intended to require nothing more than basic algebra and trigonometry, while the final six rely on some familiarity with calculus. Alongside the historical sidebars, most of which give brief biographies of influential figures such as François Viète or Abraham De Moivre, there should be plenty of material to interest readers of all levels. Indeed, there are numerous fascinating historical and mathematical gems, such as a demonstration of the ingenious method used by the Egyptians to calculate difficult multiplications in the Rhind Papyrus, or the story of the perilous 18th century expeditions to triangulate Lapland and Peru in order to determine the shape of the earth. A particular highlight for me comes from the sidebar on Johann Müller, alias Regiomontanus. A problem he posed in 1471 concerns finding the maximum apparent size (or "visual angle") of a perpendicularly suspended rod, and can be reinterpreted as finding the ideal distance to stand from a building in order to see into the first floor window. Anyone with some experience of calculus might be tempted to dive straight into a standard optimisation method, but the algebraic and geometric methods that would have been his only recourse at the time are remarkably elegant and significantly more rewarding. Discovering and solving this problem, as well as subsequently presenting it to pupils of my own, was precisely the sort of delightful experience I anticipated on starting this book, and there were many more scattered throughout its fifteen chapters.

Unfortunately, not every section enthralled me as much as those just mentioned, and I imagine that most readers would experience similar peaks and troughs in their interest. For instance, someone hoping to find some engaging applications of trigonometry might



be nonplussed by chapter three, which gives a rather dry account of the etymologies of the six standard trigonometric functions, while anyone trusting the author's assertion that the first nine chapters only require basic algebra and trigonometry would be surprised by chapter four, which discusses topics such as De Moivre's theorem, partial differential equations and Fourier series with little help offered to the uninitiated. I'm sure that even the most obscure historical content will be of great interest to some, and both complex numbers and Fourier analysis are revisited more thoroughly later in the book in a manner that could be followed by a keen reader with a decent grasp of calculus. However, the first quarter of the book in particular is heavy on historical details and light on mathematical ones, meaning that it will present a barrier to entry for many readers. School students will find it especially tough going at times, despite being half the intended audience, and even the "basic trigonometry" often requires a surprising level of fluency with compound angle and sum-to-product formulae.

Despite these flaws, persistence will certainly be rewarded. Highlights such as an exploration of Lissajous figures, the intriguing story of the so-called Witch of Agnesi and some rather ingenious geometric constructions of infinite series are all waiting in the latter half of the book, while a particularly satisfying description of how to teach the trigonometric functions and laws using projections doesn't appear until the appendix. The lack of handholding has its advantages as well, as those with the ability to keep up will be impressed by the elegance and efficiency of much of the mathematics. The chapter on epicycloids and hypocycloids springs to mind, as the curves are defined, parametrised and fully explored over the course of seven pages, and similar levels of mathematical dexterity are on display in numerous other sections. The story of trigonometry is also made more engaging by the inclusion of historical excerpts and diagrams which clearly show how the subject has evolved alongside mathematics, and any reader will leave the book with a renewed appreciation for trigonometry's lasting importance.

Overall, the book was rewarding and enjoyable to read, but I had to work my way through the first few chapters before I really started to feel engaged. I would suggest that the book is not particularly well suited to the author's intended audience of school and undergraduate students, and that it would be better appreciated by an experienced mathematician with a keen interest in history. A motivated teacher would then be able to pick and choose highlights to share with pupils, supplemented with some extra scaffolding and stripped of some of the unnecessary historical detail. In the right hands, the book could definitely help to dispel the notion that trigonometry is a dry and dull branch of mathematics. In my opinion, however, the book would struggle to do so on its own.

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The doctrine of triangles by Glen Van Brummelen, pp. 376, £25 (hard), ISBN 978-0-69117-941-4, Princeton University Press (2021)

The word 'Trigonometrie' first appeared in the English language in 1614, which was the year of the first edition of the book *Trigonometrie: or The Doctrine of Triangles*, translated by Raphe Handson. The third edition (1642) has the following title page:

Trigonometrie: or the doctrine of triangles. First written in Latin, by Bartholomew Pitiscus of Grunberg in Silesia, and now translated into English by Raphe Handson. Whereunto is added (for the mariners use)